Spin hydrodynamics in ultra-relativistic heavy-ion collisions

Hidetoshi TAYA

(Fudan University)

Ref: Hattori, Hongo, Huang, Matsuo, HT, Phys. Lett. B795, 100 (2019) [arXiv:1901.06615]

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Ultra-relativistic heavy ion collisions (HIC)

Purpose: Study **quark-gluon plasma (QGP)**
(⇒ early Universe, origin of mass, QCD …)

Lesson: QGP behaves like a perfect liquid
and **hydrodynamics works very well**

Strong vorticity in HIC (1/2)

Huge angular momentum: $\boldsymbol{L} \sim \boldsymbol{p} \times \boldsymbol{x} \sim 10^5 \hbar$

⇒ QGP should be "rotating quickly"

⇒ QGP should have strong vorticity $\boldsymbol{\omega} = \mathrm{rot}\, \boldsymbol{v} \gg 1\ (?)$

Strong vorticity in HIC (2/2)

Galaxies

$\sim 10^{-15}\text{Hz}$

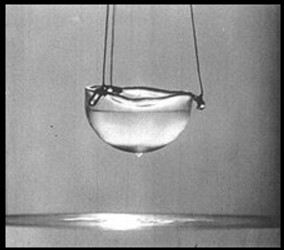
Typhoon

$< 10^{-1}\ \text{Hz}$

Washing machine

$< 10^{+3}\ \text{Hz}$

Helium 4
(strongest in lab.)



[Gomez *et al.* (2014)]

10^{+7}Hz

QGP in HIC

10^{+20}Hz
$= 1 \sim 10\ \text{MeV}$

[Jiang, Lin, Liao (2017)] [Li *et al.* (2017)]
[Xia *et al* (2018)] [Wei, Deng, Huang (2019)] …

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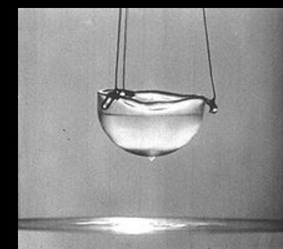

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QGP in HIC

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[Jiang, Lin, Liao (2017)] [Li *et al.* (2017)]
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The STRONGEST vorticity ever in our Universe !

- ✓ Very **NEW & UNIQUE** opportunity to study phys. of strong vorticity
- ✓ Vorticity appears in various systems ⇒ **interdisciplinary interest**

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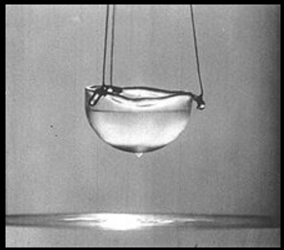

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First step: What happens to QGP ?

cf. talk by Yang, Yamamoto

Expectation: QGP is polarized along ω

[Liang, Wang (2005)] [Voloshin (2004)]

Expectation: QGP is polarized along $\boldsymbol{\omega}$

[Liang, Wang (2005)] [Voloshin (2004)]

because of **spin-vorticity coupling**

✓ $E \to E - \omega \cdot s$ (~ LS force $\delta E \propto L \cdot s$)

[Vilenkin (1980)] [Hehl, Ni (1990)]
[Matsuo *et al.* (2011)] [Beccatini (2012)] [Beccatini *et al.* (2013)] [Hattori *et al .* (2019)] …

✓

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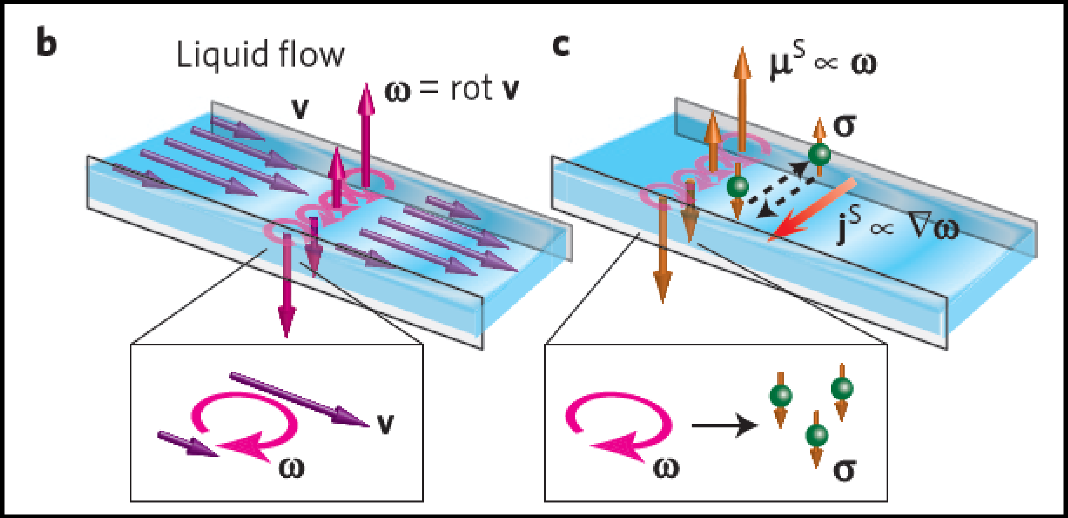
[Takahashi *et al* (2015)] cf. talk by Matsuo

✓ such a coupling is already confirmed in spintronics



Experimental fact ➡ Observed

[STAR, PRC 98, 014910 (2018)]

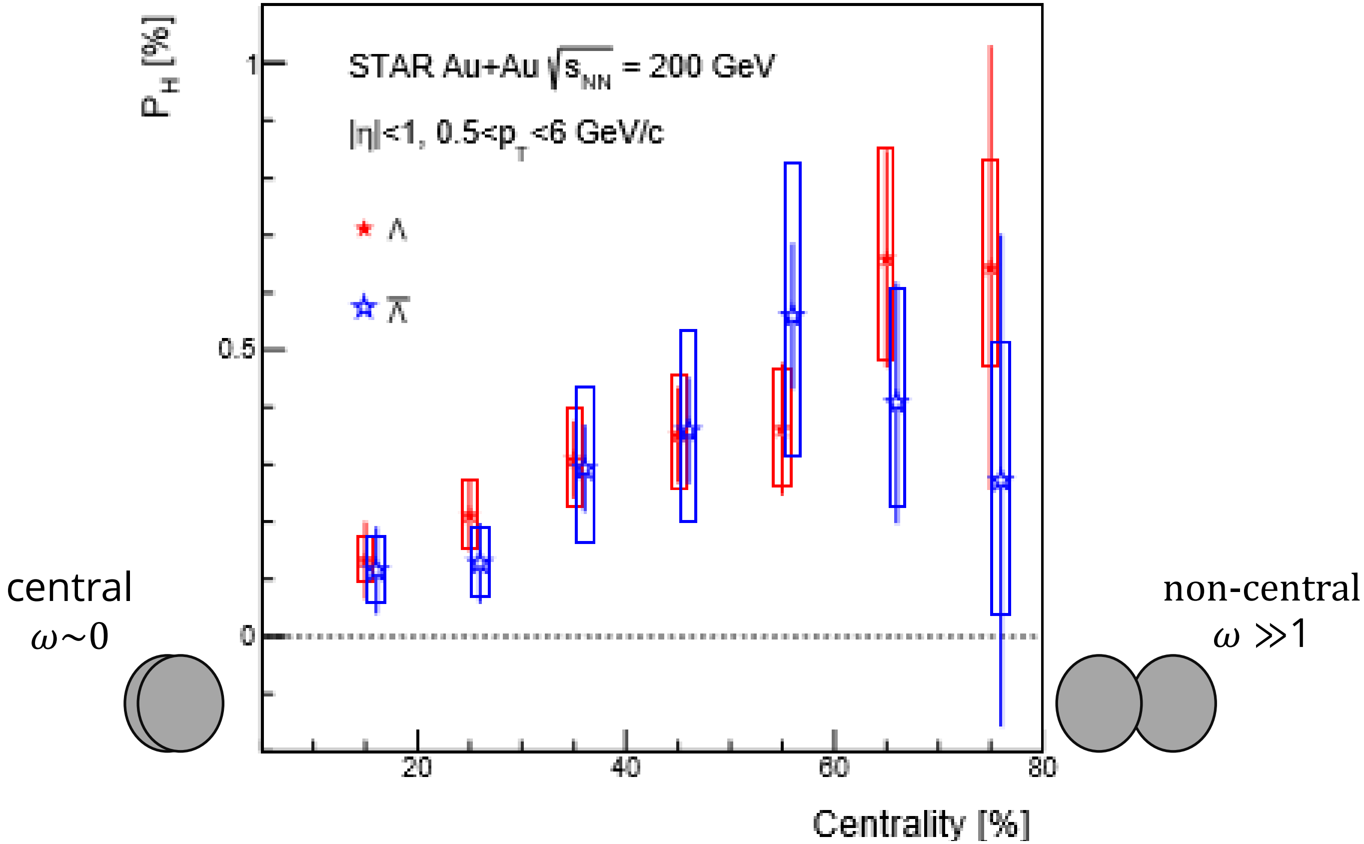


FIG. 5. Λ ($\bar{\Lambda}$) polarization as a function of the collision centrality in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Open boxes and vertical lines show systematic and statistical uncertainties. The data points for $\bar{\Lambda}$ are slightly shifted for visibility.

Exp. suggest $\omega = O(10^{20}\ \mathrm{Hz}) = O(1 \sim 10\ \mathrm{MeV})$ consistent w/ theor. estimates

How about theory?

Hydro for spin polarized QGP?

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Hydro for spin polarized QGP?

Far from complete

∵ Relativistic spin hydro has NOT even been formulated !

Current status of formulation of spin hydro

✓ Non-relativistic case

[Eringen (1998)] [Lukaszewicz (1999)]

Already established (e.g. micropolar fluid)

- applied to pheno. and is successful e.g. spintronics: [Takahashi et al. (2015)]
- **spin must be dissipative** because of mutual conversion b/w spin and orbital angular momentum

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✓ Relativistic case

[Florkowski, Ryblewski, Kumar, …]

Some preceding works do exist, but

- only for “ideal” fluid (**no dissipative corrections**)
- some claim **spin should be conserved (!?)**

Purpose of this talk

- ✓ Formulate **relativistic spin hydro with 1st order dissipative corrections** for the first time
- ✓ Clarify spin must be **dissipative**

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✓ Clarify spin must be **dissipative**

Outline

1. ~~Introduction~~
2. Formulation based on entropy-current analysis
3. Linear mode analysis
4. Summary

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Introduction to rela. hydro w/o spin (1/3)

Hydro = **An effective theory** that describes spacetime evolution of **long-wavelength modes (hydro modes)**

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✓ **Phenomenological formulation**

See textbook by Landau & Lifshitz

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Step 1: **Write down the conservation law**: $0 = \partial_\mu T^{\mu\nu}$ 4 eqs

Step 2:

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- define hydro variables: $\{\beta, u^\mu\}$ $\begin{pmatrix} u^\mu = \gamma(1, \boldsymbol{v}/c) \\ u^2 = -1 \end{pmatrix}$ 1 + (4-1) = 4 DoGs "chemical potential" for P^μ
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- write down all the possible tensor structures of $T^{\mu\nu}$

$$T^{\mu\nu} = f_1(\beta) g^{\mu\nu} + f_2(\beta) u^\mu u^\nu$$
$$+ f_3(\beta) \epsilon^{\mu\nu\rho\sigma} \partial_\rho u_\sigma + f_4(\beta) \partial^\mu u^\nu + f_5(\beta) \partial^\nu u^\mu$$
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- simplify the tensor structures by (**assumptions** in hydro)
  1. symmetry
  2. power counting ➡ **gradient expansion**
  3. other physical requirements ➡ **thermodynamics** *(see next slide)*

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✓ **Hydrodynamic eq. = conservation law + constitutive relation**

Introduction to rela. hydro w/o spin (2/3)

✓ **Constraints by thermodynamics**

Introduction to rela. hydro w/o spin (2/3)

✓ Constraints by thermodynamics

Expand $T^{\mu\nu}$ i.t.o derivatives

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + O(\partial^2) \quad \text{where} \quad T^{\mu\nu}_{(0)} = e u^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu)$$

because $T^{\mu\nu} \xrightarrow[\text{static eq.}]{} T^{\mu\nu}_{(0)} = (e, p, p, p)$

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1st law of thermodynamics says

$$ds = \beta de, \;\; s = \beta(e + p)$$

With EoM $0 = \partial_\mu T^{\mu\nu}$, div. of entropy current $S^\mu = s u^\mu + O(\partial)$ can be evaluated as

$$\partial_\mu S^\mu = -T^{\mu\nu}_{(1)} \partial_\mu (\beta u_\nu) + O(\partial^3)$$

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2st law of thermodynamics says $\partial_\mu S^\mu \geq 0$, which is guaranteed
if RHS is expressed as a semi-positive bilinear as

$$-T^{\mu\nu}_{(1)} \partial_\mu(\beta u_\nu) = \sum_{X_i \in T_{(1)}} \lambda_i X_i^{\mu\nu} X_{i\,\nu\mu} \geq 0 \text{ with } \lambda_i \geq 0$$

(strong constraint !!)

Introduction to rela. hydro w/o spin (2/3)

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ex) heat current: $2h^{(\mu}u^{\nu)} \equiv h^\mu u^\nu + h^\nu u^\mu \in T^{\mu\nu}_{(1)} \ \ (u_\mu h^\mu = 0)$

$$\Rightarrow T^{\mu\nu}_{(1)} \partial_\mu(\beta u_\nu) = -\beta h^\mu \left(\beta \partial_{\perp\mu} \beta^{-1} + u^\nu \partial_\nu u^\mu\right) \geq 0$$

$$\Rightarrow h^\mu = -\kappa\left(\beta \partial_{\perp\mu} \beta^{-1} + u^\nu \partial_\nu u^\mu\right) \text{ with } \kappa \geq 0$$

Introduction to rela. hydro w/o spin (3/3)

✓ **Constitutive relation up to 1st order w/o spin**

$$T^{\mu\nu}_{(0)} = eu^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu)$$

$$T^{\mu\nu}_{(1)} = -2\kappa\left(Du^{(\mu} + \beta\partial_\perp^{(\mu}\beta^{-1}\right)u^{\nu)} - 2\eta\partial_\perp^{<\mu}u^{\nu>} - \zeta(\partial_\mu u^\mu)\Delta^{\mu\nu}$$

heat current **shear viscosity** **bulk viscosity**

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heat current **shear viscosity** **bulk viscosity**

✓ Relativistic hydrodynamic eq. w/o spin

Hydrodynamic eq. = conservation law + constitutive relation

Euler eq.	$0 = \partial_\mu T^{\mu\nu}$	$T^{\mu\nu} = T^{\mu\nu}_{(0)}$
Navier-Stokes eq.	$0 = \partial_\mu T^{\mu\nu}$	$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)}$
⋮	⋮	⋮

(*) Correctly reproduce the non-relativistic eqs. w/ $u \to (1, \boldsymbol{v}/c) + O((v/c)^2)$

Formulation of rela. hydro with spin (1/4)

✓ **Strategy is the same**

✓ **Phenomenological formulation**

Step 1: Write down the conservation law

Step 2: Construct a constitutive relation

- define hydro variables
- write down all the possible tensor structures
- simplify the tensor structures by e.g. thermodynamics

Formulation of rela. hydro **with** spin (2/4)

Step 1: Write down the conservation law

(1) energy conservation

$$0 = \partial_\mu T^{\mu\nu}$$

(canonical)

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Formulation of rela. hydro with spin (2/4)

Step 1: Write down the conservation laws

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(2) total angular momentum conservation

$$\psi(x) \to S(\Lambda)\psi(\Lambda^{-1}x)$$

$$0 = \partial_\mu M^{\mu,\alpha\beta}$$

$$= \partial_\mu \left(L^{\mu,\alpha\beta} + \Sigma^{\mu,\alpha\beta}\right)$$

$$= \partial_\mu \left(x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha} + \Sigma^{\mu,\alpha\beta}\right)$$

$$\therefore \quad \partial_\mu \Sigma^{\mu,\alpha\beta} = T^{\alpha\beta} - T^{\beta\alpha}$$

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- ✓ Spin is **not** conserved if (canonical) $T^{\mu\nu}$ has anti-symmetric part $T^{\mu\nu}_{(\mathrm{a})}$
- ✓ There's **no** a priori reason (canonical) $T^{\mu\nu}$ must be symmetric

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$$\begin{aligned} 0 &= \partial_\mu M^{\mu,\alpha\beta} \\ &= \partial_\mu \left(L^{\mu,\alpha\beta} + \Sigma^{\mu,\alpha\beta}\right) \\ &= \partial_\mu \left(x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha} + \Sigma^{\mu,\alpha\beta}\right) \end{aligned}$$

$$\therefore \quad \partial_\mu \Sigma^{\mu,\alpha\beta} = T^{\alpha\beta} - T^{\beta\alpha}$$

- ✓ Spin is **not** conserved if (canonical) $T^{\mu\nu}$ has anti-symmetric part $T^{\mu\nu}_{(\mathrm{a})}$
- ✓ There's **no** a priori reason (canonical) $T^{\mu\nu}$ must be symmetric

Consequence

(1) Spin must not be a hydro mode in a strict sense

(2) Nevertheless, it behaves *like* a hydro mode if $T^{\mu\nu}_{(a)} \ll 1$

➡ **inclusion of dissipative nature is crucially important**

Formulation of rela. hydro **with** spin (3/4)

Step 2: Construct a constitutive relation for $T^{\mu\nu}, \Sigma^{\mu,\alpha\beta}$

(1) define hydro variables

4 DoGs

$\{\beta, u^{\mu}\}$

(2) simplify the tensor structure by thermodynamics

Formulation of rela. hydro with spin (3/4)

Step 2: Construct a constitutive relation for $T^{\mu\nu}, \Sigma^{\mu,\alpha\beta}$

(1) define hydro variables

4 + 6 = 10 DoGs = # of EoMs

Introduce **spin chemical potential** $\{\beta, u^\mu, \boldsymbol{\omega^{\mu\nu}}\}$ with $\omega^{\mu\nu} = -\omega^{\nu\mu}$

✓ $\{\beta, u^\mu, \omega^{\mu\nu}\}$ are independent w/ each other at this stage ($\omega^{\mu\nu} \neq$ thermal vorticity)

(2) simplify the tensor structure by thermodynamics

Formulation of rela. hydro with spin (3/4)

Step 2: Construct a constitutive relation for $T^{\mu\nu}, \Sigma^{\mu,\alpha\beta}$

(1) define hydro variables

4 + 6 = 10 DoGs = # of EoMs

Introduce **spin chemical potential** $\{\beta, u^\mu, \boldsymbol{\omega^{\mu\nu}}\}$ with $\omega^{\mu\nu} = -\omega^{\nu\mu}$

✓ $\{\beta, u^\mu, \omega^{\mu\nu}\}$ are independent w/ each other at this stage ($\omega^{\mu\nu} \neq$ thermal vorticity)

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Expand $T^{\mu\nu}, \Sigma^{\mu,\alpha\beta}$, i.t.o derivatives

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Generalize **1st law of thermodynamics with spin** as

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$$\partial_{\mu} S^{\mu} = -T^{\mu\nu}_{(1s)} \frac{\partial_{\mu}(\beta u_{\nu}) + \partial_{\nu}(\beta u_{\mu})}{2} - T^{\mu\nu}_{(1a)} \left\{ \frac{\partial_{\mu}(\beta u_{\nu}) - \partial_{\nu}(\beta u_{\mu})}{2} - 2\beta\omega_{\mu\nu} \right\} + O(\partial^3)$$

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✓ 2nd law of thermodynamics $\partial_\mu S^\mu \geq 0$ gives strong constraint on $T^{\mu\nu}_{(1)}$

✓ In global equil. $\partial_\mu S^\mu = 0$, so ω = vorticity (**spin-vorticity coup.**).

Formulation of rela. hydro **with** spin (4/4)

✓ Constitutive relation for $T^{\mu\nu}$ up to 1st order with spin

$$T^{\mu\nu}_{(0)} = eu^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu)$$

heat current　shear viscosity　bulk viscosity

$$T^{\mu\nu}_{(1)} = -2\kappa\left(Du^{(\mu} + \beta\partial_\perp^{(\mu}\beta^{-1}\right)u^{\nu)} - 2\eta\partial_\perp^{<\mu}u^{\nu>} - \zeta(\partial_\mu u^\mu)\Delta^{\mu\nu}$$

$$-2\lambda\left(-Du^{[\mu} + \beta\partial_\perp^{[\mu}\beta^{-1} + 4u_\rho\omega^{\rho[\mu}\right)u^{\nu]} - 2\gamma\left(\partial_\perp^{[\mu}u^{\nu]} - 2\Delta^\mu_\rho\Delta^\nu_\lambda\omega^{\rho\lambda}\right)$$

"boost heat current"　"rotational (spinning) viscosity

NEW !

e.g. Eringen (1998); Lukaszewicz (1999)

- ✓ Relativistic generalization of a non-relativistic micropolar fluid
- ✓ "boost heat current" is a relativistic effect

✓ Relativistic hydrodynamic eq. up to 1st order with spin

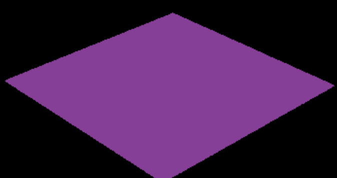
$$0 = \partial_\mu(T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + O(\partial^2)) \qquad \partial_\mu(u^\mu\sigma^{\alpha\beta}) = T^{\alpha\beta}_{(1)} - T^{\beta\alpha}_{(1)} + O(\partial^2)$$

Outline

1. ~~Introduction~~
2. ~~Formulation based on entropy-current analysis~~
3. **Linear mode analysis**
4. Summary

Linear mode analysis (1/2)

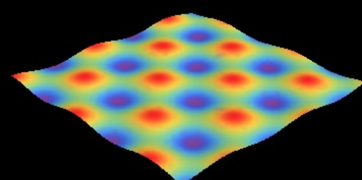
Setup: small perturbations on top of global therm. equilibrium



$\beta = \beta_0$

$u^\mu = (1, \mathbf{0})$

$\omega^{\mu\nu} = 0$



$\beta = \beta_0 + \delta\beta$

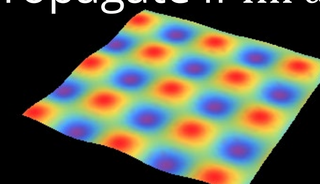
$u^\mu = (1, \mathbf{0}) + \delta u^\mu$

$\omega^{\mu\nu} = 0 + \delta\omega^{\mu\nu}$

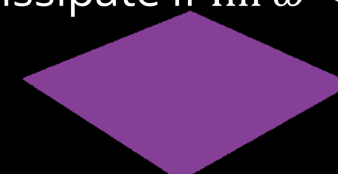
$$i\partial_t \begin{pmatrix} \delta\beta \\ \delta u^\mu \\ \delta\omega^{\mu\nu} \end{pmatrix} = M \begin{pmatrix} \delta\beta \\ \delta u^\mu \\ \delta\omega^{\mu\nu} \end{pmatrix} + O(\delta^2)$$

hydro evolution

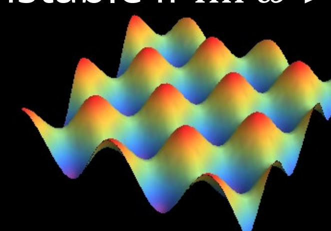
propagate if $\mathrm{Im}\,\omega = 0$



dissipate if $\mathrm{Im}\,\omega < 0$



unstable if $\mathrm{Im}\,\omega > 0$



Linear mode analysis (2/2)

✓ **Hydro w/o spin** $\{\beta, u^\mu\}$

4 gapless modes

2 sound modes $\omega = \pm c_s k + O(k^2)$

2 shear modes $\omega = 0 + O(k^2)$

where $c_s^2 \equiv \partial p / \partial e$

✓ **Hydro with spin** $\{\beta, u^\mu, \omega^{\mu\nu}\}$

Linear mode analysis (2/2)

✓ Hydro w/o spin $\{\beta, u^\mu\}$

4 gapless modes

2 sound modes $\omega = \pm c_s k + O(k^2)$

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✓ Hydro with spin $\{\beta, u^\mu, \omega^{\mu\nu}\}$

4 gapless modes

2 sound modes $\omega = \pm c_s k + O(k^2)$

2 shear modes $\omega = 0 + O(k^2)$

+ 6 dissipative gapped modes

3 "boost" modes $\omega = -2i\tau_\mathrm{b}^{-1} + O(k^2)$

3 "spin" modes $\omega = -2i\tau_\mathrm{s}^{-1} + O(k^2)$

where $\tau_\mathrm{s} \equiv \frac{\partial\sigma^{ij}/\partial\omega^{ij}}{4\gamma}, \tau_\mathrm{b} \equiv \frac{\partial\sigma^{i0}/\partial\omega^{i0}}{4\lambda}$

- ✓ We explicitly confirmed that **spin is dissipative**
- ✓ Time-scale of the dissipation is controlled by the new viscous constants γ, λ

Outline

1. ~~Introduction~~
2. ~~Formulation based on entropy-current analysis~~
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4. **Summary**

Summary

- ✓ Spin polarization in QGP is one of the hottest topics in HIC. But, its theory, in particular hydrodynamic framework, is still under construction
- ✓ **Relativistic spin hydrodynamics with 1st order dissipative corrections is formulated for the first time** based on the phenomenological entropy-current analysis
- ✓ **Spin must be dissipative** because of the mutual conversion between the orbital angular momentum and spin
- ✓ Linear mode analysis of the spin hydrodynamic equation also suggests that spin must be dissipative, whose time-scale is controlled by the new viscous constants γ, λ

Outlook: extension to 2nd order, Kubo formula, MHD, application to cond-mat, numerical simulations

BACKUP

Linearlized hydro eq.

$$M\,\delta\vec{c} = 0$$

where

$$M = \left(\begin{array}{c|ccccccc} A_{3\times3} & & & & O & & & \\ \hline & -i\omega+(\gamma_\perp+\gamma')k_z^2 & +iD_s k_z & 0 & 0 & 0 & 0 & 0 \\ & -2i\gamma' k_z & -i\omega+2D_s & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & -i\omega+(\gamma_\perp+\gamma')k_z^2 & -iD_s k_z & 0 & 0 & 0 \\ O & 0 & 0 & 2i\gamma' k_z & -i\omega+2D_s & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & -i\omega+2D_b & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & -i\omega+2D_b & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & -i\omega+2D_s \end{array}\right)$$

$$A_{3\times3} = \begin{pmatrix} -i\omega+2c_s^2\lambda' k_z^2 & ik_z & -2iD_b k_z \\ ic_s^2 k_z & -i\omega+\gamma_\parallel k_z^2 & 0 \\ 2ic_s^2\lambda' k_z & 0 & -i\omega+2D_b \end{pmatrix}$$

$$\delta\vec{c} \equiv (\delta\tilde{e}, \delta\tilde{\pi}^z, \delta\tilde{S}^{0z}, \delta\tilde{\pi}^x, \delta\tilde{S}^{zx}, \delta\tilde{\pi}^y, \delta\tilde{S}^{yz}, \delta\tilde{S}^{0x}, \delta\tilde{S}^{0y}, \delta\tilde{S}^{xy})^t$$

Dispersion relations

$$\omega = -2iD_s,$$

$$\omega = -2iD_b,$$

$$\omega = \begin{cases} -2iD_s - i\gamma' k_z^2 + \mathcal{O}(k_z^4), \\ -i\gamma_\perp k_z^2 + O(k_z^4),, \end{cases}$$

$$\omega = \begin{cases} \pm c_s k_z - i\frac{\gamma_\parallel}{2} k_z^2 + \mathcal{O}(k_z^3), \\ -2iD_b - 2ic_s^2 \lambda' k_z^2 + \mathcal{O}(k_z^4). \end{cases}$$

Further simplification by EoM

The 1st order constitutive relation reads

$$\Theta^{\mu\nu}_{(1s)} = 2h^{(\mu}u^{\nu)} + \tau^{\mu\nu}$$

$$\Theta^{\mu\nu}_{(1a)} = 2q^{[\mu}u^{\nu]} + \phi^{\mu\nu}$$

$$h^\mu = -\kappa(Du^\mu + \beta\partial^\mu_\perp T),$$

$$\tau^{\mu\nu} = -2\eta\partial^{\langle\mu}_\perp u^{\nu\rangle} - \zeta\theta\Delta^{\mu\nu},$$

$$q^\mu = -\lambda\big(-Du^\mu + \beta\partial^\mu_\perp T - 4\omega^{\mu\nu}u_\nu\big),$$

$$\phi^{\mu\nu} = -2\gamma\big(\partial^{[\mu}_\perp u^{\nu]} - 2\Delta^\mu_\rho\Delta^\nu_\lambda\omega^{\rho\lambda}\big),$$

By using LO hydro eq.,

$$(e+p)Du^\mu = -\partial^\mu_\perp p + \mathcal{O}(\partial^2)$$

we can further simplify *h,q* as

$$h^\mu = -\kappa\left[\frac{-\partial^\mu_\perp p}{e+p} + \beta\partial^\mu_\perp T + \mathcal{O}(\partial^2)\right] = \mathcal{O}(\partial^2),$$

$$q^\mu = -\lambda\left[\frac{2\partial^\mu_\perp p}{e+p} - 4\omega^{\mu\nu}u_\nu\right] + \mathcal{O}(\partial^2).$$