# The Schwinger mechanism with perturbative electric fields

**Hidetoshi TAYA**
(Fudan University)

[HT, PRD 99, 056006 (2019)]
[X.-G. Huang, HT, PRD 100, 016013 (2019)]
[X.-G. Huang, M. Matsuo, HT (to appear in PTEP)]

# Overview

| | |
|---|---|
| **Problem** | **Dynamically assisted Schwinger mechanism**<br>⇒ spontaneous particle production from the vacuum by strong slow E-field + weak fast E-field w/ <u>**arbitrary**</u> time-dep. |
| **Technical results** | **Analytical formula for <u>arbitrary</u> time-dep. weak fast E is derived based on pert. theory in Furry picture**<br>⇒ has wider applicability compared to conventional formulas based on, e.g. WKB, worldline instanton method |
| **Physical results** | • **Dynamically assisted Schwinger mech. in high-energy = Franz-Keldysh effect in cond-mat**<br>• **Spin-dependence appears** |

# INTRODUCTION

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SUMMARY

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  e.g.) heavy ion collisions, high-Z atoms (Z>173), intense lasers, early Universe

# Particle production by strong E-field

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- **is expected to occur in various systems under extreme conditions**

  e.g.) heavy ion collisions, high-Z atoms (Z>173), intense lasers, early Universe

- **can be classified into 3 depending on size of freq. Ω: $E = E_0 \cos(\Omega t)$**

  (1) slow E-field (small Ω) — Schwinger mechanism [Sauter (1932)] [Schwinger (1951)] [Heisenberg, Euler (1936)]

  (2) fast E-field (large Ω) — multi-photon pair production [Brezin, Izykson (1970)] [Popov (1971)] [HT, Fujii, Itakura (2014)]

  (3) both (slow + fast E) — dynamically assisted Schwinger mechanism [Dunne, Gies, Schutzhold (2008), (2009)] [Piazza et al (2009)] [Monin, Voloshin (2010)]

# (1) Slow E-field ⇒ Schwinger mechanism

[Sauter (1932)] [Heisenberg, Euler (1936)] [Schwinger (1951)]

E-field tilts the band ⇒ level crossing ⇒ **tunneling (non-pert.)**

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## ☺ well understood

**(1) An analog of "electrical breakdown (Landau-Zener trans.)" in cond-mat** [Landau (1932)] [Zener (1932)] [Majorana (1932)] [Stueckelberg (1932)]

**(2) Analytical formula exists (Gaussian + spin-independent)**

Schwinger formula: $\frac{d^3N_\uparrow}{d\boldsymbol{p}^3} = \frac{d^3N_\downarrow}{d\boldsymbol{p}^3} = \frac{V}{(2\pi)^3}\exp[-\frac{\pi(m^2+\boldsymbol{p}_T^2)}{eE}]$

# (2) Fast E-field ⇒ multi-photon pair production

[Brezin, Izykson (1970)]
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interact as a particle (photon) ⇒ **perturbative scattering**

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## ☺ well understood

**(1) An analog of “photo-electric effect” in cond-mat**

[Einstein (1905)]

**(2) Analytical formula exists (delta func. + spin-independent)**

LO perturbation theory: $\frac{d^3N_\uparrow}{d\boldsymbol{p}^3} = \frac{d^3N_\downarrow}{d\boldsymbol{p}^3} = \# \times \left(\frac{eE}{m^2+\boldsymbol{p}^2}\right)^2 \delta(2\sqrt{m^2+\boldsymbol{p}^2} - \Omega)$

# (3) Slow + Fast E-field ⇒ dynamically assisted Schwinger mechanism

[Dunne, Gies, Schutzhold (2008), (2009)] [Piazza et al (2009)]
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**tunneling (non-pert.)** by slow E **+** **scattering (pert.)** by fast E

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## 😨 NOT understood well

**(1) No analog in cond-mat (?)**

**(2) No general analytical formula**

- **No analytical formula for weak fast E with <u>arbitrary</u> time-dep.**
- Usually formulated w/i semi-classical methods (e.g. WKB, worldline)
  ⇒ **limited applicability:** E must be adiabatic (i.e., valid for tiny $\omega \ll 2m$)
- Less attention has been paid to spin DoG

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## Note: important to phenomenology

**ex.1) heavy ion collisions**

- at *glasma* phase, a lot of (mini-)jets exists ⇒ int. b/w glasma & jets ⇒ **dyn. ass. Sch. mech.**
  (~ strong slow EM field) (~ weak fast EM field)
- event generators (e.g. PYTHIA) assume a complete separation b/w soft (by string breaking ~ Schwinger mech.) & hard (by pert. collisions) contributions
  ⇒ **dynamical assistance is completely neglected**

**ex.2) laser** [Dunne, Gies, Schutzhold (2008), (2009)][Piazza et al (2009)][Monin, Voloshin (2010)]

- available E-field is too weak ⇒ **needs dynamical assistance to observe in exp.**

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**A** Derive a general analytical formula

**B** Claim "Franz-Keldysh effect" is the cond-mat analog

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**Apply strong slow E-field and a photon (~ weak fast E-field) onto a semi-conductor, and measure photo-absorption rate**

$\Rightarrow$ photo-aborp. rate $\sim$ Im[1-loop action] $\sim$ particle prod. rate

$\Rightarrow$ looks very similar to the dynamically assisted Schwinger mech.

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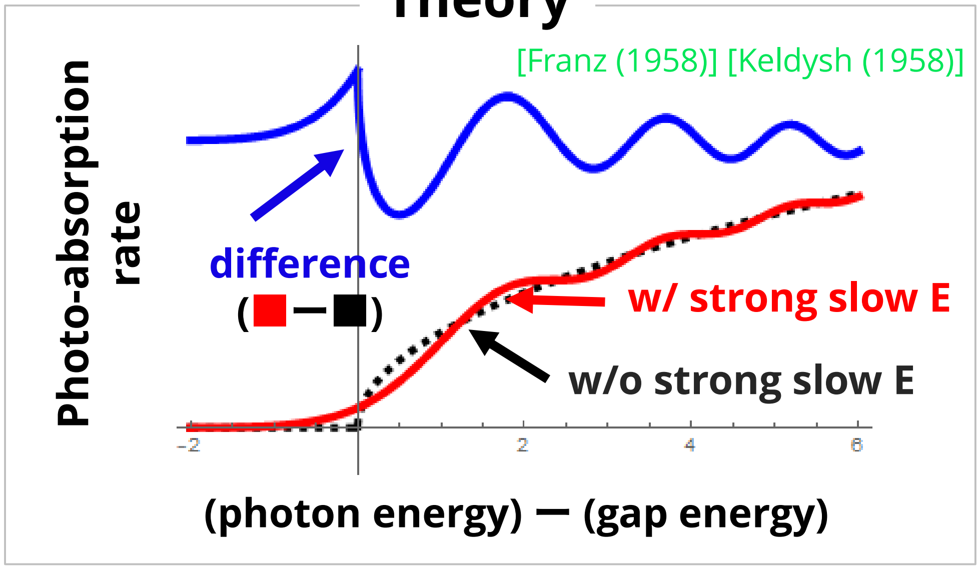


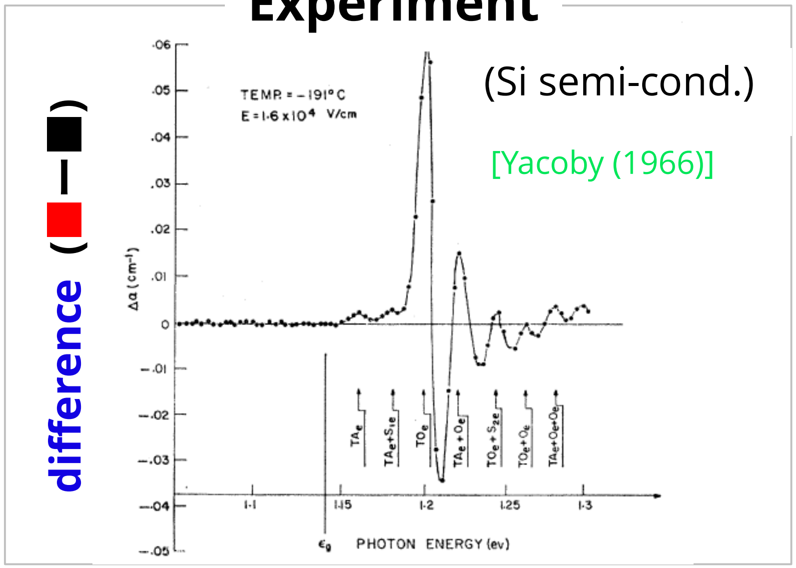


**⇒ (a) enhancement below gap; (b) oscillation above gap**

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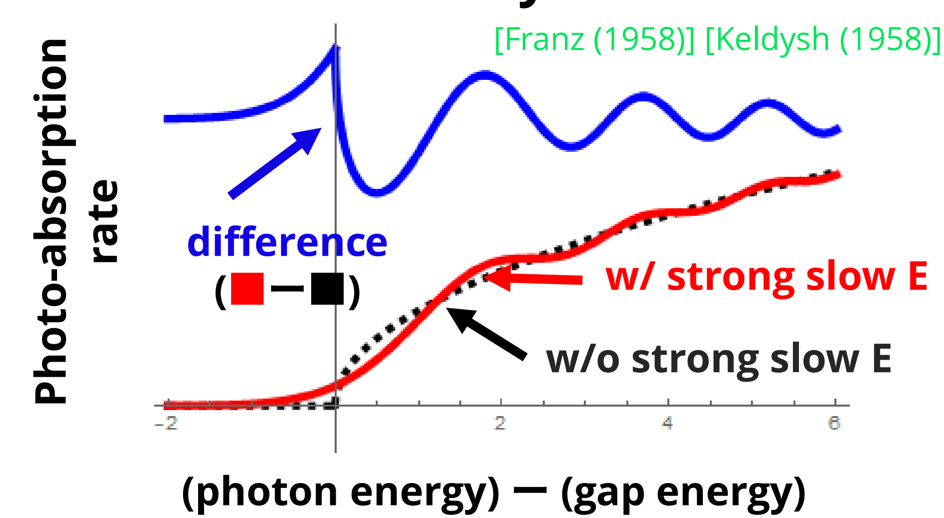
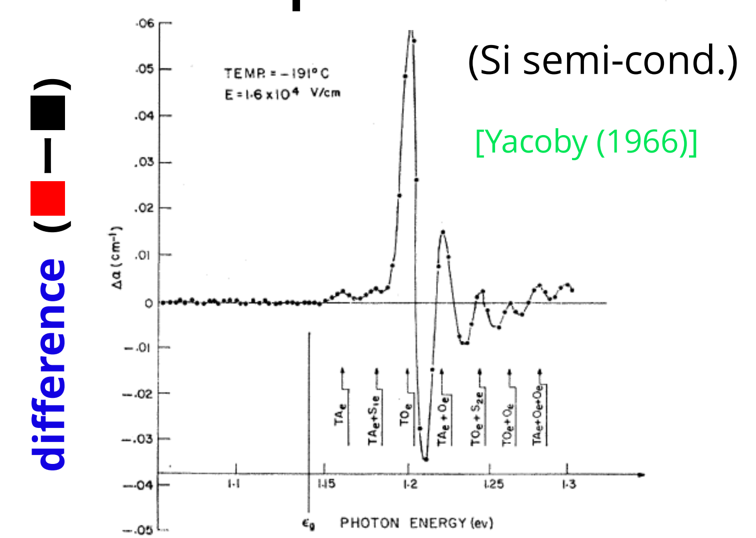
**Theory**




**Experiment**




⇒ **(a) enhancement below gap; (b) oscillation above gap**

**To-do**

**Show Franz-Keldysh effect is the correct analog [B] by**

**(1) Deriving an analytical formula for the production [A]**

**(2) Using that formula to explicitly demonstrate (a), (b) occur in the dynamically assisted Schwinger mech.**

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# Perturbation theory in Furry picture (1/4)

**Goal**

**Evaluate** $\frac{d^3 N_s}{dp^3} = \langle \mathrm{vac} | \hat{a}^\dagger_{\boldsymbol{p},s} \, \hat{a}_{\boldsymbol{p},s} | \mathrm{vac} \rangle$ **in the presence of strong slow** $\boldsymbol{E}_{\mathbf{s}}$ **& weak fast** $\boldsymbol{\mathcal{E}}_{\mathbf{f}}$

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strong slow E is non-perturbative

while **weak fast E is just perturbative**

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## Idea

**Perturbative expansion** $\hat{a}_{\boldsymbol{p},s}$ **i.t.o. weak fast** $\mathcal{E}_{\mathbf{f}}$

[Furry (1951)] [Fradkin, Gitman, Shvartsman (1991)] [Torgrimsson, Schneider, Shutzhold (2017)] [HT (2019)]

# Perturbation theory in Furry picture (2/4)

**STEP 1** **Separate the total E-field $E$ into strong slow $E_s$ and weak fast $\mathcal{E}_f$**

$$E = E_s + \mathcal{E}_f \qquad (E_s \gg \mathcal{E}_f)$$

/

# Perturbation theory in Furry picture (2/4)

**STEP 1** **Separate the total E-field $\boldsymbol{E}$ into strong slow $\boldsymbol{E}_{\mathrm{s}}$ and weak fast $\boldsymbol{\mathcal{E}}_{\mathrm{f}}$**

$$\boldsymbol{E} = \boldsymbol{E}_{\mathrm{s}} + \boldsymbol{\mathcal{E}}_{\mathrm{f}} \qquad (\boldsymbol{E}_{\mathrm{s}} \gg \boldsymbol{\mathcal{E}}_{\mathrm{f}})$$

**STEP 2** **Solve Dirac eq. non-pert. w.r.t. $\boldsymbol{E}_{\mathrm{s}}$, but pert. w.r.t. $\boldsymbol{\mathcal{E}}_{\mathrm{f}}$**

$$[i\partial\!\!\!/ - e A\!\!\!/_{\mathrm{s}} - m]\hat{\psi} = e\mathcal{A}\!\!\!/_{\mathrm{f}}\hat{\psi}$$

$$\Rightarrow \quad \hat{\psi}(x) = \hat{\psi}^{(0)}(x) + \int_{-\infty}^{\infty} \mathrm{d}y^4 S_{\mathrm{R}}(x,y) e\mathcal{A}\!\!\!/_{\mathrm{f}}(y)\hat{\psi}^{(0)}(y) + O(|e\mathcal{A}_{\mathrm{f}}|^2)$$

Here, $\hat{\psi}^{(0)}$ and $S_{\mathrm{R}}$ are **non-perturbatively dressed by** $\boldsymbol{E}_{\mathrm{s}}$ as

$$[i\partial\!\!\!/ - e A\!\!\!/_{\mathrm{s}} - m]\hat{\psi}^{(0)} = 0$$

$$[i\partial\!\!\!/ - e A\!\!\!/_{\mathrm{s}} - m]S_{\mathrm{R}}(x,y) = \delta^4(x-y)$$

# Perturbation theory in Furry picture (3/4)

**STEP 3** **Compute in/out annihilation operators $\hat{a}^{\text{in/out}}_{p,s}$, $\hat{b}^{\text{in/out}}_{p,s}$ from $\hat{\psi}$**

$$\begin{pmatrix} \hat{a}^{\text{in/out}}_{\boldsymbol{p},s} \\ \hat{b}^{\text{in/out}\dagger}_{-\boldsymbol{p},s} \end{pmatrix} \equiv \lim_{t\to-\infty/+\infty} \int \mathrm{d}^3\boldsymbol{x} \begin{pmatrix} (u_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}})^\dagger \\ (v_{\boldsymbol{p},s}\mathrm{e}^{+i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}})^\dagger \end{pmatrix} \hat{\psi}(x)$$

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$\Rightarrow$ $\hat{o}^{\text{in}}_{\boldsymbol{p},s}$, $\hat{o}^{\text{out}}_{\boldsymbol{p},s}$ **are inequivalent** $\hat{o}^{\text{in}}_{\boldsymbol{p},s} \neq \hat{o}^{\text{out}}_{\boldsymbol{p},s}$ and related with each other by a Bogoliubov transformation

$$\begin{pmatrix} \hat{a}^{\text{out}}_{\boldsymbol{p},s} \\ \hat{b}^{\text{out}\dagger}_{-\boldsymbol{p},s} \end{pmatrix} = \sum_{s'} \int \mathrm{d}^3\boldsymbol{p}' \begin{pmatrix} \alpha_{\boldsymbol{p},s;\boldsymbol{p}',s'} & \beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} \\ -\beta^*_{\boldsymbol{p},s;\boldsymbol{p}',s'} & \alpha^*_{\boldsymbol{p},s;\boldsymbol{p}',s'} \end{pmatrix} \begin{pmatrix} \hat{a}^{\text{in}}_{\boldsymbol{p}',s'} \\ \hat{b}^{\text{in}\dagger}_{-\boldsymbol{p}',s'} \end{pmatrix}$$

where, up to 1st order in $e\mathcal{A}_{\text{f}}$,

$$\alpha_{\boldsymbol{p},s;\boldsymbol{p}',s'} = \int \mathrm{d}^3\boldsymbol{x}\; {}_{+}\psi^{(0)\text{out}\dagger}_{\boldsymbol{p},s}\; {}_{+}\psi^{(0)\text{in}}_{\boldsymbol{p}',s'} - i\int d^4x\; {}_{+}\bar{\psi}^{(0)\text{out}}_{\boldsymbol{p},s} e\not{\mathcal{A}}_{\text{f}}\; {}_{+}\psi^{(0)\text{in}}_{\boldsymbol{p}',s'} + O(|e\mathcal{A}_{\text{f}}|^2)$$

$$\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} = \int \mathrm{d}^3\boldsymbol{x}\; {}_{-}\psi^{(0)\text{out}\dagger}_{\boldsymbol{p},s}\; {}_{+}\psi^{(0)\text{in}}_{\boldsymbol{p}',s'} - i\int d^4x\; {}_{-}\bar{\psi}^{(0)\text{out}}_{\boldsymbol{p},s} e\not{\mathcal{A}}_{\text{f}}\; {}_{+}\psi^{(0)\text{in}}_{\boldsymbol{p}',s'} + O(|e\mathcal{A}_{\text{f}}|^2)$$

Here, ${}_{\pm}\psi^{(0)\text{in/out}}_{\boldsymbol{p},s}$ are sol. of Dirac eq. **dressed by** $eA_{\text{s}}$ w/ different B.C.

$$[i\not{\partial} - e\not{A}_{\text{s}} - m]\; {}_{\pm}\psi^{(0)\text{in/out}}_{\boldsymbol{p},s} = 0 \quad \text{w/} \lim_{t\to-\infty/+\infty} \begin{pmatrix} {}_{+}\psi^{(0)\text{in/out}}_{\boldsymbol{p},s} \\ {}_{-}\psi^{(0)\text{in/out}}_{\boldsymbol{p},s} \end{pmatrix} = \begin{pmatrix} u_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}} \\ v_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}} \end{pmatrix}$$

# Perturbation theory in Furry picture (4/4)

**STEP 4** **Evaluate the in-vacuum expectation value of # operator**

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} \equiv \langle \mathrm{vac;in} | a^{\mathrm{out}\dagger}_{\boldsymbol{p},s} a^{\mathrm{out}}_{\boldsymbol{p},s} | \mathrm{vac;in} \rangle = \sum_{s'} \int \mathrm{d}^3\boldsymbol{p}' \left| \beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} \right|^2$$

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Assume $\boldsymbol{E}_{\mathrm{s}}$ is sufficiently slow (i.e., static) and spatially uniform

⇒ analytical sol. of Dirac eq. ${}_{\pm}\psi^{(0)\mathrm{in/out}}_{\boldsymbol{p},s}$ is known

⇒ one can evaluate $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$ **exactly !** [HT, (2019)] [Huang, HT, (2019)]

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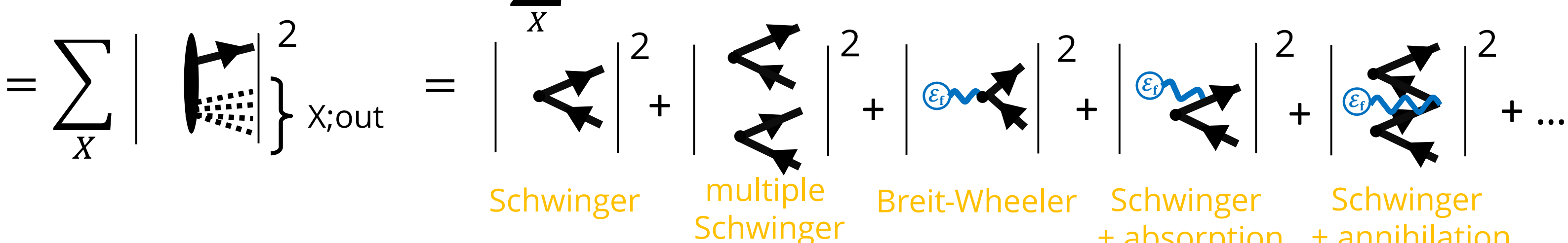
## Remarks:

### (1) Directly computing VEV of # operator

[Baltz, McLerran (2001)]

- **inclusive quantity** that includes all the processes up to 1st order in $\boldsymbol{\mathcal{E}}_{\mathrm{f}}$

$$\langle \mathrm{vac;in}| a^{\mathrm{out}\dagger}_{\boldsymbol{p},s} a^{\mathrm{out}}_{\boldsymbol{p},s} |\mathrm{vac;in}\rangle = \sum_X \left|\langle e_{\boldsymbol{p},s} X;\mathrm{out}|\mathrm{vac;in}\rangle\right|^2$$




### (2) No approximation in evaluating $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$

cf) within 0-th order WKB [Torgrimsson *et al* (2017)]

- important to discuss an analog of Franz-Keldysh effect in Schwinger mechanism

# Formula (1/2): for $E_S \,/\!/\, \mathcal{E}_{\mathbf{f}}$

[HT, (2019)]

$$\frac{d^3 N_e}{d\boldsymbol{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_{\mathrm{s}}}\right]$$

$$\times \left| 1 + \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}} \int_0^\infty d\omega \frac{\tilde{\mathcal{E}}_{\mathbf{f}}(\omega)}{E_{\mathrm{s}}} \exp\left[-\frac{i}{4}\frac{\omega^2+4\omega p_\parallel}{eE_{\mathrm{s}}}\right] {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}; 2; \frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right) \right|^2$$

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**Schwinger mech. by slow $E_s$**

**Dynamical assistance by fast $\mathcal{E}_f$**

- slow limit $\omega/\sqrt{eE_s} \ll 1$: ■ dominates ➡ **usual Schwinger formula**

$$\frac{d^3N_e}{d\boldsymbol{p}^3} \sim \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_s}\right]\left|1+\frac{\pi}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s}\frac{\mathcal{E}_f}{E_s}\right|^2 \sim \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{e(E_s+\mathcal{E}_f)}\right]$$

- fast limit $\omega/\sqrt{eE_s} \gg 1$: ■ dominates ➡ **multi-photon pair prod. (LO)**

$$\frac{d^3N_e}{d\boldsymbol{p}^3} \sim \frac{V}{(2\pi)^3}\frac{1}{4}\frac{m^2+\boldsymbol{p}_\perp^2}{\omega_{\boldsymbol{p}}^2}\frac{|e\tilde{\mathcal{E}}_f(2\omega_{\boldsymbol{p}})|^2}{\omega_{\boldsymbol{p}}^2}$$

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✓ **Derived an analytical formula for the dynamically assisted Schwinger mech. for arbitrary time-dep. $\mathcal{E}_\mathrm{f}$**

# Formula (2/2): for $E_S \nparallel \mathcal{E}_f$

[Huang, HT, (2019)]

$$\frac{d^3N_e}{d\boldsymbol{p}^3} = \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_s}\right]$$

$$\times\left[\left| 1 + \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_s}} \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s}\frac{\widetilde{\boldsymbol{\mathcal{E}}}_f(\omega)\cdot\boldsymbol{E}_s}{E_s^2}\mathrm{e}^{-i\frac{\omega^2}{4eE_s}}{}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};2;\frac{i}{2}\frac{\omega^2}{eE_s}\right)\right.\right.$$

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$$\left.+s\times i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_s}} \frac{(\widetilde{\boldsymbol{\mathcal{E}}}_f(\omega)\times\boldsymbol{p}_\perp)\cdot\boldsymbol{E}_s}{E_s^2\omega}\mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_s}}{}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};1;\frac{i}{2}\frac{\omega^2}{eE_s}\right)\right]\right|^2$$

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- becomes complicated (red = new terms), but the basic structure is the same

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$$+s\times i\int_0^\infty d\omega\, {\rm e}^{-i\frac{\omega p_\parallel}{eE_{\rm s}}}\frac{(\tilde{\boldsymbol{\mathcal{E}}}_{\rm f}(\omega)\times\boldsymbol{p}_\perp)\cdot\boldsymbol{E}_{\rm s}}{E_{\rm s}^2\omega}{\rm Im}\left[{\rm e}^{-i\frac{\omega^2}{4eE_{\rm s}}}{}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}};1;\frac{i}{2}\frac{\omega^2}{eE_{\rm s}}\right)\right]\Bigg|^2$$

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Sch. mech. by slow $E_{\rm s}$

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**Dynamical assistance by fast $\mathcal{E}_f$**

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**Dynamical assistance by fast $\mathcal{E}_{\mathrm{f}}$**

**Sch. mech. by slow $E_{\mathrm{s}}$**

- becomes complicated (red = new terms), but the basic structure is the same
- **spin-dependence appears even without magnetic fields**

  ∵ Dirac particle has a spin-orbit coupling $\boldsymbol{s}\cdot(\boldsymbol{p}\times\boldsymbol{E})$ [Foldy, Wouthuysen (1950)] [Tani (1951)]

  e.g. application to spintronics [Huang, Matsuo, HT (2019)]

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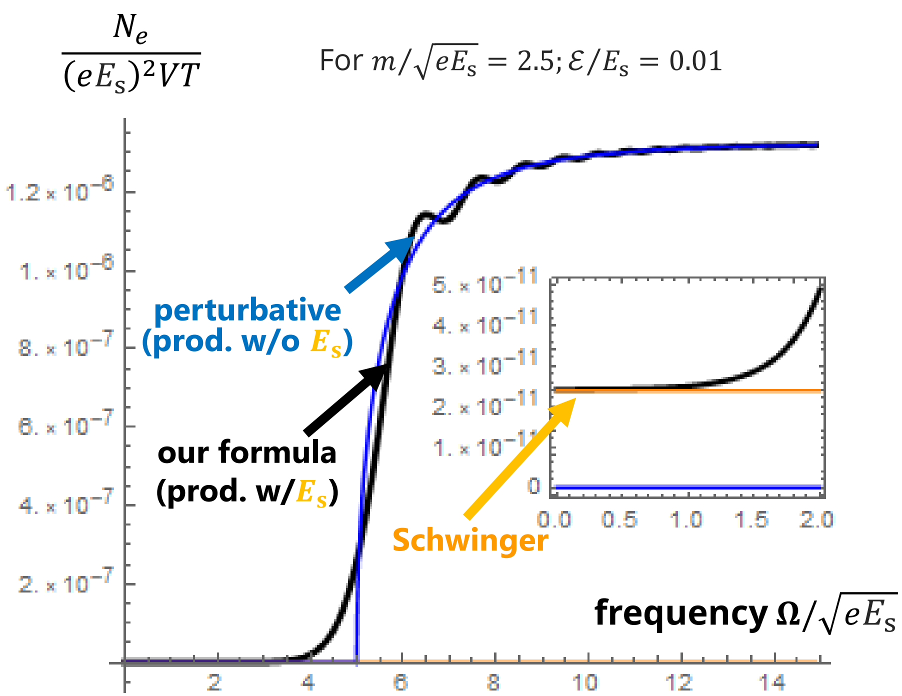
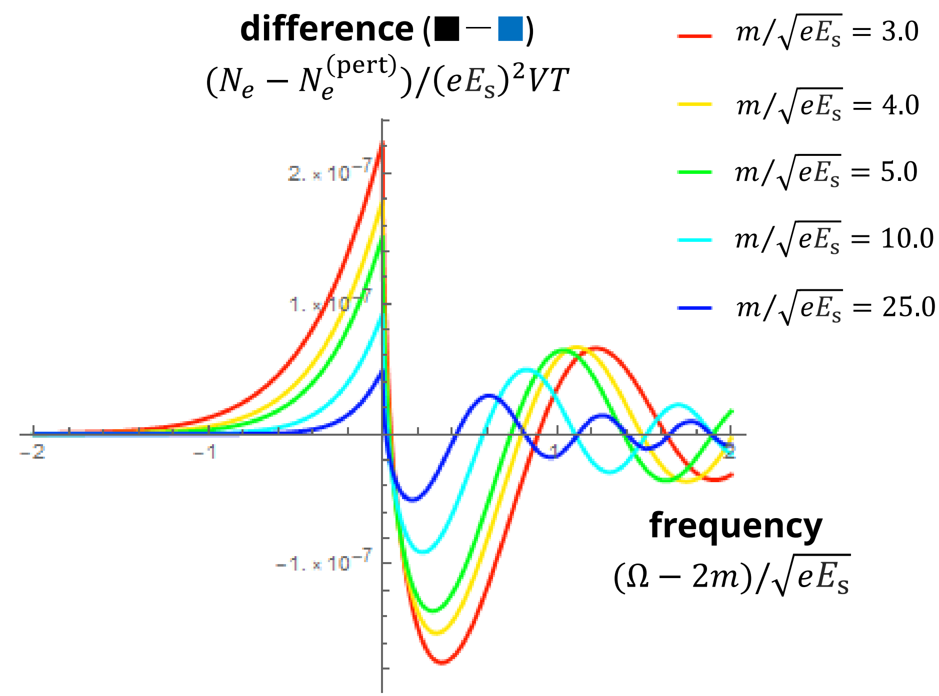
# Results (1/3): Total prod. # $N$

Parallel field configuration: $\boldsymbol{E} = \begin{pmatrix} 0 \\ 0 \\ E_\mathrm{s} + \mathcal{E}\cos\Omega t \end{pmatrix}$



# Results (1/3): Total prod. # N

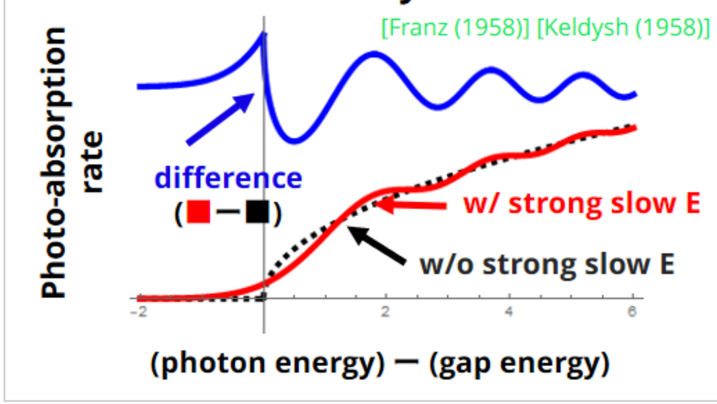
# Franz-Keldysh effect

**Apply strong slow E-field and a photon (~ weak fast E-field) onto a semi-conductor, and measure photo-absorption rate**

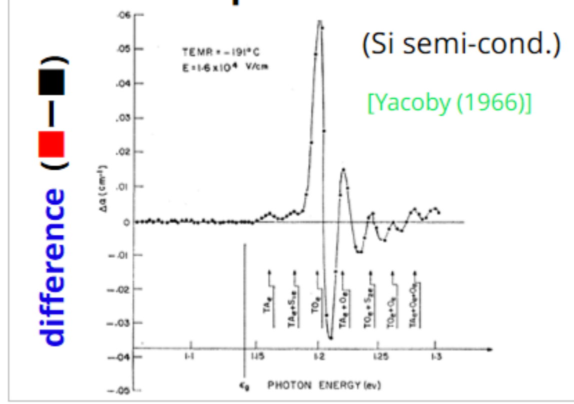
⇒ photo-aborp. rate ∼ Im[1-loop action] ∼ particle prod. rate

⇒ looks very similar to the dynamically assisted Schwinger mech.

**Theory**




**Experiment**




**⇒ (a) enhancement below gap; (b) oscillation above gap**

**To-do**

**Show Franz-Keldysh effect is the correct analog [B] by**
**(1) Deriving an analytical formula for the production [A]**
**(2) Using that formula to explicitly demonstrate (a), (b) occur in the dynamically assisted Schwinger mech.**

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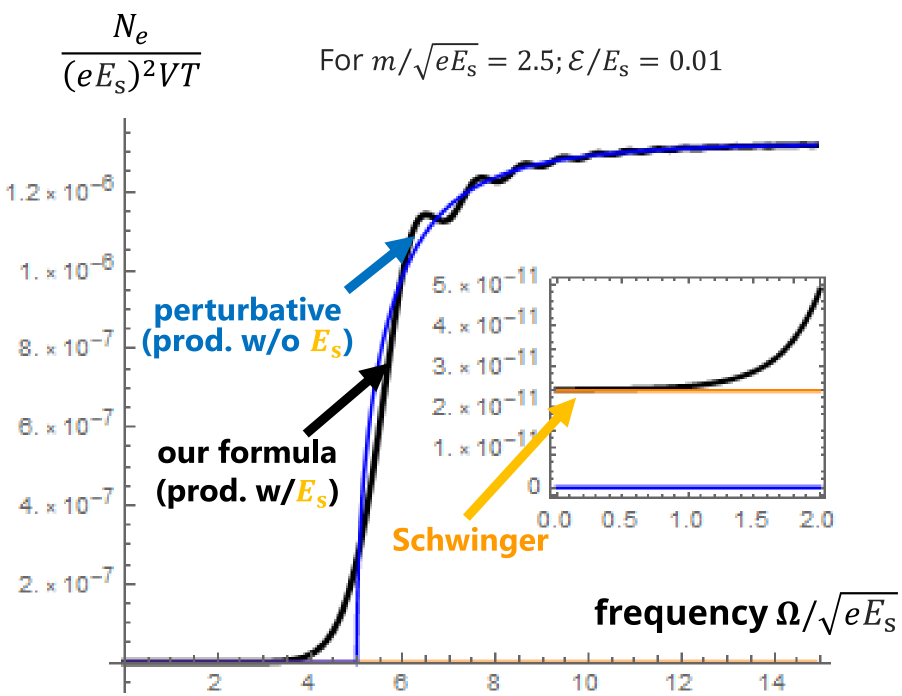


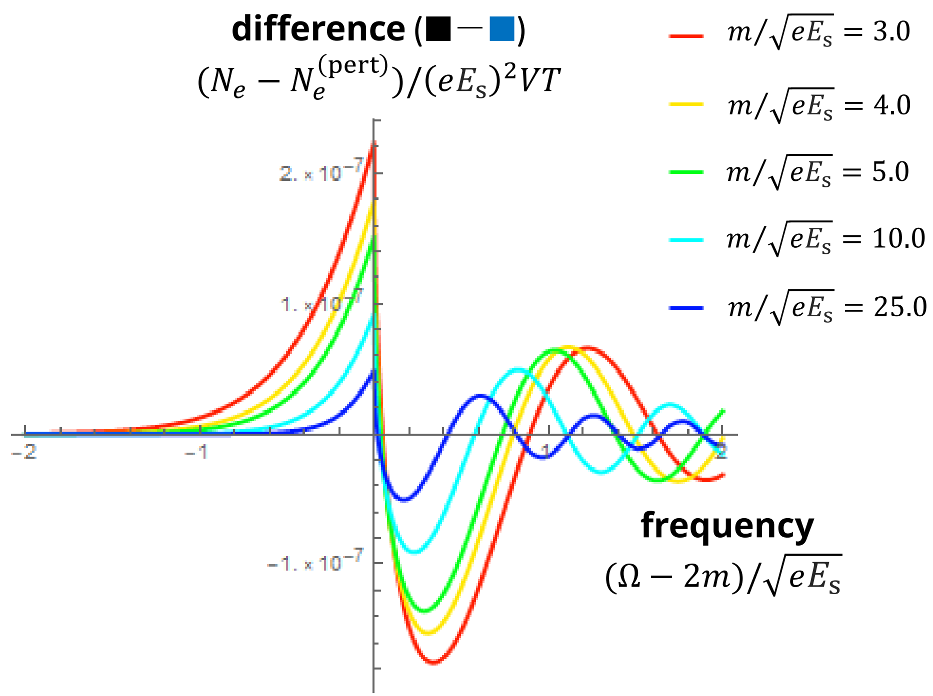


**Completely the same as the Franz-Keldysh effect !**

- enhancement below the gap [Dunne, Gies, Schutzhold (2008), (2009)]
- oscillation above the gap

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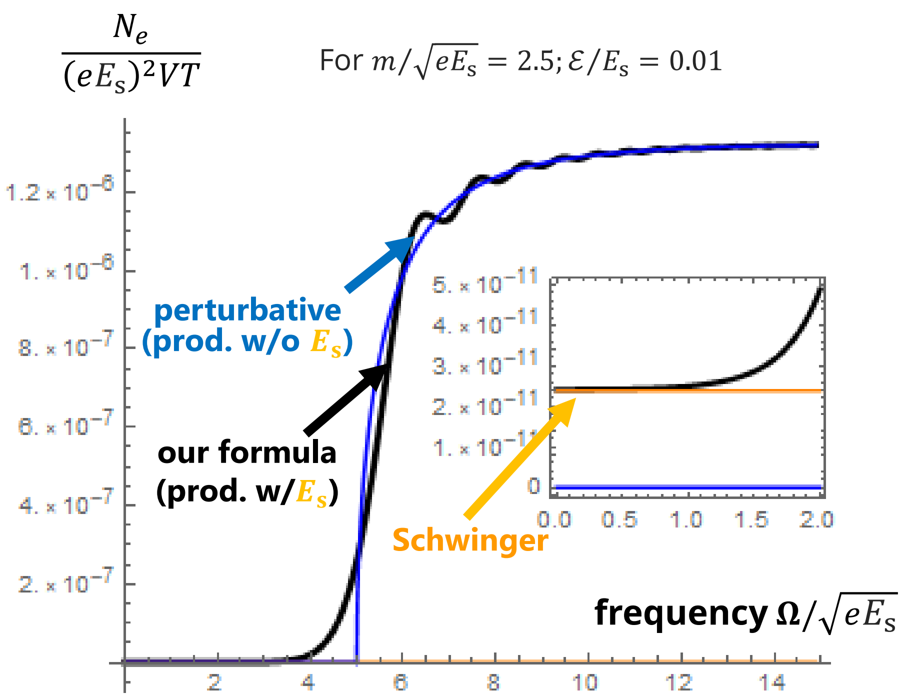
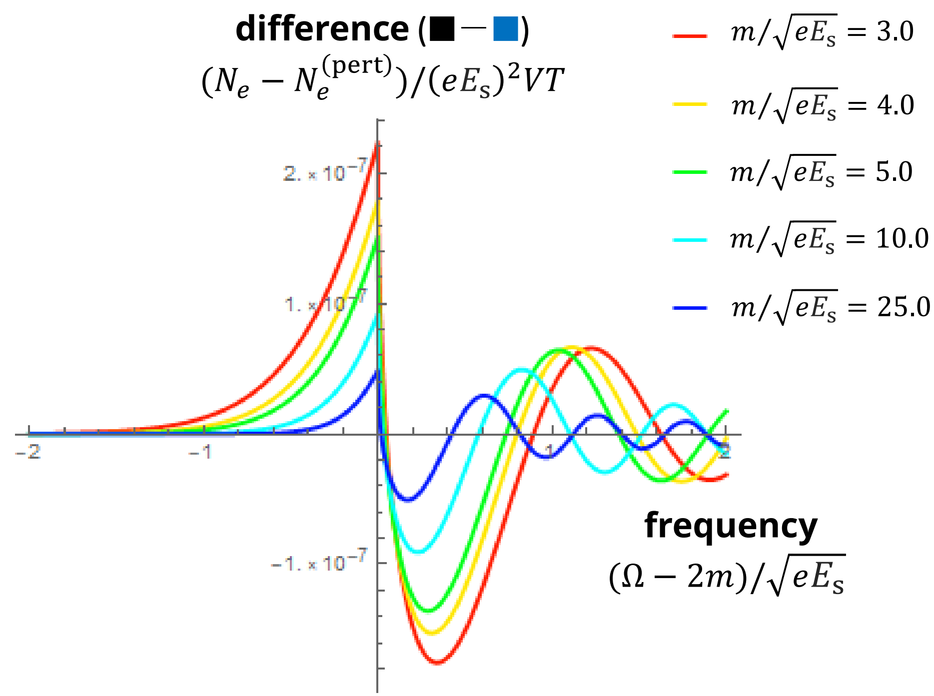


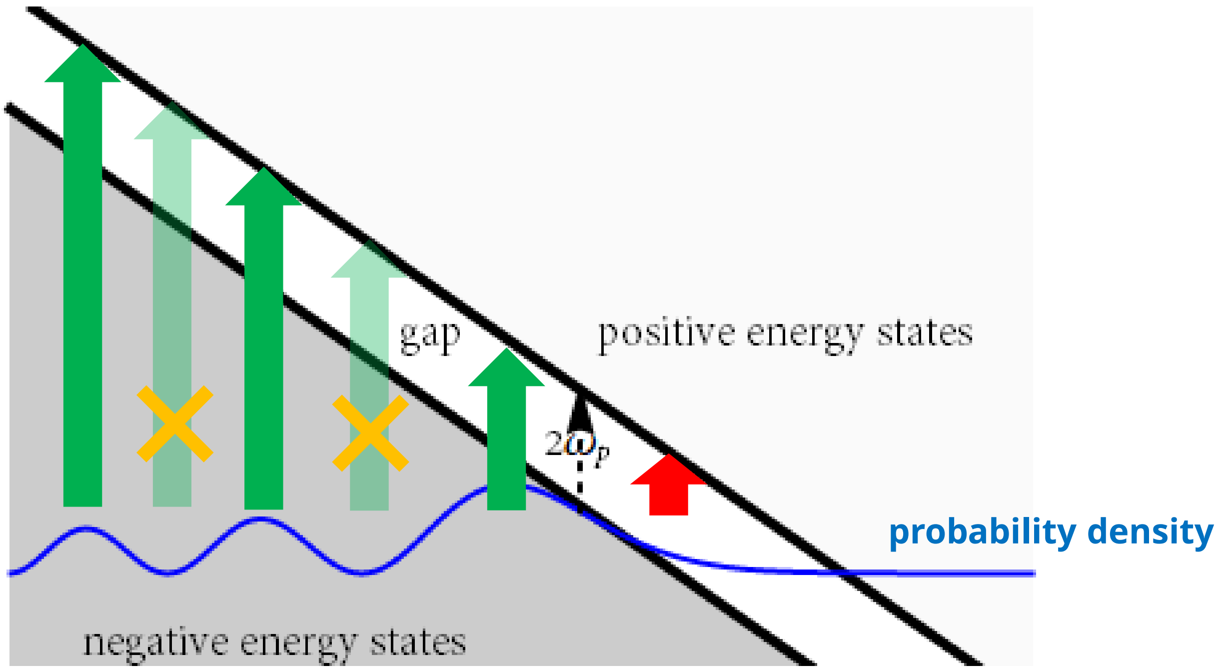




**Completely the same as the Franz-Keldysh effect !**

- enhancement below the gap [Dunne, Gies, Schutzhold (2008), (2009)]
- oscillation above the gap

**Franz-Keldysh effect = dyn. ass. Schwinger mech.**

# Intuitive explanation




- **quantum tunneling ⇒ enhancement**
- **quantum reflection ⇒ oscillation**

  - non-uniform prob. dist. due to interference b/w in-coming and reflected waves

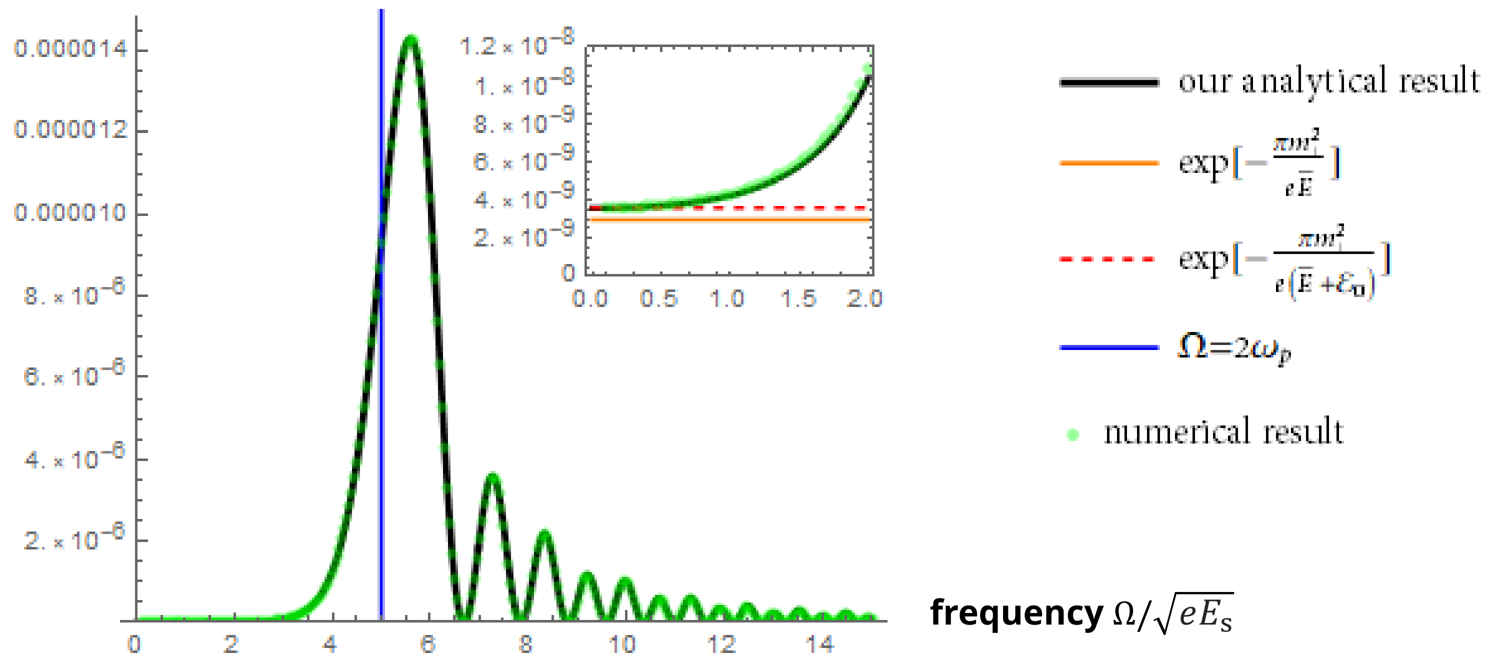
  - production occurs most efficiently at the maxima

# Results (2/3): Momentum dist. $\mathrm{d}^3 N_e / \mathrm{d}\boldsymbol{p}^3$

Parallel field configuration: $\boldsymbol{E} = \begin{pmatrix} 0 \\ 0 \\ E_s + \mathcal{E} \cos \Omega t \end{pmatrix}$

**momentum dist.** $\frac{(2\pi)^3}{V} \frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3}$

For $m/\sqrt{eE_s} = 2.5; \mathcal{E}/E_s = 0.01$




**frequency** $\Omega/\sqrt{eE_s}$

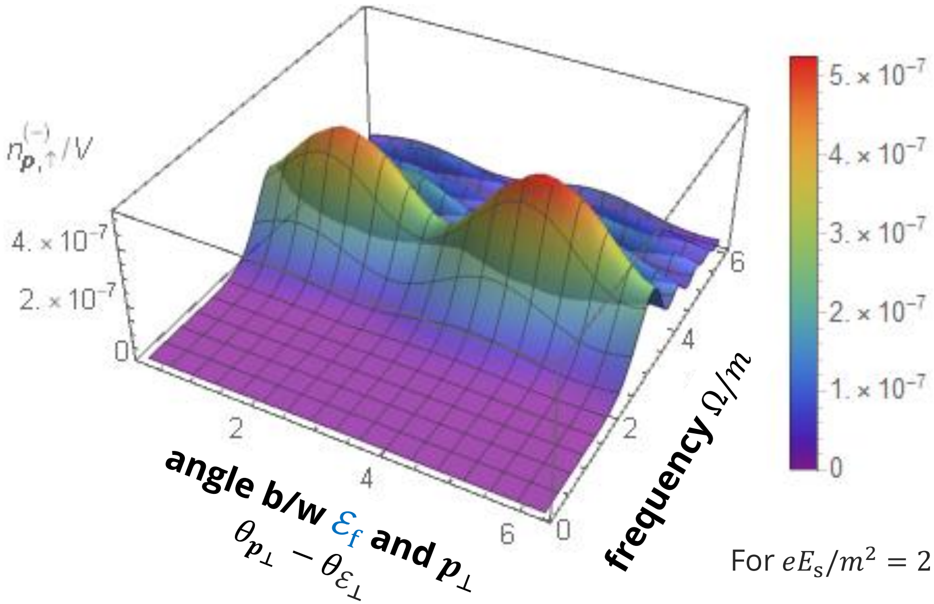
ef) effective mass [Kohlfurst, Gies, Alkofer (2014)]

- **enhancement below gap; oscillation above gap**
- **the pert. peak is slightly above the gap** $\Omega = 2\omega_{\boldsymbol{p}}$ due to reflection
- **excellent agreement** b/w our analytical formula and the numerics

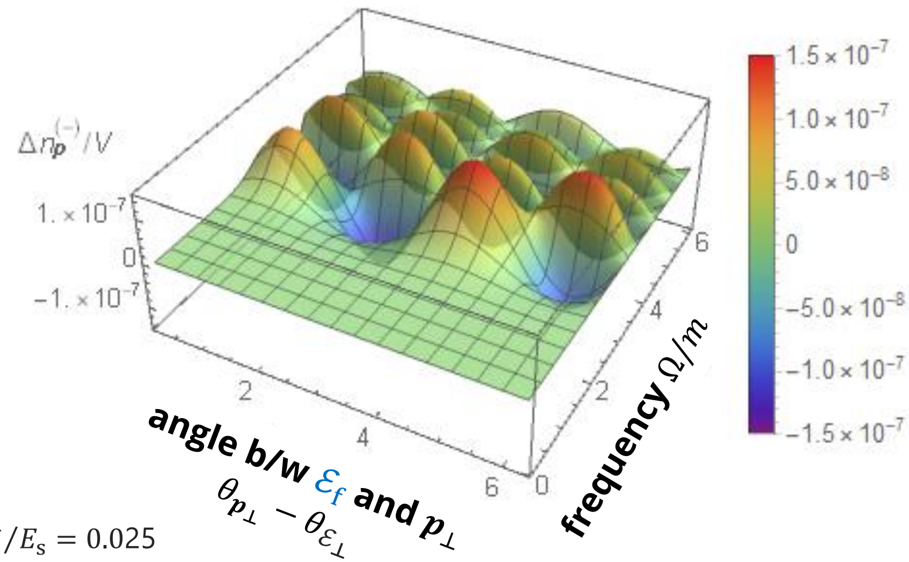
# Results (3/3): Spin-dependence

<u>Perpendicular</u> field configuration: $\boldsymbol{E} = \begin{pmatrix} \mathcal{E}\cos\theta_{\mathcal{E}_\perp} \times \cos\Omega t \\ \mathcal{E}\sin\theta_{\mathcal{E}_\perp} \times \cos\Omega t \\ E_{\mathrm{s}} \end{pmatrix}$

**momentum dist. of spin <u>up</u>** $\frac{1}{V}\frac{\mathrm{d}^3 N_\uparrow}{\mathrm{d}\boldsymbol{p}^3}$




**diff. b/w spin up and down** $\frac{1}{V}\frac{\mathrm{d}^3 N_\uparrow}{\mathrm{d}\boldsymbol{p}^3} - \frac{1}{V}\frac{\mathrm{d}^3 N_\downarrow}{\mathrm{d}\boldsymbol{p}^3}$




For $eE_{\mathrm{s}}/m^2 = 2.5$; $\mathcal{E}/E_{\mathrm{s}} = 0.025$

- Basically the same as the parallel case: **enhancement/oscillation below/above gap**
- **Spin-dependence appears** ⇒ O(10%) effect ⇒ not negligible
  - $\theta_{p_\perp}$-dependent because of the spin-orbit interaction $\boldsymbol{s} \cdot (\boldsymbol{p} \times \boldsymbol{\mathcal{E}})$

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# Summary

[HT, PRD 99, 056006 (2019)]
[X.-G. Huang, HT, PRD 100, 016013 (2019)]
[X.-G. Huang, M. Matsuo, HT (to appear in PTEP)]

**Problem**

**Dynamically assisted Schwinger mechanism**

⇒ spontaneous particle production from the vacuum by strong slow E-field + weak fast E-field w/ **arbitrary** time-dep.

**Technical results**

**Analytical formula for arbitrary time-dep. weak fast E is derived based on pert. theory in Furry picture**

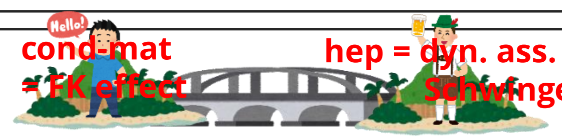
⇒ reproduces the numerics so well and has wider applicability compared to conventional methods (e.g. WKB, worldline)

**Physical results**

- **Dyn. ass. Schwinger mech. = FK effect in cond-mat**
  ⇒ enhancement/oscillation below/above the gap energy
- **Spin-dependence appears**
  ⇒ not negligible ~ O(10%) effect

**Outlook**

- **Interact w/ cond-mat**
- **Phenomenological applications. (e.g. HIC, laser, ...)**




dynamical FK, exciton effect, modulation spectroscopy ...

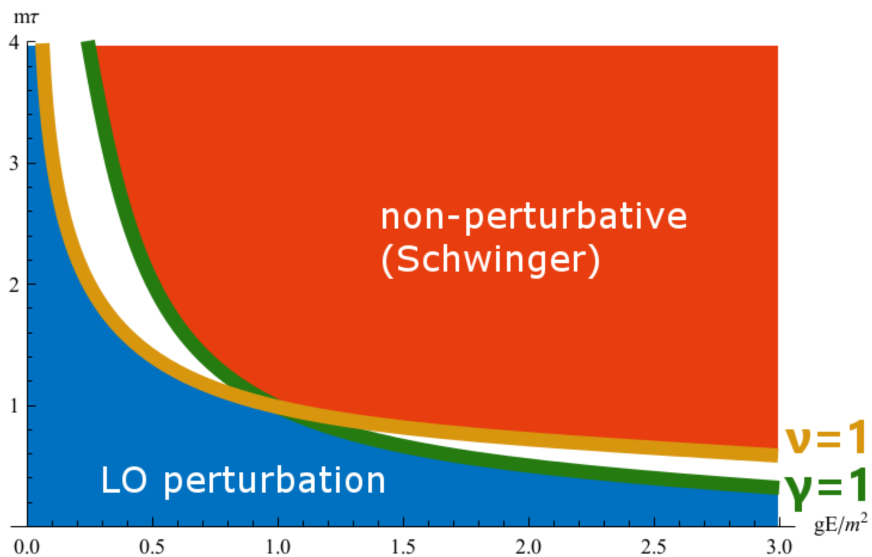
worldline method, Furry picture, 2PI, resurgence ...

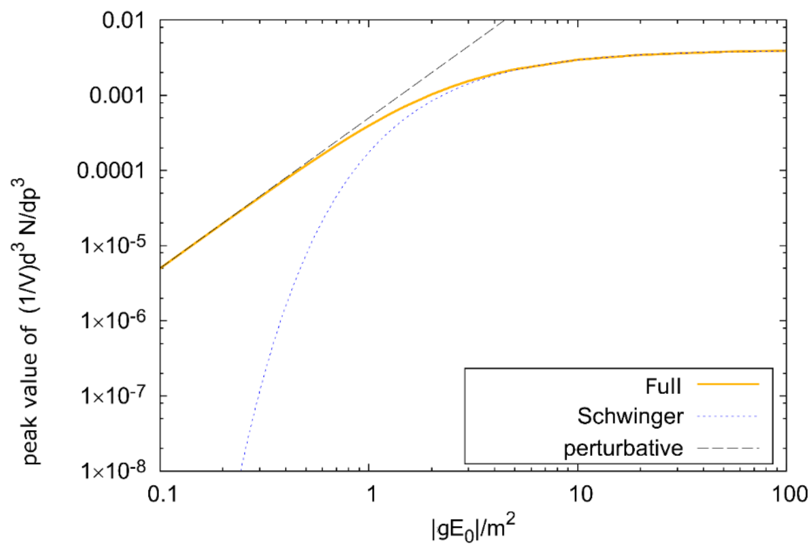
# BACK UP

# Interplay b/w pert. & non-pert. prod.

[HT, Fujii, Itakura (2014)]

## Sauter E-field with lifetime $\tau$ & strength $E_0$: $E = E_0/\cosh^2(t/\tau)$




- **Analytically solvable** [Sauter (1932)]
- **2 dimensionless parameters $\gamma = gE_0\tau/m$, $\nu = gE_0\tau^2$ controls the interplay**

  - because there are 3 dimensionfull parameters