Franz-Keldysh effect in strong-field QED

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Based on [HT, PRD 99, 056006 (2019)]
[X.-G. Huang, HT, 1904.08200 (to appear in PRD)]

@LPHYS2019 2019/7/9

Brief summary

What

Spontaneous particle production from the vacuum (Schwinger mech.) by a strong slow E-field superimposed by a weak fast E-field

How

Derive an analytical formula for the production based on the perturbation theory in the Furry picture

Result

A QED analog of Franz-Keldysh effect occurs

- ✓ a generalization of the dynamically assisted Schwinger mechanism
- ✓ non-trivial oscillating pattern in the production

INTRODUCTION

⇓

THEORY

⇓

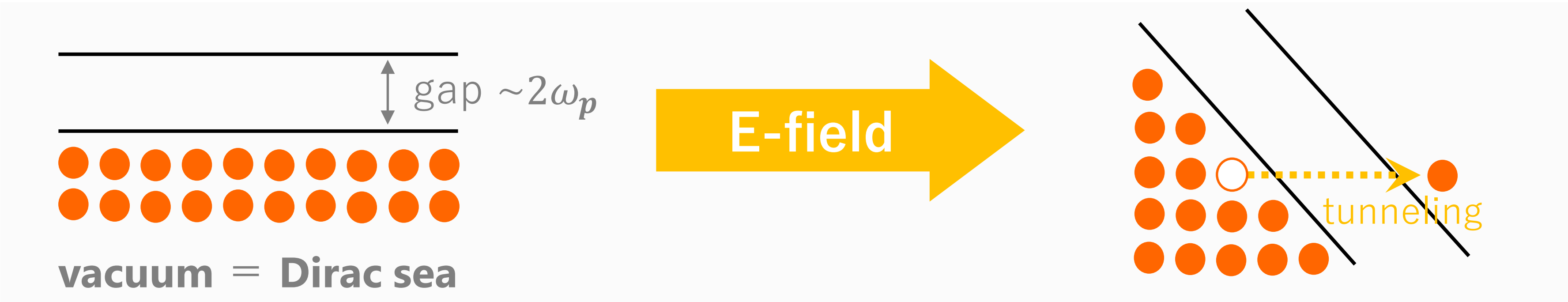
RESULTS

⇓

SUMMARY

Schwinger mechanism

Particles are spontaneously produced from the vacuum in the presence of a strong slow electric field



✓ **QED analog of electrical breakdown** of a semi-conductor in cond-mat (Landau-Zener transition)

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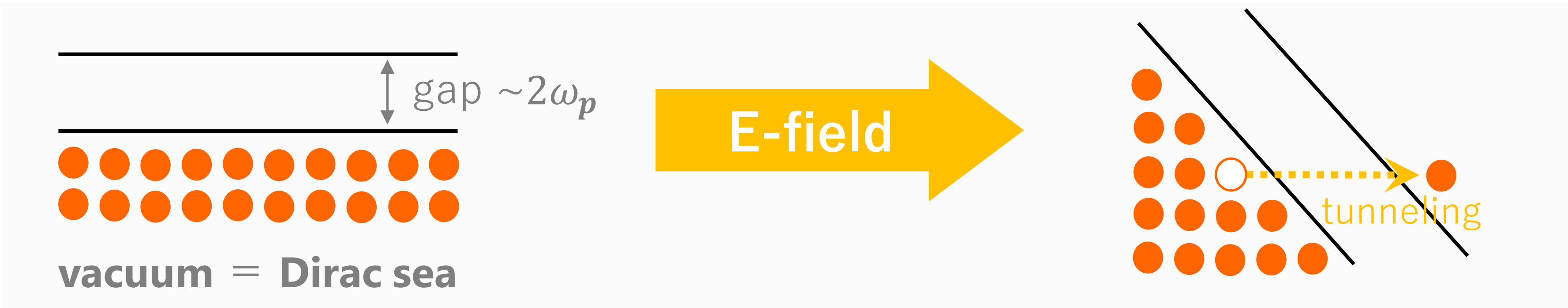

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[Schwinger 1951]

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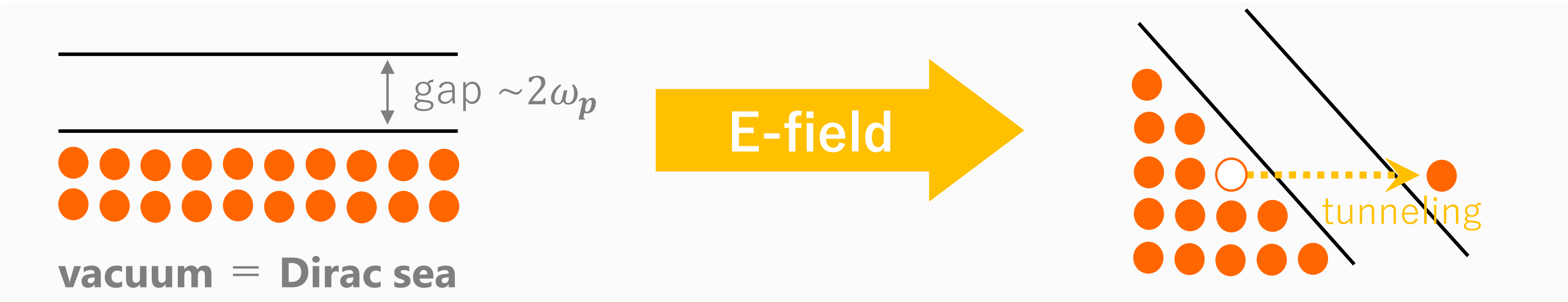

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- **attainable E-field is too weak** compared electron's mass $eE \ll m_e^2$

cf) Guinness world record = HERCULES laser = $eE \sim \sqrt{10^{22}\mathrm{W/cm^2}} \ll \sqrt{10^{28}\mathrm{W/cm^2}} \sim m_e^2$

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Q1: Can we enhance the production rate ?

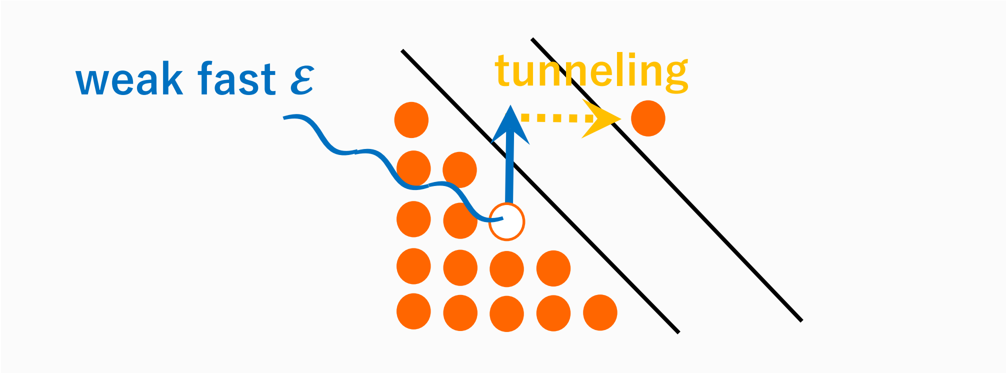
Q2: What does happen if E becomes time-dep. ?

Enhancement by weak fast E-field

[Schutzhold, Gies, Dunne 2008]

Dynamically assisted Schwinger mechanism

= Schwinger mechanism is dynamically enhanced by superimposing
a weak fast time-depending $\mathcal{E}$-field on top of **a strong slow $\bar{E}$-field**



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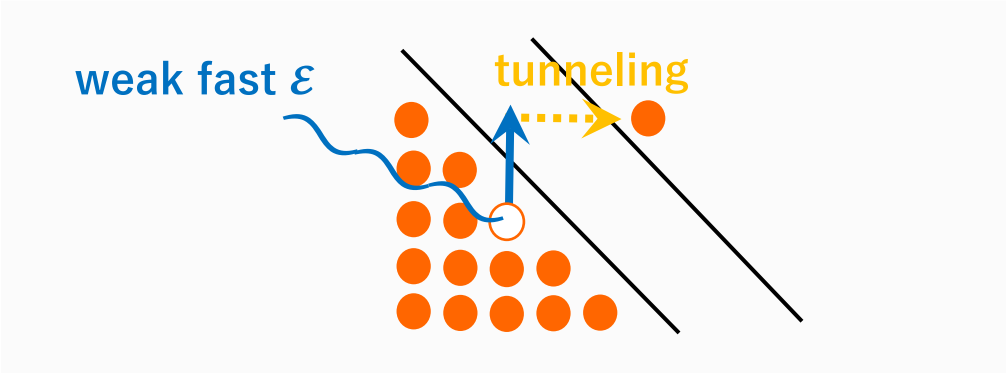

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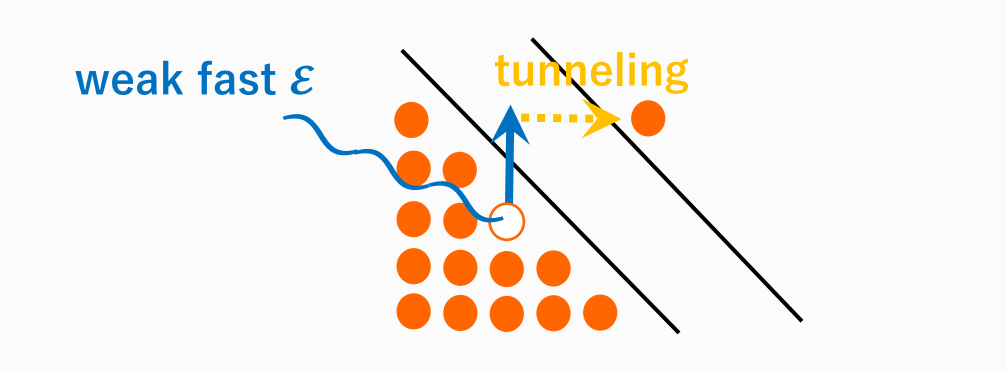

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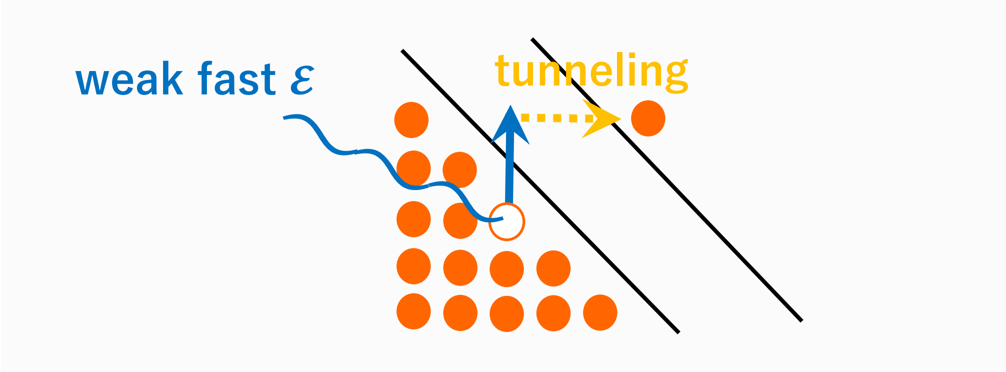

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Deepening our understanding of dyn. ass. Schwinger mech. is important

✔ **for the up-coming laser experiments (e.g. ELI, HiPER)**

✔ **for going beyond the Schwinger pair production for a static E-field**

Idea: Franz-Keldysh effect in cond-mat

A very similar situation has been studied both theoretically and experimentally more than 60 years in cond-mat !!!

- ✓ Apply **a strong slow $\bar{E}$-field** and **a weak fast time-depending $\mathcal{E}$-field (or a dynamical photon)** onto a semi-conductor
- ✓ Measure photo-absorption rate (instead of e-h pair production number)

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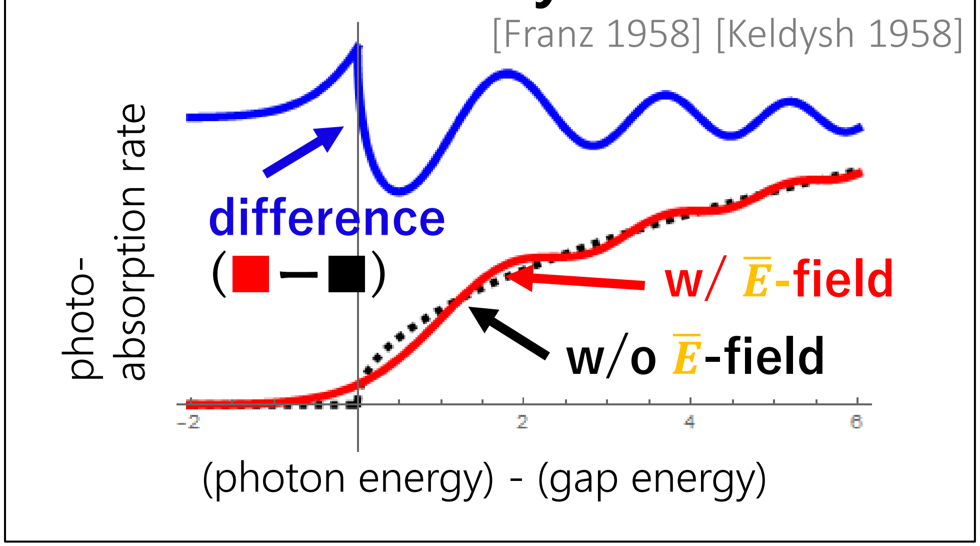
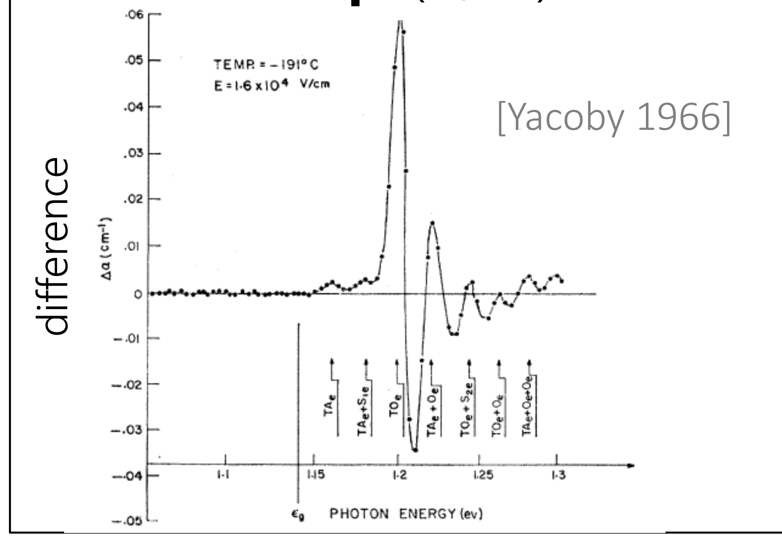


Exp. (w/ Si)



- ✓ abs. rate is **enhanced below the threshold** (looks like dyn. ass. Schwinger mech. (?))
- ✓ not only enhancement, but also **oscillation above threshold** (FK oscillation)

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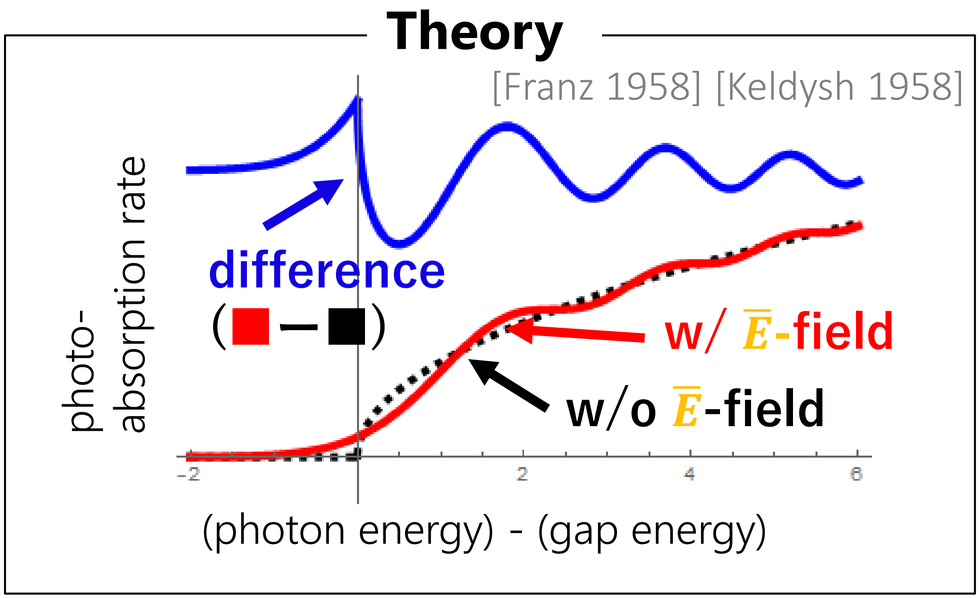

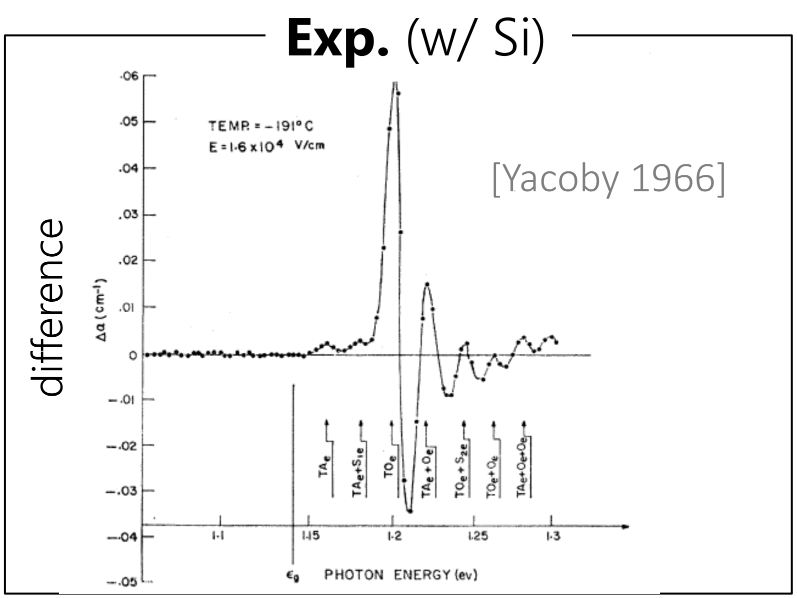

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Q: Is this really similar? Does FK effect equally occur for QED?

Aim of this study

Analytically show if FK effect occurs in QED

- ✓ clarify if dyn. ass. Schwinger mech. can be understood as a part of FK effect
- ✓ clarify if FK oscillation above the threshold $\omega > 2m$ occurs in QED
- ✓ clarify how FK effect modifies the momentum spectrum

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OK

valid only when
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valid as long as
$\mathcal{E}$ is weak $e\mathcal{E} \ll m^2$, $e\bar{E}$

[Furry 1951]
[Fradkin *et al*. 1991]
[Torgrimsson *et al*. 2017]

Semi-classical methods (e.g. worldline, WKB)

Perturbation theory in Furry picture

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Perturbation theory in Furry picture (1/2)

Compute $\mathrm{d}^3N/\mathrm{d}\boldsymbol{p}^3 = \langle a^\dagger_{\boldsymbol{p},s} a_{\boldsymbol{p},s} \rangle$ non-perturbatively w.r.t. strong slow $\bar{\boldsymbol{E}}$, but perturbatively w.r.t. weak fast $\boldsymbol{\mathcal{E}}$

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STEP 1 **Separate the total E-field $\boldsymbol{E}$ into strong $\bar{\boldsymbol{E}}$ and weak $\boldsymbol{\mathcal{E}}$**

$$\boldsymbol{E} = \bar{\boldsymbol{E}} + \boldsymbol{\mathcal{E}} \text{ with } \bar{\boldsymbol{E}} \gg \boldsymbol{\mathcal{E}}$$

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STEP 2 **Solve the Dirac eq. non-perturbatively w.r.t $\bar{\boldsymbol{E}}$, but perturbatively w.r.t. $\boldsymbol{\mathcal{E}}$**

$$[i\partial\!\!\!/ - e\bar{A\!\!\!/} - m]\hat{\psi} = e\mathcal{A\!\!\!/}\hat{\psi}$$

$$\Rightarrow \hat{\psi}(x) = \hat{\psi}^{(0)}(x) + \int_{-\infty}^{\infty} \mathrm{d}y^4 S(x,y) e\mathcal{A\!\!\!/}(y)\hat{\psi}^{(0)}(y) + O(|e\mathcal{A}|^2)$$

Here, $\hat{\psi}^{(0)}$ and S are **non-perturbatively dressed by** $\bar{\boldsymbol{E}}$ as

$$[i\partial\!\!\!/ - e\bar{A\!\!\!/} - m]\hat{\psi}^{(0)} = 0$$

$$[i\partial\!\!\!/ - e\bar{A\!\!\!/} - m]S(x,y) = \delta^4(x-y)$$

Perturbation theory in Furry picture (2/2)

STEP 3 Compute in/out annihilation operators $\hat{a}_{p,s}^{\text{in/out}}$, $\hat{b}_{p,s}^{\text{in/out}}$ from $\hat{\psi}$

$$\begin{pmatrix} \hat{a}_{\boldsymbol{p},s}^{\text{in/out}} \\ \hat{b}_{-\boldsymbol{p},s}^{\text{in/out}\dagger} \end{pmatrix} \equiv \lim_{t\to-\infty/+\infty} \int \mathrm{d}^3\boldsymbol{x} \begin{pmatrix} (u_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}})^\dagger \\ (v_{\boldsymbol{p},s}\mathrm{e}^{+i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}})^\dagger \end{pmatrix} \hat{\psi}(x)$$

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➡ $\hat{o}^{\rm in}_{\boldsymbol{p},s}$, $\hat{o}^{\rm out}_{\boldsymbol{p},s}$ **are inequivalent** $\hat{o}^{\rm in}_{\boldsymbol{p},s} \neq \hat{o}^{\rm out}_{\boldsymbol{p},s}$ and related with each other by a Bogoliubov transformation

$$\begin{pmatrix} \hat{a}^{\rm out}_{\boldsymbol{p},s} \\ \hat{b}^{\rm out\dagger}_{-\boldsymbol{p},s} \end{pmatrix} = \sum_{s'} \int \mathrm{d}^3\boldsymbol{p}' \begin{pmatrix} \alpha_{\boldsymbol{p},s;\boldsymbol{p}',s'} & \beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} \\ -\beta^*_{\boldsymbol{p},s;\boldsymbol{p}',s'} & \alpha^*_{\boldsymbol{p},s;\boldsymbol{p}',s'} \end{pmatrix} \begin{pmatrix} \hat{a}^{\rm in}_{\boldsymbol{p}',s'} \\ \hat{b}^{\rm in\dagger}_{-\boldsymbol{p}',s'} \end{pmatrix}$$

where

$$\alpha_{\boldsymbol{p},s;\boldsymbol{p}',s'} = \int \mathrm{d}^3\boldsymbol{x}\, {}_+\psi^{(0)\rm out\dagger}_{\boldsymbol{p},s}\, {}_+\psi^{(0)\rm in}_{\boldsymbol{p}',s'} - i\int d^4x\, {}_+\bar{\psi}^{(0)\rm out}_{\boldsymbol{p},s}\, e\mathcal{A}\, {}_+\psi^{(0)\rm in}_{\boldsymbol{p}',s'} + O(|e\mathcal{A}|^2)$$

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STEP 4 Compute the in-vacuum expectation value of # operator

$$\frac{\mathrm{d}^3N_e}{\mathrm{d}\boldsymbol{p}^3} \equiv \langle \text{vac; in}| a^{\text{out}\dagger}_{\boldsymbol{p},s} a^{\text{out}}_{\boldsymbol{p},s} |\text{vac; in}\rangle = \sum_{s'} \int \mathrm{d}^3\boldsymbol{p}' \left|\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}\right|^2$$

c.f.) evaluation based on WKB (valid for adiabatic case) [Torgrimsson *et al*. 2017]

Formula

Assume $\bar{\boldsymbol{E}}$ is sufficiently slow (i.e., static) and spatially uniform

➡ analytical sol. of Dirac eq. ${}_{\pm}\psi_{\boldsymbol{p},s}^{(0)\mathrm{in/out}}$ is known

➡ one can evaluate $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$ analytically !!! [HT 2019]

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$$\times \left| 1 + \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{e\bar{E}} \int_0^\infty d\omega \frac{\tilde{\mathcal{E}}(\omega)}{\bar{E}} \exp\left[-\frac{i}{4}\frac{\omega^2+4\omega p_\parallel}{e\bar{E}}\right] {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{e\bar{E}}; 2; \frac{i}{2}\frac{\omega^2}{e\bar{E}}\right) \right|^2$$

NOTE: $\bar{E} \parallel \mathcal{E}$ and $\mathcal{E} = \mathcal{E}(\mathrm{t})$ (i.e., spatially uniform) are assumed;
for generalization, see [Huang, HT 2019], [Itakura, HT (in prep)]

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"perturbative assistance" by $\mathcal{E}$

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➡ analytical sol. of Dirac eq. ${}_{\pm}\psi^{(0)\mathrm{in/out}}_{\boldsymbol{p},s}$ is known

➡ one can evaluate $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$ analytically !!! [HT 2019]

$$\frac{d^3N_e}{d\boldsymbol{p}^3} = \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{e\bar{E}}\right]$$
$$\times\left|1+\frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{e\bar{E}}\int_0^\infty d\omega\,\frac{\tilde{\mathcal{E}}(\omega)}{\bar{E}}\exp\left[-\frac{i}{4}\frac{\omega^2+4\omega p_\parallel}{e\bar{E}}\right]{}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{e\bar{E}};2;\frac{i}{2}\frac{\omega^2}{e\bar{E}}\right)\right|^2$$

NOTE: $\bar{E} \parallel \mathcal{E}$ and $\mathcal{E} = \mathcal{E}(t)$ (i.e., spatially uniform) are assumed; for generalization, see [Huang, HT 2019], [Itakura, HT (in prep)]

usual Schwinger formula by static $\bar{E}$

"perturbative assistance" by $\mathcal{E}$

✓ **slow limit $\omega/\sqrt{e\bar{E}} \ll 1$: ■ dominates ➡ usual Schwinger pair production**

$$\frac{d^3N_e}{d\boldsymbol{p}^3} \sim \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{e\bar{E}}\right]\left|1+\frac{\pi}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{e\bar{E}}\frac{\mathcal{E}}{\bar{E}}\right|^2 \sim \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{e(\bar{E}+\mathcal{E})}\right]$$

✓ **fast limit $\omega/\sqrt{e\bar{E}} \gg 1$: ■ dominates ➡ perturbative production**

$$\frac{d^3N_e}{d\boldsymbol{p}^3} \sim \frac{V}{(2\pi)^3}\frac{1}{4}\frac{m^2+\boldsymbol{p}_\perp^2}{\omega_{\boldsymbol{p}}^2}\frac{|e\tilde{\mathcal{E}}(2\omega_{\boldsymbol{p}})|^2}{\omega_{\boldsymbol{p}}^2}$$

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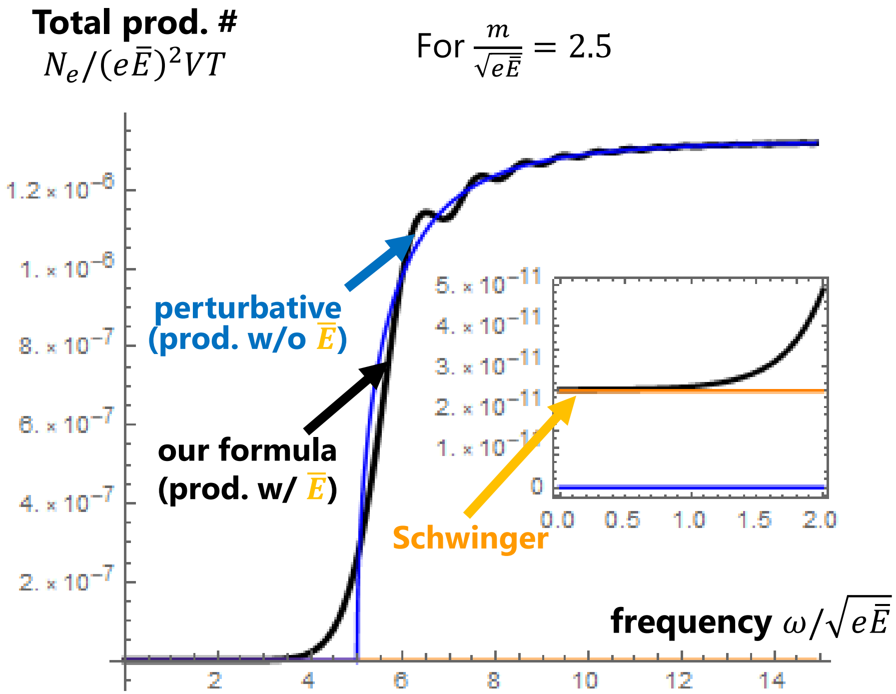
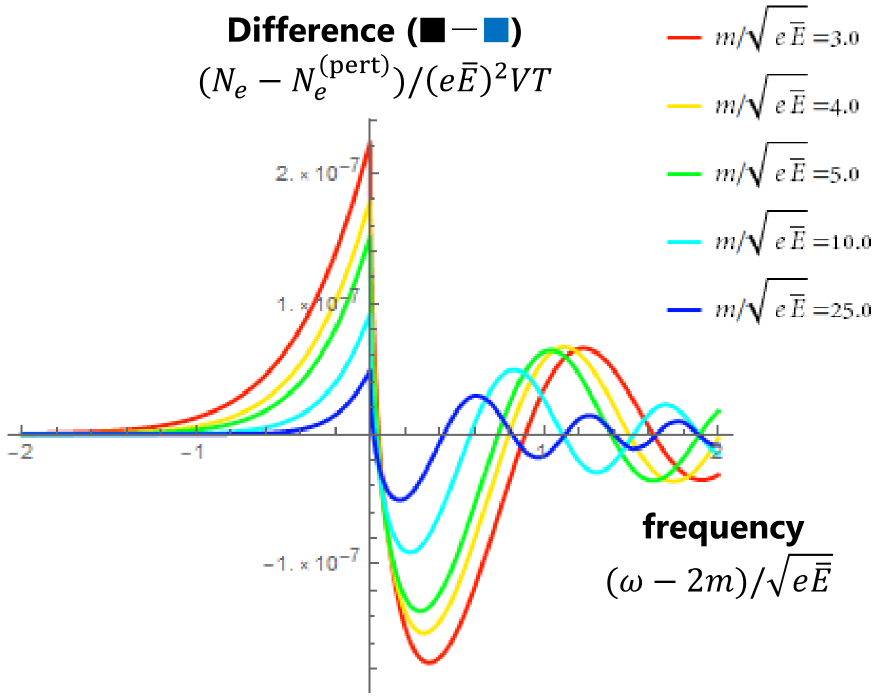
Total production number *N*

✓ For a monochromatic E-field $\mathcal{E} = \mathcal{E}_0 \cos \omega t$





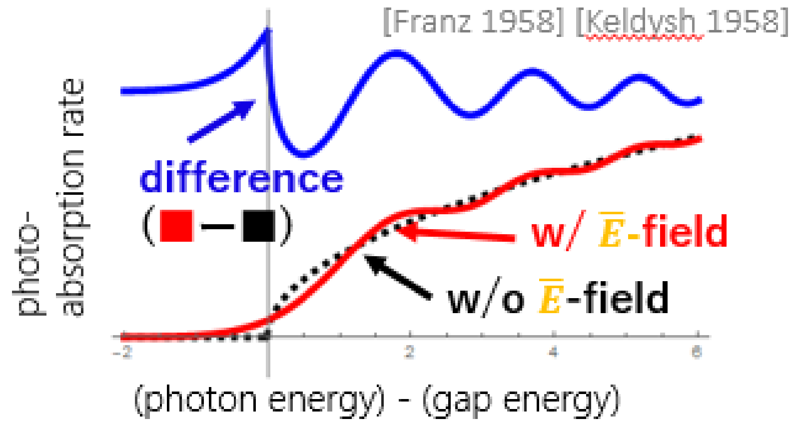
Idea: Franz-Keldysh effect in cond-mat

A very similar situation has been studied both theoretically and experimentally more than 60 years in cond-mat !!!

- ✓ Apply **a strong slow $\bar{E}$-field** and **a weak fast time-depending $\mathcal{E}$-field (or a dynamical photon)** onto a semi-conductor
- ✓ Measure photo-absorption rate (instead of e-h pair production number)

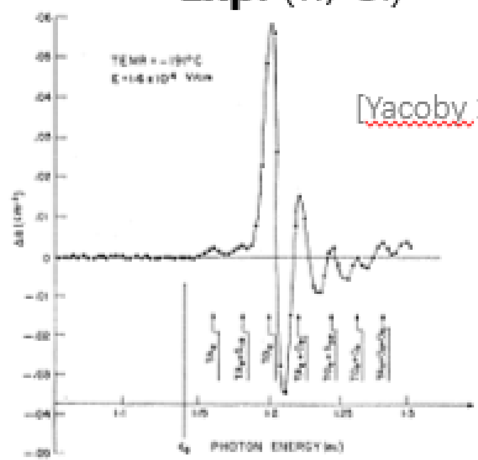
Theory

[Franz 1958] [Keldysh 1958]



Exp. (w/ Si)

[Yacoby 1966]

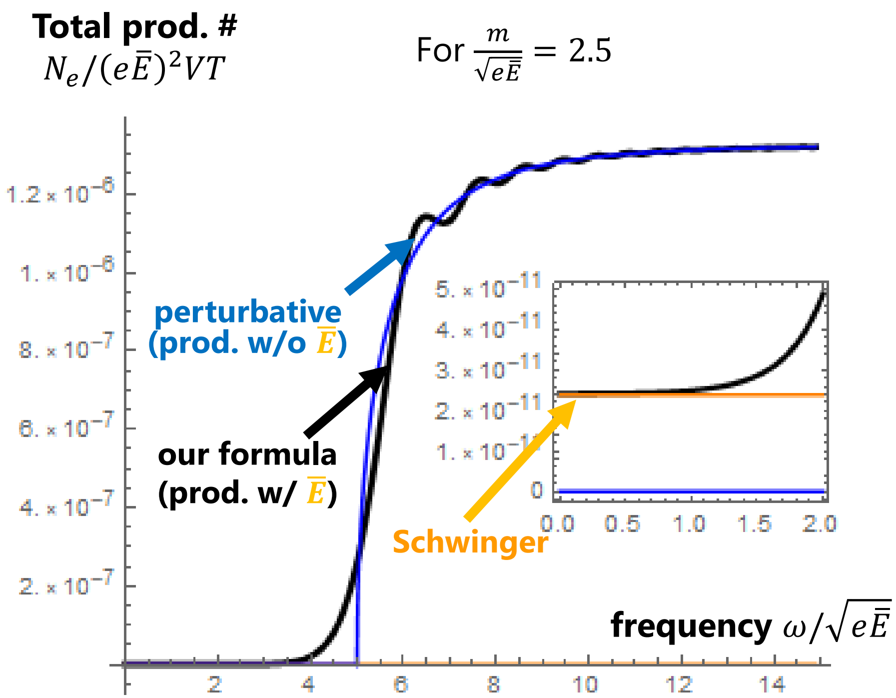


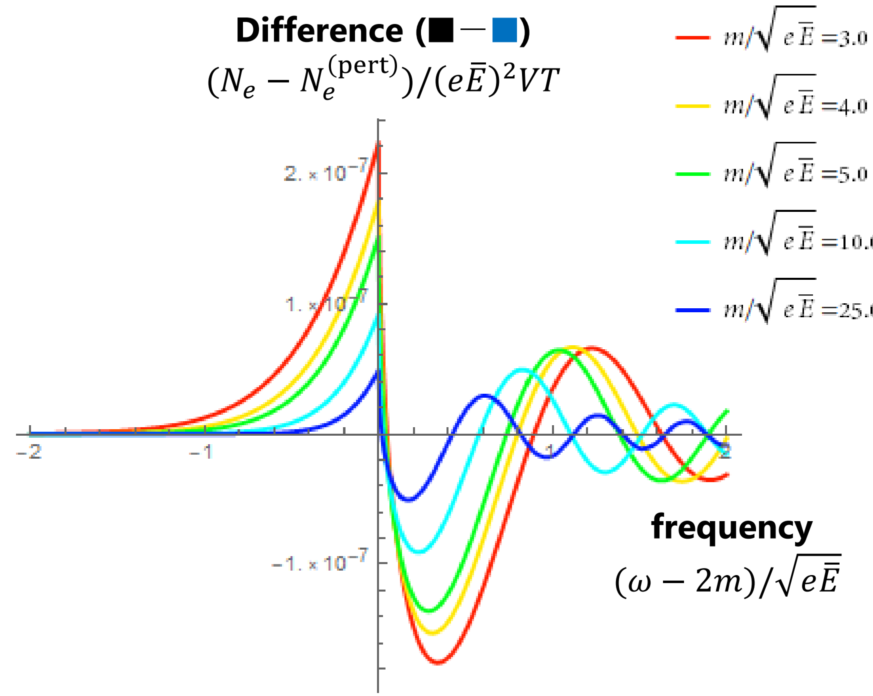
- ✓ abs. rate is **enhanced below the threshold** (looks like dyn. ass. Schwinger mech. (?))
- ✓ not only enhancement, but also **oscillation above threshold** (FK oscillation)

Q: Is this really similar? Does FK effect equally occur for QED?

Total production number *N*

✓ For a monochromatic E-field $\mathcal{E} = \mathcal{E}_0 \cos \omega t$



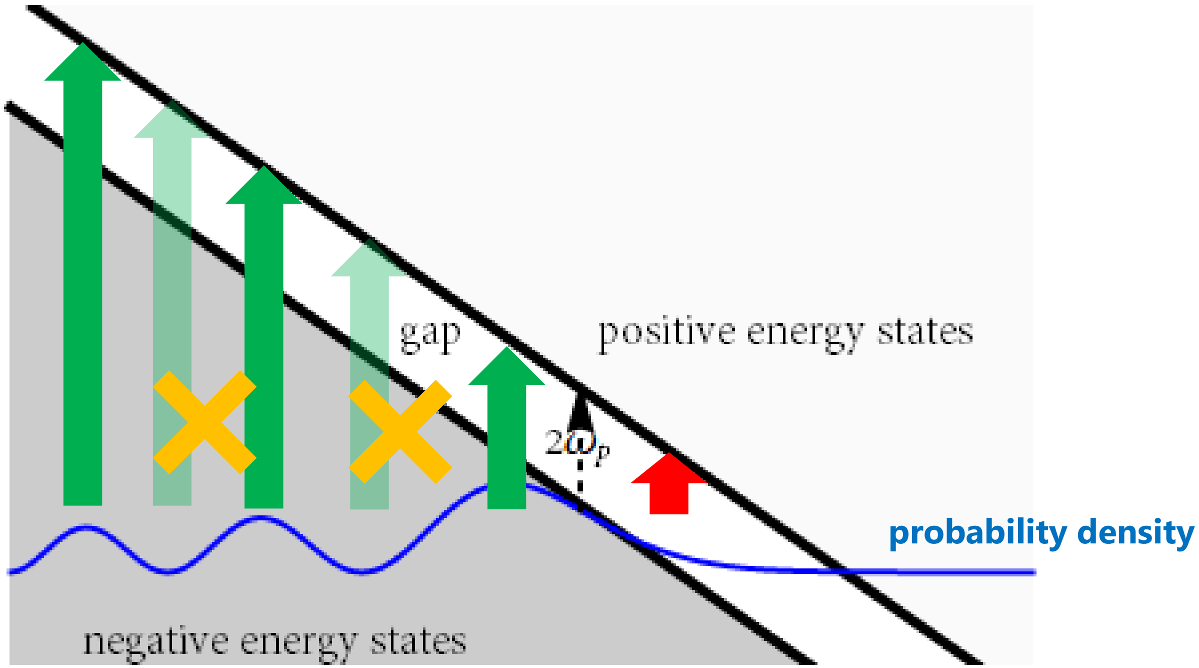


✓ **exactly the same as cond-mat ➡ FK effect does occur in QED!**

- production is **enhanced below the threshold** (dyn. ass. Schwinger mech.)
- not only enhancement, but also **oscillation above threshold** (~ FK oscillation)
- **very sharp peak at the threshold** in the number difference

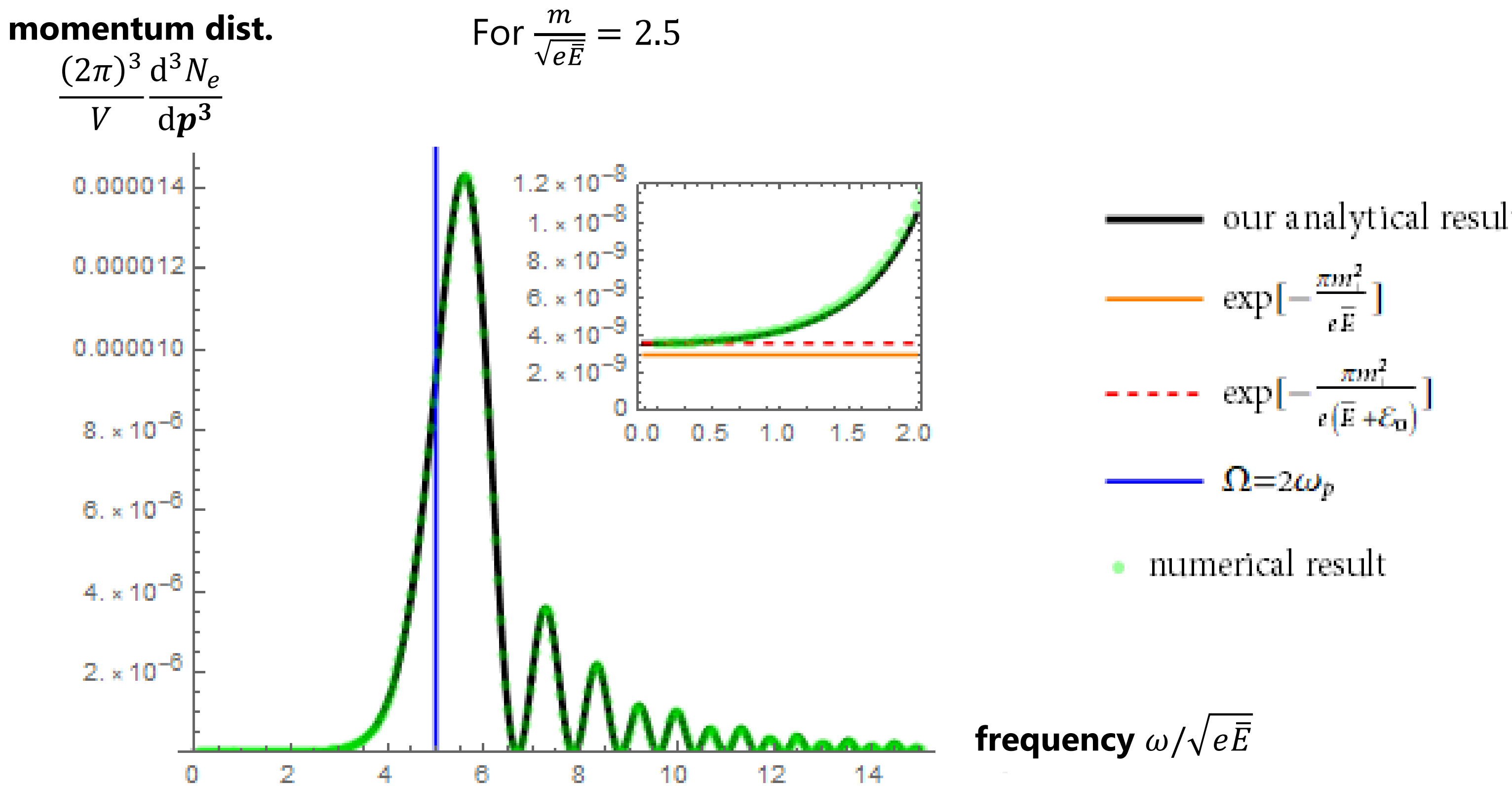
c.f.) modulation spectroscopy

Interpretation of the oscillation



- ✓ **quantum tunneling** ⇒ **dynamically assisted Schwinger mechanism**
- ✓ **quantum reflection** ⇒ **FK oscillation**
  - non-uniform prob. dist. due to interference b/w in-coming and reflected waves
  - production occurs most efficiently at the maxima

Momentum distribution $\mathrm{d}^3 N_e / \mathrm{d}\boldsymbol{p}^3$



- ✓ **FK effect:** enhancement below threshold & oscillation above threshold
- ✓ **the location of the perturbative peak is modified** due to reflection
- ✓ **excellent agreement b/w our analytical formula and numerical results**

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Summary

- ✓ **I discussed spontaneous particle production from the vacuum (Schwinger mechanism) in the presence of a strong slow E-field superimposed by a weak fast E-field**

- ✓ **I derived an analytical formula for the production number based on the perturbation theory in the Furry picture**
  - reproduces the numerical results very well as long as $\bar{E} \gg \mathcal{E}$

- ✓ **I claimed that a QED analog of Franz-Keldysh effect occurs:**
  - enhancement below the threshold (dyn. ass. Schwinger mech.)
  - oscillation above the threshold (FK oscillation)
  - the location of the perturbative peak in $\mathrm{d}^3 N_e / \mathrm{d}\boldsymbol{p}^3$ is modified

- ✓ Not only quantum tunneling, but also reflection is important