How time-dependent electric fields affect the Schwinger mechanism?

Hidetoshi TAYA

Keio U. → RIKEN (from this April)

What is the Schwinger mechanism?

[Sauter (1932)] [Heisenberg, Euler (1936)] [Schwinger (1951)]

✔ **Vacuum pair production occurs in the presence of strong fields**

Intuitive picture for a slow electric field = **quantum tunneling**

What is the Schwinger mechanism?

[Sauter (1932)] [Heisenberg, Euler (1936)] [Schwinger (1951)]

✔ Vacuum pair production occurs in the presence of strong fields

Intuitive picture for a slow electric field = **quantum tunneling**

✔ For constant strong E-field $E(t,x) = E$, it's understood well (at least, theoretically)

$$N_{e^\pm} = \frac{(eE)^2 VT}{(2\pi)^3} \times \exp\left[-\pi\frac{m^2}{eE}\right] \sim \exp[-\# \times (\text{gap height}) \times (\text{gap length})]$$

- Non-perturbative ⇒ Interesting, since it is the unexplored region of QED (or QFT)
- Vacuum process ⇒ Fundamental, since all the physical processes occur on top of vacuum
- However, exponentially suppressed i.t.o. mass ⇒ Not confirmed by experiments yet…

cf.) Guinness world record: [Yanovsky et al (2008)]

HERCULES laser $eE \sim \sqrt{10^{22}\,\mathrm{W/cm^2}} \sim (0.01\,\mathrm{MeV})^2 \ll m_e^2 \sim \sqrt{10^{29}\,\mathrm{W/cm^2}} \sim (0.5\,\mathrm{MeV})^2$

Timeliness

✔ **Now is the best time to study the Schwinger mechanism !**

Developments in experimental technologies ⇒ **Novel strong-field sources**

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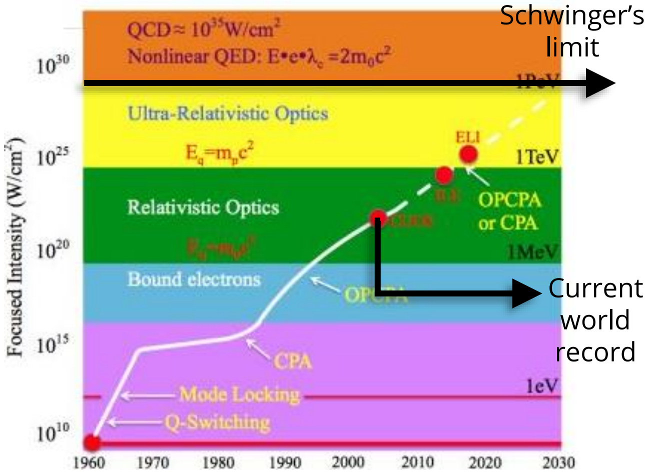
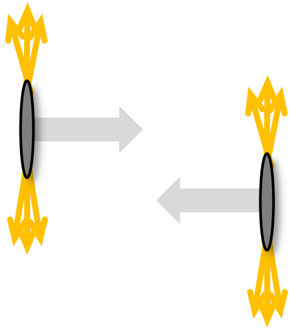
ex.1) Intense lasers



[Mourou, Tajima, Bulanov (2006)]

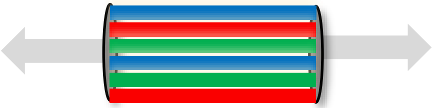
ex.2) Heavy-ion collisions



- (ultra)peripheral collisions

$$eE, eB \sim \frac{\alpha Z v \gamma}{r^2} \sim \alpha Z \gamma \times m_\pi^2$$

[Skokov, Illarionov, Toneev (2009)]
[Deng, Huang (2012)]



- glasma (color flux tube)

$$gE_{\text{color}}, gB_{\text{color}} \sim Q_s^2 \sim O(1)\ \text{GeV}^2$$

[Lappi, McLerran (2006)]

- Also in other geometries:
  - asymmetric coll. [Hirono, Hongo, Hirano (2014)] [Voronyuk, et al (2014)]
  - low-energy coll. [Review: Rafelski, et al (2014)] [Maltsev et al (2019)]

[Allor, Cohen, McGady (2008)] [Solinas, Amoretti, Giazotto (2021)]

ex.3) Cond-mat analogues: Graphene, Cold atom, Superconductor, Semiconductor, ...

[Szpak, Shutzhold (2012)] [Thesis by Linder; 1807.08050]

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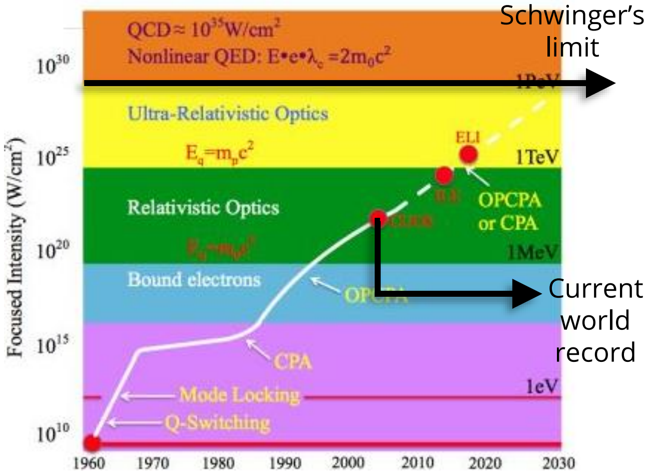

Schwinger's limit

Current world record

[Mourou, Tajima, Bulanov (2006)]

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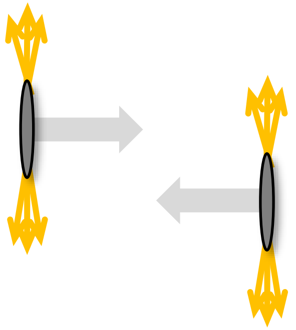

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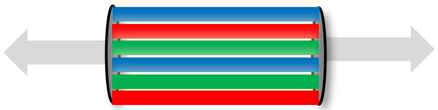

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✔ Schwinger's result is insufficient for actual problems ⇒ **Beyond Schwinger !**

- Inhomogeneous field (e.g., time- & space-dependence, more efficient field config., …)
- Realtime dynamics (e.g., backreaction, intermediate particle number, …)
- Higher loop effects (e.g., radiation, mass shift, …)
- Other observables (e.g., spin, chirality, high-harmonic generation…)
- …

Today's talk

Discuss the Schwinger mech. under time-depending E-field

Part I: Interplay b/w non-pert. & pert. production mechanisms

[HT, Fujii, Itakura, PRD (2014)]
[HT et al., JHEP (2021)]

Part II: Dynamically assisted Schwinger mechanism

[HT, PRD (2019)]
[Huang, HT, PRD (2019)]
[Huang, Matsuo, HT, PTEP (2019)]

Part III: Dynamical assistance to chirality production [HT, PRR (2020)]

Introduction

⇓

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Interplay b/w non-pert. & pert. production mechanisms

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Dynamically assisted Schwinger mechanism

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Part III:

Dynamical assistance to chirality production

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Summary

Background (1/2): Interplay b/w non-pert. & pert. mech.

✔ **Consider time-dependent E-field, having strength eE_0 and frequency Ω**

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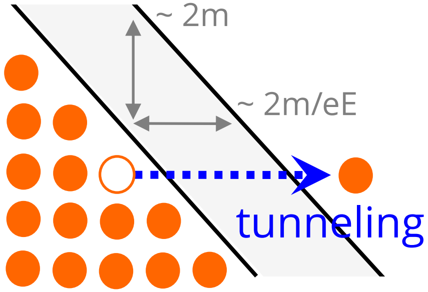
- The tunneling picture should break down if the frequency Ω is large

Non-pert. tunneling $N \sim \exp[\#/eE_0]$



Tunneling time $\Delta t \sim \frac{2m}{eE}$

$\Rightarrow$ E-field should be slower than Δt

$\Rightarrow \Omega^{-1} \gtrsim \Delta t$

$\Rightarrow 1 \gtrsim \frac{\Delta t}{\Omega^{-1}} = \frac{\Omega m}{eE} \equiv \gamma$ (Keldysh parameter)

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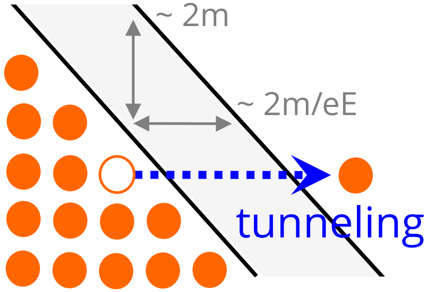

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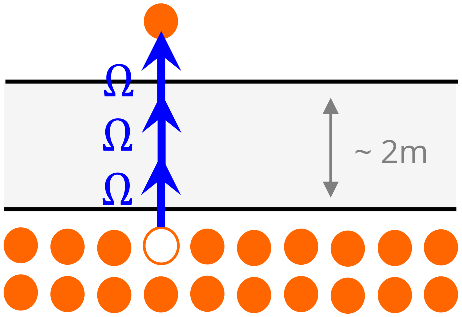
Pert. multi-photon $N \sim eE_0^{2n}$



$n > \frac{2m}{\Omega}$ is # of photons involved

- For large Ω, E-field may behave like a photon and interact incoherently (perturbatively)

Background (2/2): Semi-classical analysis

✔ The interplay has been "confirmed" by (but only by) semi-classical analysis

- Steepest descent analysis [Brezin, Itzykson (1970)]
- Imaginary-time method [Popov (1972)]
- Divergent asymptotic series method [Berry (1989)]
- Worldline instanton method [Dunne, Shubert (2005)]
- (exact) WKB [HT, Fujimori, Misumi, Nitta, Sakai (2020)]
- …

- Trans-series expansion in terms of $\hbar$

$$N = \sum_{n,m} N_{n,m} \hbar^n \mathrm{e}^{-mS/\hbar} = (N_{0,1} + O(\hbar))\, \mathbf{e}^{-S/\hbar} + O(\mathrm{e}^{-2S/\hbar})$$

- Valid in the slow limit $\hbar \ll 1 \Leftrightarrow \Omega \ll 1$: $\because\ \mathrm{i}\hbar\partial_t\psi = H(\Omega t)\psi \xrightarrow[\tau \equiv t/\hbar]{} \mathrm{i}\partial_\tau\psi = H(\hbar\Omega\tau)\psi$

- Production number N is controlled solely by the Keldysh parameter $\gamma \equiv \dfrac{m\Omega}{eE_0}$ [Keldysh (1965)]

$$S = \pi\frac{m^2}{eE_0}g(\gamma) \xrightarrow[\text{Example: } eE(t) = eE_0\cos(\Omega t)]{} \begin{cases} \pi\dfrac{m^2}{eE_0} + O(\gamma^1) & \text{(slow limit } \gamma \ll 1) \\ \dfrac{2m}{\Omega}\ln\gamma^2 + \mathrm{const.} + O(\gamma^{-1}) & \text{(fast limit } \gamma \gg 1) \end{cases}$$

\# of photons

pert. power-dep.
after exponentiation

See, e.g., [Dunne, Gies, Schubert, Wang (2006)]
for an explicit expression for $g(\gamma)$

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See, e.g., [Dunne, Gies, Schubert, Wang (2006)] for an explicit expression for $g(\gamma)$

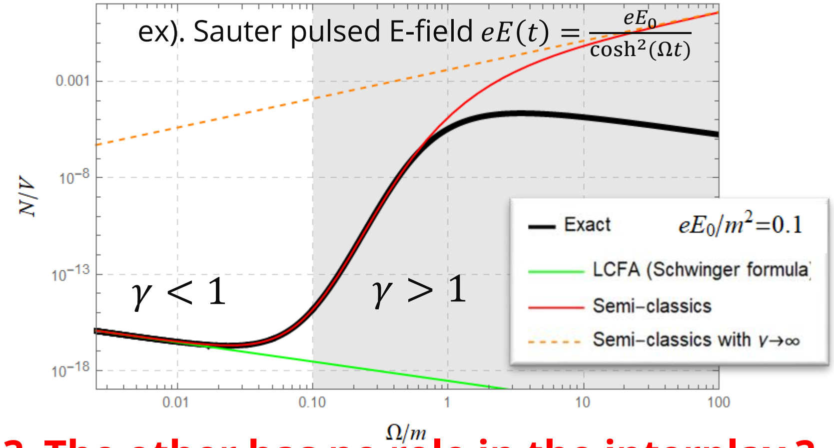
# of photons

pert. power-dep. after exponentiation

✔ However, you should not be convinced with the semi-classical argument

- Fast limit $\gamma \gg 1$ is dangerous

⇒ Q1: When are semi-classical methods really OK i.t.o. Ω ?


ex). Sauter pulsed E-field $eE(t) = \frac{eE_0}{\cosh^2(\Omega t)}$

- Another dimensionless parameter should exist

$\because$ 3 dimension**ful** parameters m, eE_0, Ω

⇒ 2 dimension**less** parameters **⇒ Q2: Why only γ ? The other has no role in the interplay ?**

Idea: One-photon process

- ✓ **One-photon process is the key to answer Q1 & Q2 !**

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✔ **One-photon process is the key to answer Q1 & Q2 !**

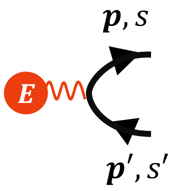
✔ **Larger Ω, more perturbative ⇒ one-photon dominates eventually**

- because there is no time to interact many times, no matter how strong eE_0 is

multiple scatterings

finite # of scatterings

- should cover different region, as photon # should be huge $\frac{2m}{\Omega} \to \infty$ in the semi-classics
- analytical formula is available (for any field config. !)

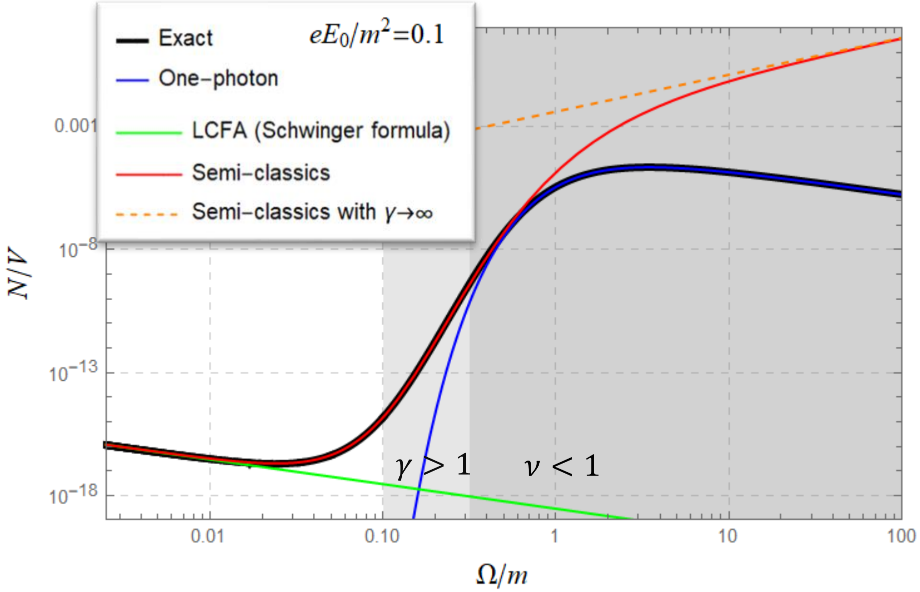


$$N_{1-\mathrm{photon}} = \sum_{s,s'} \int \mathrm{d}^3\boldsymbol{p}\,\mathrm{d}^3\boldsymbol{p}' \left| \quad \right|^2 = \frac{V}{(4\pi)^2} \int_{2m}^{\infty} \mathrm{d}\omega \sqrt{1 - \frac{4m^2}{\omega^2}}\, \frac{1}{3} \left(2 + \frac{4m^2}{\omega^2} \right) \left| e\tilde{E}(\omega) \right|^2$$

[HT, Fujii, Itakura (2014)] [QFT textbook by Itzykson, Zuber]

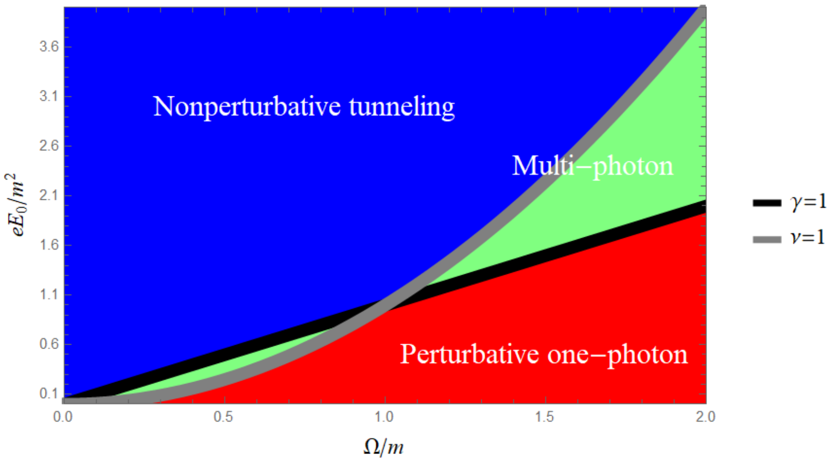
Result: Interplay b/w tunneling, multi-, one-photon

✔ **An explicit demonstration for Sauter pulsed E-field $eE(t) = \frac{eE_0}{\cosh^2(\Omega t)}$**



- **One-photon dominates for large Ω**, where semi-classical methods fail,
- **One-photon is more efficient** than tunneling ; N is the largest at $\Omega \sim 2m$

✔ **Two dimensionless parameters $\gamma = \frac{m\Omega}{eE_0}$, $\nu \equiv \frac{eE_0}{\Omega^2}$ control the interplay**



- Sauter field is solvable, which can be compared w/ Schwinger & one-photon

$$\gamma \gg 1,\ \ \nu \ll 1 \Rightarrow \text{ pert. one−photon}$$
$$\gamma \ll 1,\ \ \nu \gg 1 \Rightarrow \text{non−pert. tunneling}$$

- $\nu = \frac{eE_0/\Omega}{\Omega} = \frac{\text{(work done by E-field)}}{\text{(photon energy)}}$ $= \text{(\# of photons involved)}$

Message of Part I

(1) The interplay b/w non-pert. & pert. production mechanisms is controlled

by $\gamma = \frac{m\Omega}{eE}$ (Keldysh parameter) and also by $\nu = \frac{eE}{\Omega^2}$ (~ # of photons involved)

(2) Semi-classical methods (e.g., worldline) are dangerous for $\gamma \gg 1$;

It breaks down for $\nu \gtrsim 1$, where one-photon process dominates

(3) One-photon production is very efficient, compared to non-pert. tunneling

[HT, Fujii, Itakura, PRD (2014)]
[HT, Fujimori, Misumi, Nitta, Sakai, JHEP (2021)]

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Summary

Background (1/2): Dynamically assisted Schwinger mechanism

- ✔ In Part I, I discussed E-field having a single frequency mode Ω
- ✔ What if E-field is bi-frequent; superposition of slow strong + weak fast E-fields ?

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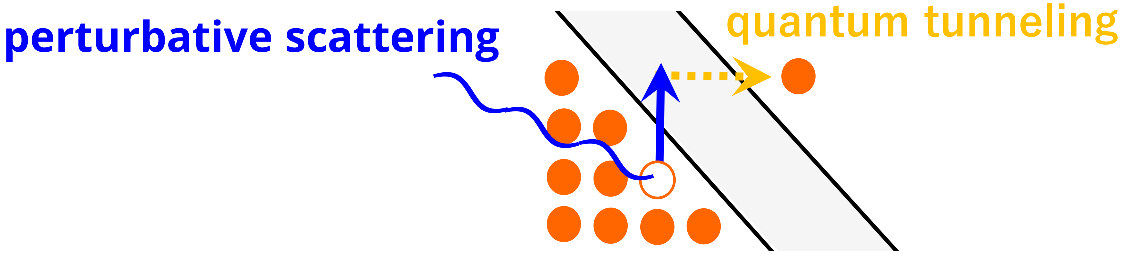
✔ **What if E-field is bi-frequent; superposition of slow strong + weak fast E-fields ?**

- Combinatorial effects of non-pert. & pert. mechs. ⇒ **dynamically assisted Schwinger mech.**

[Dunne, Gies, Schutzhold (2008), (2009)]
[Piazza et al (2009)] [Monin, Voloshin (2010)]



$N \sim \exp[-\# \times (\text{gap height}) \times (\text{gap length})]$ ⇒ **Enhancement in production**

(even though the fast field is very weak)

reduced by pert.

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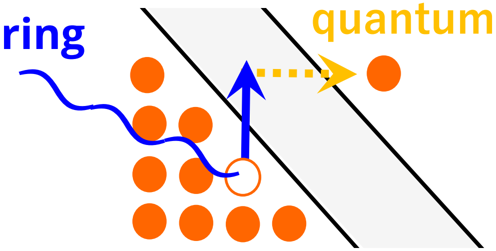


perturbative scattering

quantum tunneling

$$N \sim \exp[-\# \times (\text{gap height}) \times (\text{gap length})]$$ ⇒ **Enhancement in production**

reduced by pert.

(even though the fast field is very weak)

- Typically, analyzed within semi-classical methods [Dunne, Gies, Schutzhold (2008), (2009)]
- Phenomenological importance:

ex1) laser: available E-field is still weak ⇒ needs enhancement to observe the Schwinger mech

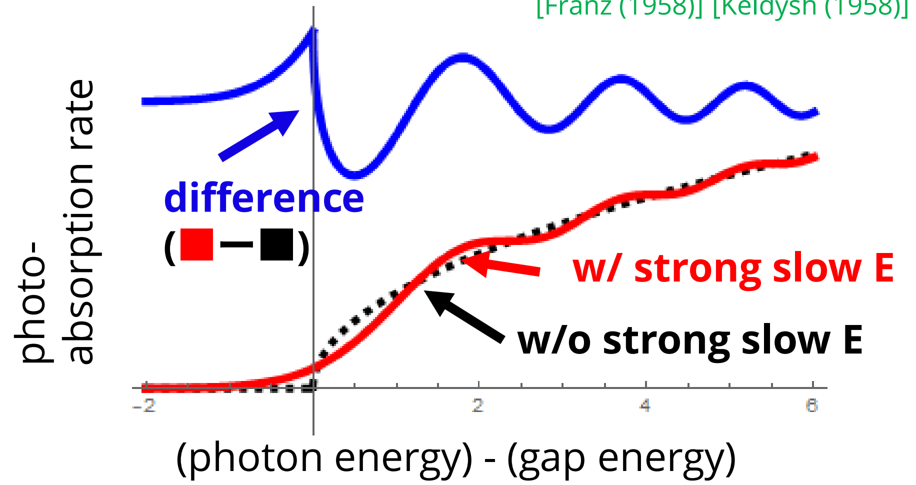
ex2) heavy-ion collisions: (Mini-)jets on top of glasma, Event generators (e.g., PYTHIA)

Background (2/2): Franz-Keldysh effect in cond-mat

✔ **Apply strong slow E-field & a photon (~ weak fast E-field) onto a semi-conductor, and measure photo-absorption rate**

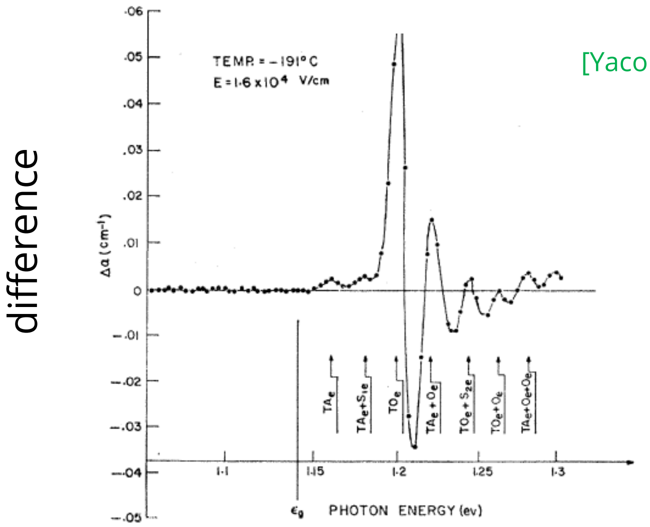
- photo-absorption rate ∼ Im[1-loop action] ∼ particle production rate

Theory



[Franz (1958)] [Keldysh (1958)]

Experiment (w/ Si)



[Yacoby (1966)]

- Enhancement below the mass gap

  ⇒ **Looks very similar to the dynamically assisted Schwinger mechanism (?)**

- Oscillation above the mass gap (Franz-Keldysh oscillation)
- Enhancement is maximized around the mass gap

What I am going to do

✔ Get a better understanding of the dynamical assistance by …

- Clarifying the relation to the Franz-Keldysh effect in cond-mat
- Establishing a novel analytical method, applicable for very fast E-field

⇐ Why such method needed ?

(1) Conventional semi-classical approaches are invalid for very fast E-field (result of Part I)

(2) Enhancement by pert. one-photon around $\Omega \sim 2m$ may be important (result of Part I)

(3) Such an enhancement is observed in Franz-Keldysh effect

(4) The Franz-Keldysh oscillation occurs for very fast E-field

- Revealing novel features, e.g., behavior at large Ω, spin generation, effective mass concept, …

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✔ Use "the perturbation theory in the Furry picture"

Goal: Evaluate $\frac{\mathrm{d}^3 N_s}{\mathrm{d}\boldsymbol{p}^3} = \langle \mathrm{vac} | \hat{a}^\dagger_{\boldsymbol{p},s} \, \hat{a}_{\boldsymbol{p},s} | \mathrm{vac} \rangle$ in the presence of strong slow E_s & weak fast $\mathcal{E}_\mathrm{f}$

Idea: Perturbative expansion $\hat{a}_{p,s}$ i.t.o. $\mathcal{E}_\mathrm{f}$, but no expansion i.t.o. E_s

[Furry (1951)] [Fradkin, Gitman, Shvartsman (1991)] [Torgrimsson, Schneider, Shutzhold (2017)]

Perturbation theory in Furry picture (1/3)

STEP 1

Separate the total E into strong slow E_S & weak fast $\mathcal{E}_\mathrm{f}$

$$E = E_\mathrm{s} + \mathcal{E}_\mathrm{f}$$

Perturbation theory in Furry picture (1/3)

STEP 1 **Separate the total E into strong slow E_s & weak fast $\mathcal{E}_f$**

$$\boldsymbol{E} = \boldsymbol{E}_{\mathbf{s}} + \boldsymbol{\mathcal{E}}_{\mathbf{f}}$$

STEP 2 **Solve Dirac eq. non-pert. w.r.t. E_s, but just pert. w.r.t. $\mathcal{E}_f$**

$$[i\slashed{\partial} - e\slashed{A}_{\mathrm{s}} - m]\hat{\psi} = e\slashed{\mathcal{A}}_{\mathrm{f}}\hat{\psi}$$

$$\Rightarrow \hat{\psi}(x) = \hat{\psi}^{(0)}(x) + \int_{-\infty}^{\infty} \mathrm{d}y^4 S_{\mathrm{R}}(x,y) e\slashed{\mathcal{A}}_{\mathrm{f}}\,(y)\hat{\psi}^{(0)}(y) + O(|e\mathcal{A}_{\mathrm{f}}|^2)$$

Here, $\hat{\psi}^{(0)}$ and S_{R} are **non-perturbatively dressed by** E_s as

$$[i\slashed{\partial} - e\slashed{A}_{\mathrm{s}} - m]\hat{\psi}^{(0)} = 0$$

$$[i\slashed{\partial} - e\slashed{A}_{\mathrm{s}} - m]S_{\mathrm{R}}(x,y) = \delta^4(x-y)$$

Perturbation theory in Furry picture (2/3)

STEP 3

Compute in/out annihilation operators $\hat{a}_{p,s}^{\mathrm{in/out}}$, $\hat{b}_{p,s}^{\mathrm{in/out}}$ from $\hat{\psi}$

$$\begin{pmatrix} \hat{a}_{\boldsymbol{p},s}^{\mathrm{in/out}} \\ \hat{b}_{-\boldsymbol{p},s}^{\mathrm{in/out}\dagger} \end{pmatrix} \equiv \lim_{t\to-\infty/+\infty} \int \mathrm{d}^3\boldsymbol{x} \begin{pmatrix} (u_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}})^\dagger \\ (v_{\boldsymbol{p},s}\mathrm{e}^{+i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}})^\dagger \end{pmatrix} \hat{\psi}(x)$$

Perturbation theory in Furry picture (2/3)

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⇒ $\hat{o}_{\boldsymbol{p},s}^{\text{in}}$, $\hat{o}_{\boldsymbol{p},s}^{\text{out}}$ **are inequivalent** $\hat{o}_{\boldsymbol{p},s}^{\text{in}} \neq \hat{o}_{\boldsymbol{p},s}^{\text{out}}$ and related with each other by a Bogoliubov transformation

$$\begin{pmatrix} \hat{a}_{\boldsymbol{p},s}^{\text{out}} \\ \hat{b}_{-\boldsymbol{p},s}^{\text{out}\dagger} \end{pmatrix} = \sum_{s'} \int \mathrm{d}^3\boldsymbol{p}' \begin{pmatrix} \alpha_{\boldsymbol{p},s;\boldsymbol{p}',s'} & \beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} \\ -\beta^*_{\boldsymbol{p},s;\boldsymbol{p}',s'} & \alpha^*_{\boldsymbol{p},s;\boldsymbol{p}',s'} \end{pmatrix} \begin{pmatrix} \hat{a}_{\boldsymbol{p}',s'}^{\text{in}} \\ \hat{b}_{-\boldsymbol{p}',s'}^{\text{in}\dagger} \end{pmatrix}$$

where, up to 1st order in $\boldsymbol{e\mathcal{A}_{\mathrm{f}}}$,

$$\alpha_{\boldsymbol{p},s;\boldsymbol{p}',s'} = \int \mathrm{d}^3\boldsymbol{x}\ {}_{+}\psi_{\boldsymbol{p},s}^{(0)\text{out}\dagger}\ {}_{+}\psi_{\boldsymbol{p}',s'}^{(0)\text{in}} - i\int d^4x\ {}_{+}\bar{\psi}_{\boldsymbol{p},s}^{(0)\text{out}} e\displaystyle{\not}\mathcal{A}_{\mathrm{f}}\ {}_{+}\psi_{\boldsymbol{p}',s'}^{(0)\text{in}} + O(|e\mathcal{A}_{\mathrm{f}}|^2)$$

$$\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} = \int \mathrm{d}^3\boldsymbol{x}\ {}_{-}\psi_{\boldsymbol{p},s}^{(0)\text{out}\dagger}\ {}_{+}\psi_{\boldsymbol{p}',s'}^{(0)\text{in}} - i\int d^4x\ {}_{-}\bar{\psi}_{\boldsymbol{p},s}^{(0)\text{out}} e\displaystyle{\not}\mathcal{A}_{\mathrm{f}}\ {}_{+}\psi_{\boldsymbol{p}',s'}^{(0)\text{in}} + O(|e\mathcal{A}_{\mathrm{f}}|^2)$$

Here, ${}_{\pm}\psi_{\boldsymbol{p},s}^{(0)\text{in/out}}$ are sol. of Dirac eq. **dressed by $\boldsymbol{eA_{\mathrm{s}}}$** w/ different B.C.

$$[i\displaystyle{\not}\partial - e\displaystyle{\not}A_{\mathrm{s}} - m]\ {}_{\pm}\psi_{\boldsymbol{p},s}^{(0)\text{in/out}} = 0 \quad \text{w/} \lim_{t\to-\infty/+\infty} \begin{pmatrix} {}_{+}\psi_{\boldsymbol{p},s}^{(0)\text{in/out}} \\ {}_{-}\psi_{\boldsymbol{p},s}^{(0)\text{in/out}} \end{pmatrix} = \begin{pmatrix} u_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}} \\ v_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}} \end{pmatrix}$$

Perturbation theory in Furry picture (3/3)

STEP 4 **Evaluate the in-vacuum expectation value of # operator**

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} \equiv \langle \mathrm{vac;in} | a^{\mathrm{out}\dagger}_{\boldsymbol{p},s} a^{\mathrm{out}}_{\boldsymbol{p},s} | \mathrm{vac;in} \rangle = \sum_{s\prime} \int \mathrm{d}^3\boldsymbol{p}' \left| \beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} \right|^2$$

Perturbation theory in Furry picture (3/3)

STEP 4 **Evaluate the in-vacuum expectation value of # operator**

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} \equiv \langle \mathrm{vac;in} | a^{\mathrm{out}\dagger}_{\boldsymbol{p},s} a^{\mathrm{out}}_{\boldsymbol{p},s} | \mathrm{vac;in} \rangle = \sum_{s'} \int \mathrm{d}^3\boldsymbol{p}' \left| \beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} \right|^2$$

Assume $\boldsymbol{E}_{\mathrm{s}}$ is so slow that it can be approximated as a static E-field

$\Rightarrow$ analytical sol. of Dirac eq. ${}_{\pm}\psi^{(0)\mathrm{in/out}}_{\boldsymbol{p},s}$ is known

$\Rightarrow$ one can evaluate $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$ **exactly !**

Perturbation theory in Furry picture (3/3)

STEP 4 **Evaluate the in-vacuum expectation value of # operator**

$$\frac{\mathrm{d}^3N_e}{\mathrm{d}\boldsymbol{p}^3} \equiv \langle \mathrm{vac;in}|a^{\mathrm{out}\dagger}_{\boldsymbol{p},s}a^{\mathrm{out}}_{\boldsymbol{p},s}|\mathrm{vac;in}\rangle = \sum_{s'}\int \mathrm{d}^3\boldsymbol{p}'\,|\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}|^2$$

Assume $\boldsymbol{E}_\mathrm{s}$ is so slow that it can be approximated as a static E-field

⇒ analytical sol. of Dirac eq. ${}_{\pm}\psi^{(0)\mathrm{in/out}}_{\boldsymbol{p},s}$ is known

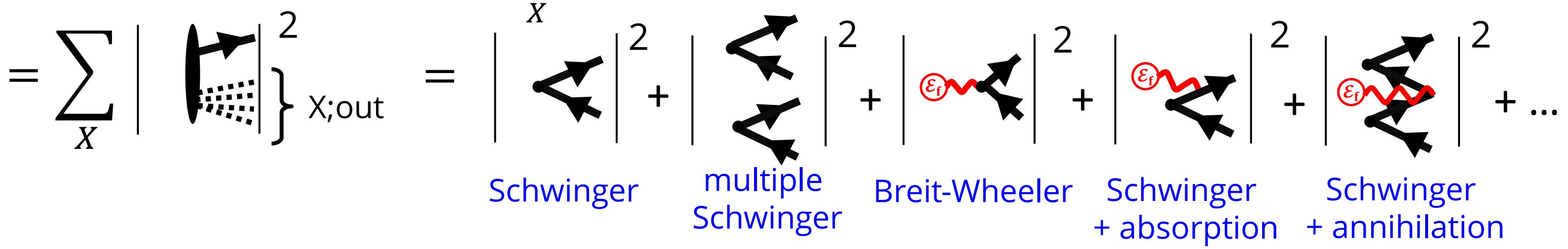
⇒ one can evaluate $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$ **exactly !**

✔ Remarks

- **Directly computing VEV of # operator** [Baltz, McLerran (2001)]

⇒ **inclusive quantity** that includes all the processes up to 1st order in $\mathcal{E}_\mathrm{f}$

$$\langle \mathrm{vac;in}|a^{\mathrm{out}\dagger}_{\boldsymbol{p},s}a^{\mathrm{out}}_{\boldsymbol{p},s}|\mathrm{vac;in}\rangle = \sum_X |\langle e_{\boldsymbol{p},s}X;\mathrm{out}|\mathrm{vac;in}\rangle|^2$$



- **No approximation in evaluating $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$** within 0-th order WKB [Torgrimsson et al (2017)]

Result (1/6): Analytical formula for parallel $E_{\rm s}$ || $\mathcal{E}_{\rm f}$

✔ **Analytical formula for $\boldsymbol{E} = (0,0,E_{\rm s} + \mathcal{E}_{\rm f}(t))$, with arbitrary time-dep.**

- is applicable even for very fast $\mathcal{E}_{\rm f}$ and reproduces numerics very well (show later)

$$\frac{{\rm d}^3 N_e}{{\rm d}\boldsymbol{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_{\rm s}}\right]$$

$$\times \left| 1 + \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}} \int_0^\infty d\omega \frac{\tilde{\mathcal{E}}_{\rm f}(\omega)}{E_{\rm s}} \exp\left[-\frac{i}{4}\frac{\omega^2+4\omega p_\parallel}{eE_{\rm s}}\right] {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}}; 2; \frac{i}{2}\frac{\omega^2}{eE_{\rm s}}\right) \right|^2$$

Result (1/6): Analytical formula for parallel $E_\mathrm{s} \,||\, \mathcal{E}_\mathrm{f}$

✔ **Analytical formula for $\boldsymbol{E} = (0,0,E_\mathrm{s} + \mathcal{E}_\mathrm{f}(t))$, with arbitrary time-dep.**

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$$\times \left| \boxed{1} + \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}} \int_0^\infty d\omega \frac{\tilde{\mathcal{E}}_\mathrm{f}(\omega)}{E_\mathrm{s}} \exp\left[-\frac{i}{4}\frac{\omega^2+4\omega p_\parallel}{eE_\mathrm{s}}\right] {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}}; 2; \frac{i}{2}\frac{\omega^2}{eE_\mathrm{s}}\right) \right|^2$$

Schwinger mech. by slow E_s

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$$\times\left|\boxed{1} + \boxed{\frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}\int_0^\infty d\omega\,\frac{\tilde{\mathcal{E}}_{\mathrm{f}}(\omega)}{E_{\mathrm{s}}}\exp\left[-\frac{i}{4}\frac{\omega^2+4\omega p_\parallel}{eE_{\mathrm{s}}}\right]{}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}};2;\frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)}\right|^2$$

Schwinger mech. by slow E_{s}

Dynamical assistance by fast $\mathcal{E}_{\mathrm{f}}$

- describes interplay b/w Schwinger & one-photon process smoothly:
  - Slow limit $\omega/\sqrt{eE_{\mathrm{s}}} \ll 1$: 🟦 dominates **⇒ Schwinger formula**

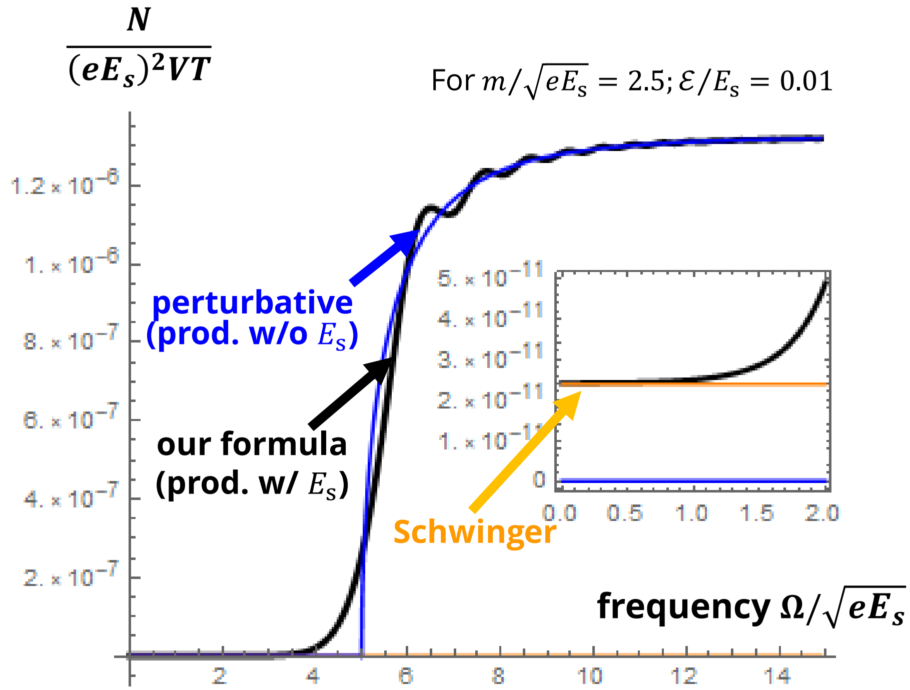
$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} \sim \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_{\mathrm{s}}}\right]\left|1+\frac{\pi}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}\frac{\mathcal{E}_{\mathrm{f}}}{E_{\mathrm{s}}}\right|^2 \sim \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{e(E_{\mathrm{s}}+\mathcal{E}_{\mathrm{f}})}\right]$$

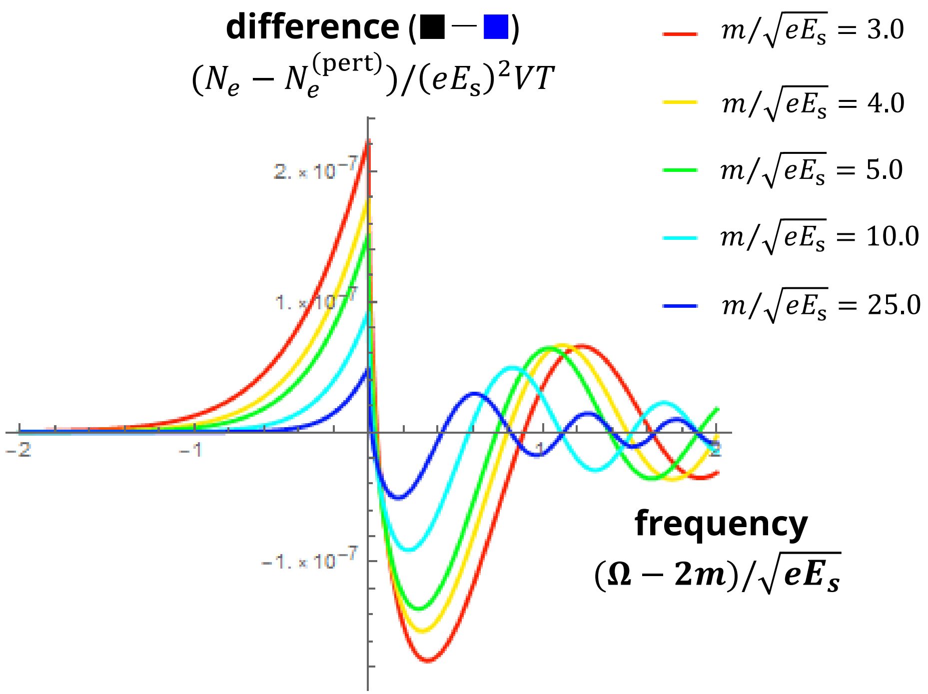
  - Fast limit $\omega/\sqrt{eE_{\mathrm{s}}} \gg 1$: 🟥 dominates **⇒ one-photon process**

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} \sim \frac{V}{(2\pi)^3}\frac{1}{4}\frac{m^2+\boldsymbol{p}_\perp^2}{\omega_{\boldsymbol{p}}^2}\frac{|e\tilde{\mathcal{E}}_{\mathrm{f}}(2\omega_{\boldsymbol{p}})|^2}{\omega_{\boldsymbol{p}}^2}$$

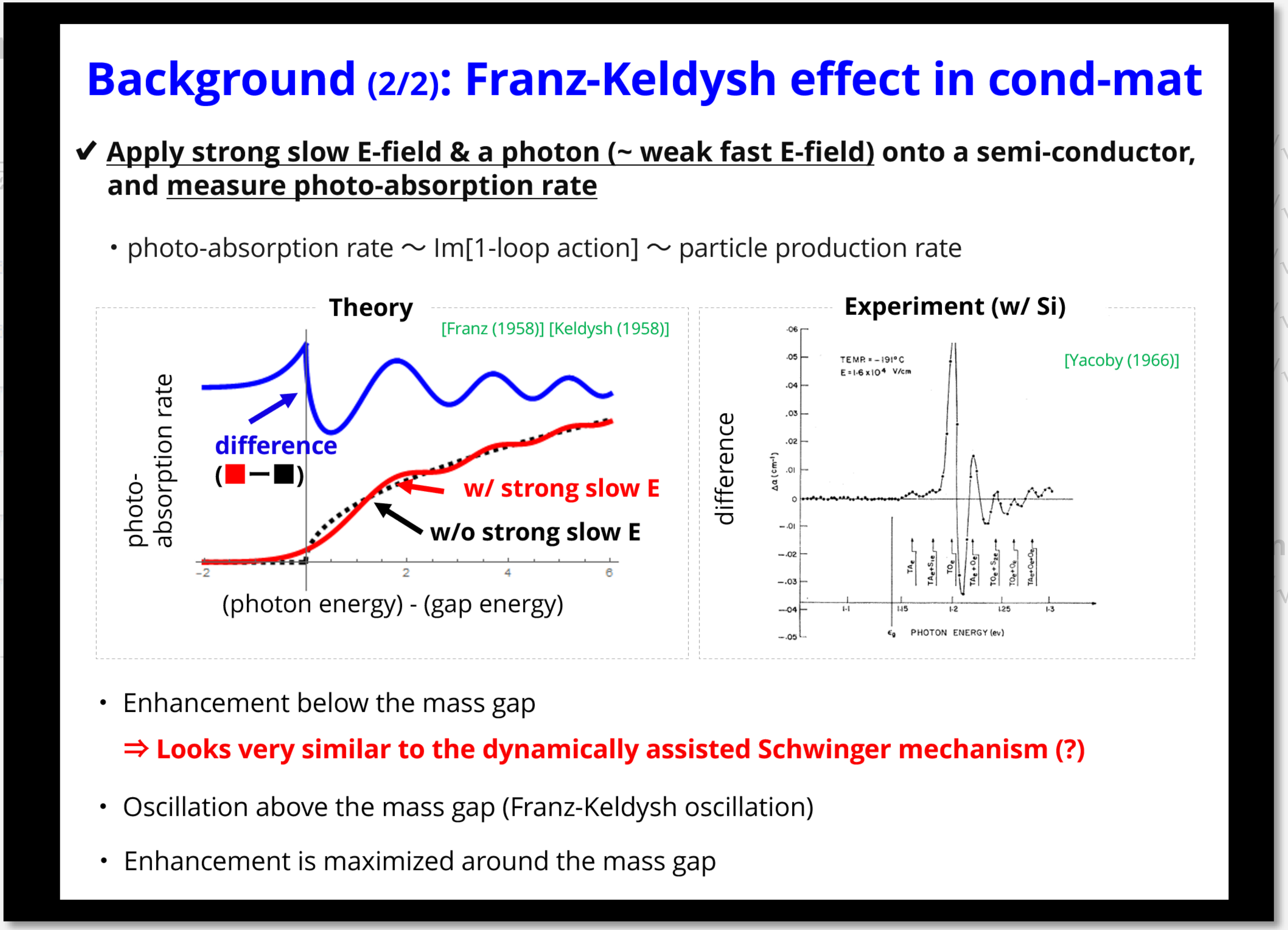
Result (2/6): Total N & Relation to Franz-Keldysh effect

✔ **For an oscillating perturbation $\mathcal{E}_{\mathrm{f}}(t) = \mathcal{E}\cos(\Omega t)$ as a demonstration**





Result (2/6): Total N & Relation to Franz-Keldysh effect



Background (2/2): Franz-Keldysh effect in cond-mat

✔ **<u>Apply strong slow E-field & a photon (~ weak fast E-field)</u> onto a semi-conductor, and <u>measure photo-absorption rate</u>**

- photo-absorption rate ∼ Im[1-loop action] ∼ particle production rate

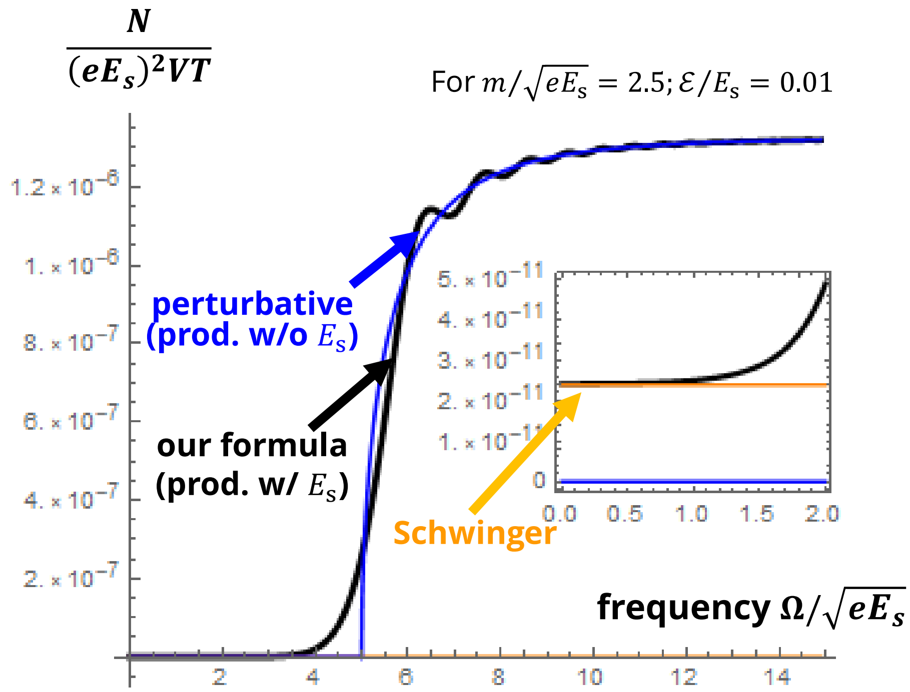
- Enhancement below the mass gap

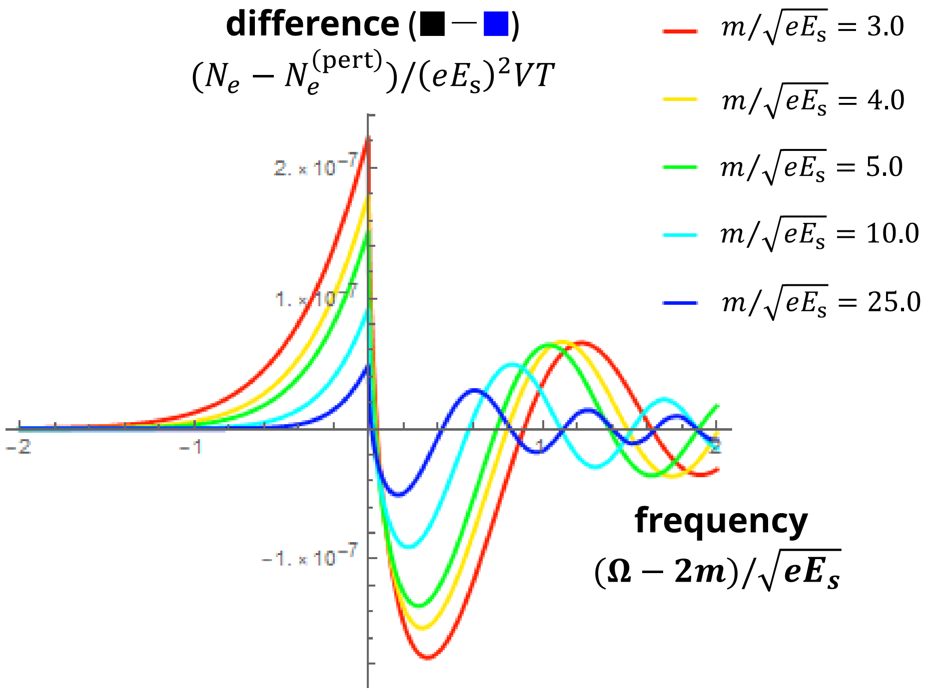
  ⇒ Looks very similar to the dynamically assisted Schwinger mechanism (?)

- Oscillation above the mass gap (Franz-Keldysh oscillation)
- Enhancement is maximized around the mass gap

Result (2/6): Total N & Relation to Franz-Keldysh effect

✔ **For an oscillating perturbation $\mathcal{E}_{\mathrm{f}}(t) = \mathcal{E}\cos(\Omega t)$ as a demonstration**





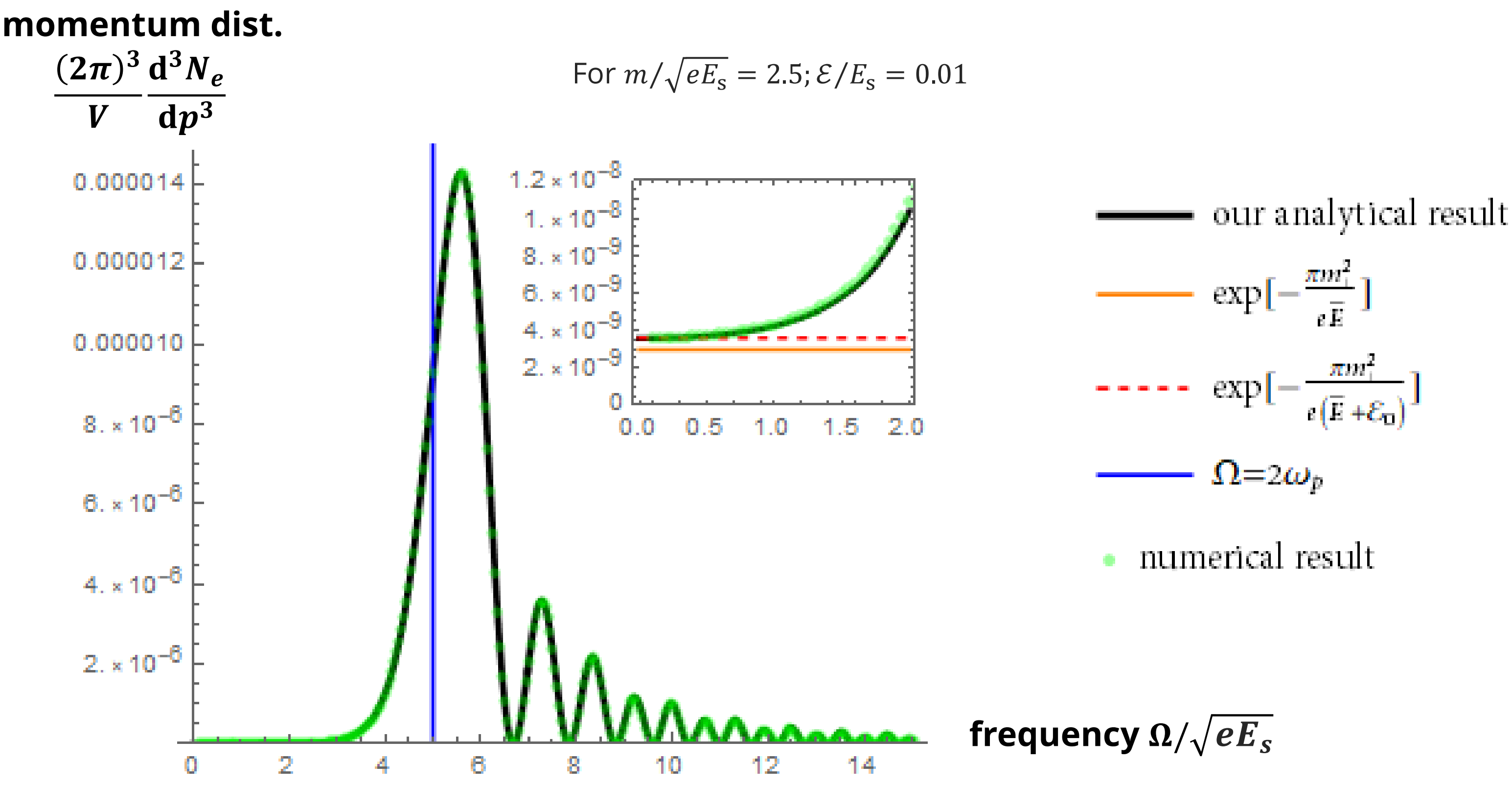
✔ **Completely the same as the Franz-Keldysh effect !**

- enhancement below the mass gap
- oscillation above the mass gap
- enhancement becomes the maximum at around the mass gap

⇒ dynamically assisted Schwinger mechanism = Franz-Keldysh effect

Result (3/6): Momentum dist. & comp. w/ numerics

✔ **Similar enhancement/oscillating behavior for the momentum dist.**



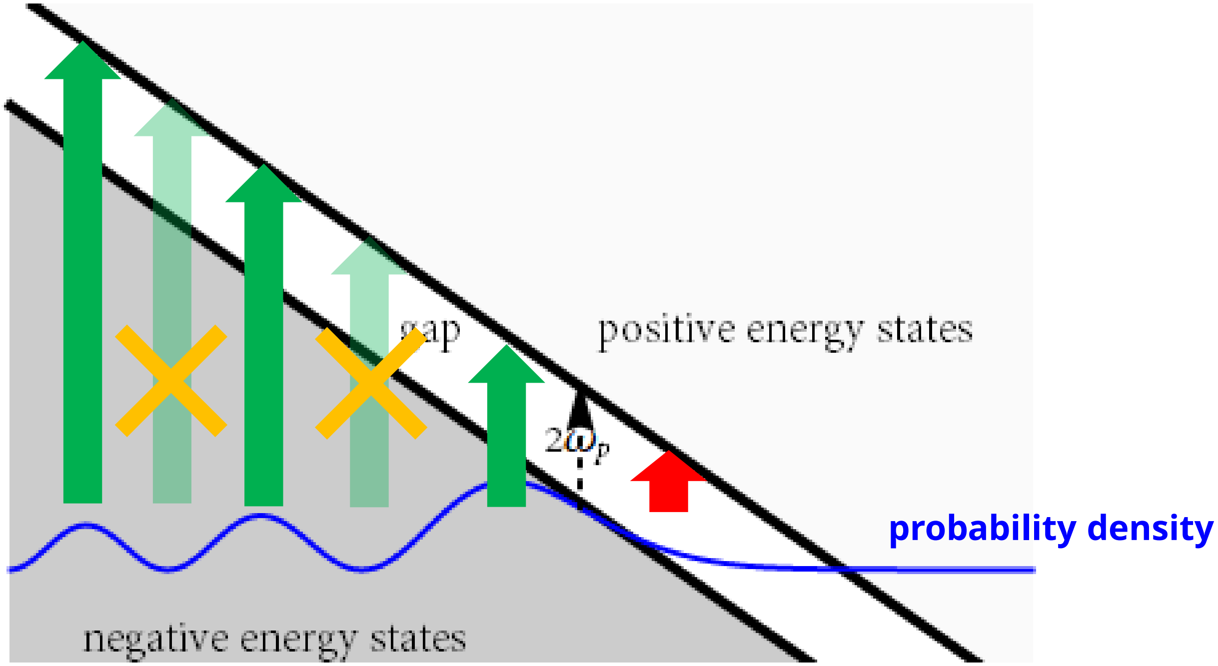
- **excellent agreement** b/w our analytical formula and the numerics

  ⇒ One-photon process it the essence of the dynamical assistance

- Another notable feature: the biggest peak (~ 1-photon peak) is above the gap $\Omega > 2\omega_p$

cf) effective mass shift in the multi-photon regime
[Kohlfurst, Gies, Alkofer (2014)]

Result (4/6): Physical interpretation



- **quantum tunneling ⇒ enhancement**
- **quantum reflection ⇒ oscillation**

⇒ production occurs most efficiently at the maxima

Result (5/6): Analytical formula for general $E_{\mathrm{s}} \nparallel \mathcal{E}_{\mathrm{f}}$

✔ **Generalization of the analytical formula for $E_{\mathrm{s}} \parallel \mathcal{E}_{\mathrm{f}} \rightarrow E_{\mathrm{s}} \nparallel \mathcal{E}_{\mathrm{f}}$**

$$
\begin{aligned}
\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_{\mathrm{s}}}\right] \\
\times \Bigg[\Bigg| \; 1 + \int_0^\infty d\omega \, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{1}{2} \frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}} \frac{\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{f}}(\omega)\cdot \boldsymbol{E}_{\mathrm{s}}}{E_{\mathrm{s}}^2} \mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}; 2; \frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right) \\
+ i\int_0^\infty d\omega \, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{f}}(\omega)\cdot \boldsymbol{p}_\perp}{E_{\mathrm{s}}\omega} \mathrm{Re}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}; 1; \frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)\right] \\
+ s \times i\int_0^\infty d\omega \, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{(\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{f}}(\omega)\times \boldsymbol{p}_\perp)\cdot \boldsymbol{E}_{\mathrm{s}}}{E_{\mathrm{s}}^2\omega} \mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}; 1; \frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)\right] \Bigg|^2 \\
+ \Bigg| \int_0^\infty d\omega \, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{m}{\omega} \frac{\tilde{\mathcal{E}}_{\mathrm{f}}^x(\omega) + is\tilde{\mathcal{E}}_{\mathrm{f}}^y(\omega)}{E_{\mathrm{s}}} \mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}; 1; \frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)\right] \Bigg|^2 \Bigg]
\end{aligned}
$$

- becomes complicated (green = new terms), but the basic structure is the same

Result (5/6): Analytical formula for general $E_{\mathrm{s}} \nparallel \mathcal{E}_{\mathrm{f}}$

✔ **Generalization of the analytical formula for $E_{\mathrm{s}} \parallel \mathcal{E}_{\mathrm{f}} \rightarrow E_{\mathrm{s}} \nparallel \mathcal{E}_{\mathrm{f}}$**

$$
\begin{aligned}
\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_{\mathrm{s}}}\right]
\times \Bigg[\Bigg| & \boxed{1} + \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}} \frac{\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{f}}(\omega)\cdot \boldsymbol{E}_{\mathrm{s}}}{E_{\mathrm{s}}^2} \mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}};2;\frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right) \\
& + i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{f}}(\omega)\cdot \boldsymbol{p}_\perp}{E_{\mathrm{s}}\omega} \mathrm{Re}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}};1;\frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)\right] \\
& + s\times i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{(\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{f}}(\omega)\times \boldsymbol{p}_\perp)\cdot \boldsymbol{E}_{\mathrm{s}}}{E_{\mathrm{s}}^2\omega} \mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}};1;\frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)\right] \Bigg|^2 \\
& + \left| \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{m}{\omega} \frac{\tilde{\mathcal{E}}_{\mathrm{f}}^x(\omega)+is\tilde{\mathcal{E}}_{\mathrm{f}}^y(\omega)}{E_{\mathrm{s}}} \mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}};1;\frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)\right] \right|^2 \Bigg]
\end{aligned}
$$

Schwinger mechanism by slow E_s

- becomes complicated (green = new terms), but the basic structure is the same

Result (5/6): Analytical formula for general $E_\mathrm{s} \nparallel \mathcal{E}_\mathrm{f}$

✔ **Generalization of the analytical formula for $E_\mathrm{s} \parallel \mathcal{E}_\mathrm{f} \rightarrow E_\mathrm{s} \nparallel \mathcal{E}_\mathrm{f}$**

Dynamical assistance by fast $\mathcal{E}_\mathrm{f}$

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_\mathrm{s}}\right]$$

$$\times \left[\left| 1 + \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_\mathrm{s}}} \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}} \frac{\widetilde{\boldsymbol{\mathcal{E}}}_\mathrm{f}(\omega)\cdot\boldsymbol{E}_\mathrm{s}}{E_\mathrm{s}^2} \mathrm{e}^{-i\frac{\omega^2}{4eE_\mathrm{s}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}};2;\frac{i}{2}\frac{\omega^2}{eE_\mathrm{s}}\right) \right.\right.$$

$$+i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_\mathrm{s}}} \frac{\widetilde{\boldsymbol{\mathcal{E}}}_\mathrm{f}(\omega)\cdot\boldsymbol{p}_\perp}{E_\mathrm{s}\omega} \mathrm{Re}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_\mathrm{s}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}};1;\frac{i}{2}\frac{\omega^2}{eE_\mathrm{s}}\right)\right]$$

$$\left. +s\times i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_\mathrm{s}}} \frac{(\widetilde{\boldsymbol{\mathcal{E}}}_\mathrm{f}(\omega)\times\boldsymbol{p}_\perp)\cdot\boldsymbol{E}_\mathrm{s}}{E_\mathrm{s}^2\omega} \mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_\mathrm{s}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}};1;\frac{i}{2}\frac{\omega^2}{eE_\mathrm{s}}\right)\right] \right|^2$$

$$\left. + \left| \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_\mathrm{s}}} \frac{m}{\omega} \frac{\tilde{\mathcal{E}}_\mathrm{f}^x(\omega)+is\tilde{\mathcal{E}}_\mathrm{f}^y(\omega)}{E_\mathrm{s}} \mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_\mathrm{s}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}};1;\frac{i}{2}\frac{\omega^2}{eE_\mathrm{s}}\right)\right] \right|^2 \right]$$

Schwinger mechanism by slow E_s

- becomes complicated (green = new terms), but the basic structure is the same

Result (5/6): Analytical formula for general $E_s \nparallel \mathcal{E}_f$

✔ **Generalization of the analytical formula for $E_s \parallel \mathcal{E}_f$ → $E_s \nparallel \mathcal{E}_f$**

Dynamical assistance by fast $\mathcal{E}_f$

Schwinger mechanism by slow E_s

$$\frac{d^3N_e}{d\boldsymbol{p}^3} = \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_s}\right]$$

$$\times\Bigg[\Bigg|\boxed{1} + \int_0^\infty d\omega\, e^{-i\frac{\omega p_\parallel}{eE_s}} \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s}\frac{\tilde{\boldsymbol{\mathcal{E}}}_f(\omega)\cdot\boldsymbol{E}_s}{E_s^2} e^{-i\frac{\omega^2}{4eE_s}}{}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};2;\frac{i}{2}\frac{\omega^2}{eE_s}\right)$$

$$+i\int_0^\infty d\omega\, e^{-i\frac{\omega p_\parallel}{eE_s}}\frac{\tilde{\boldsymbol{\mathcal{E}}}_f(\omega)\cdot\boldsymbol{p}_\perp}{E_s\omega}\mathrm{Re}\left[e^{-i\frac{\omega^2}{4eE_s}}\,{}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};1;\frac{i}{2}\frac{\omega^2}{eE_s}\right)\right]$$

$$+s\times i\int_0^\infty d\omega\, e^{-i\frac{\omega p_\parallel}{eE_s}}\frac{(\tilde{\boldsymbol{\mathcal{E}}}_f(\omega)\times\boldsymbol{p}_\perp)\cdot\boldsymbol{E}_s}{E_s^2\omega}\mathrm{Im}\left[e^{-i\frac{\omega^2}{4eE_s}}\,{}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};1;\frac{i}{2}\frac{\omega^2}{eE_s}\right)\right]\Bigg|^2$$

$$+\Bigg|\int_0^\infty d\omega\, e^{-i\frac{\omega p_\parallel}{eE_s}}\frac{m}{\omega}\frac{\tilde{\mathcal{E}}_f^x(\omega)+is\tilde{\mathcal{E}}_f^y(\omega)}{E_s}\mathrm{Im}\left[e^{-i\frac{\omega^2}{4eE_s}}\,{}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};1;\frac{i}{2}\frac{\omega^2}{eE_s}\right)\right]\Bigg|^2\Bigg]$$

- becomes complicated (green = new terms), but the basic structure is the same
- **spin-dependence appears even without magnetic fields** [Takayoshi, Wu, Oka (2020)]

∵ Dirac particle has a spin-orbit coupling $\boldsymbol{s}\cdot(\boldsymbol{p}\times\boldsymbol{E})$

[Foldy, Wouthuysen (1950)] [Tani (1951)]

- can be applied to rotating E-fields: $E = E_0(\cos(\Omega t), \sin(\Omega t), 0) \sim (\overset{E_s}{E_0}, \overset{\mathcal{E}_f}{E_0\Omega t}, 0)$

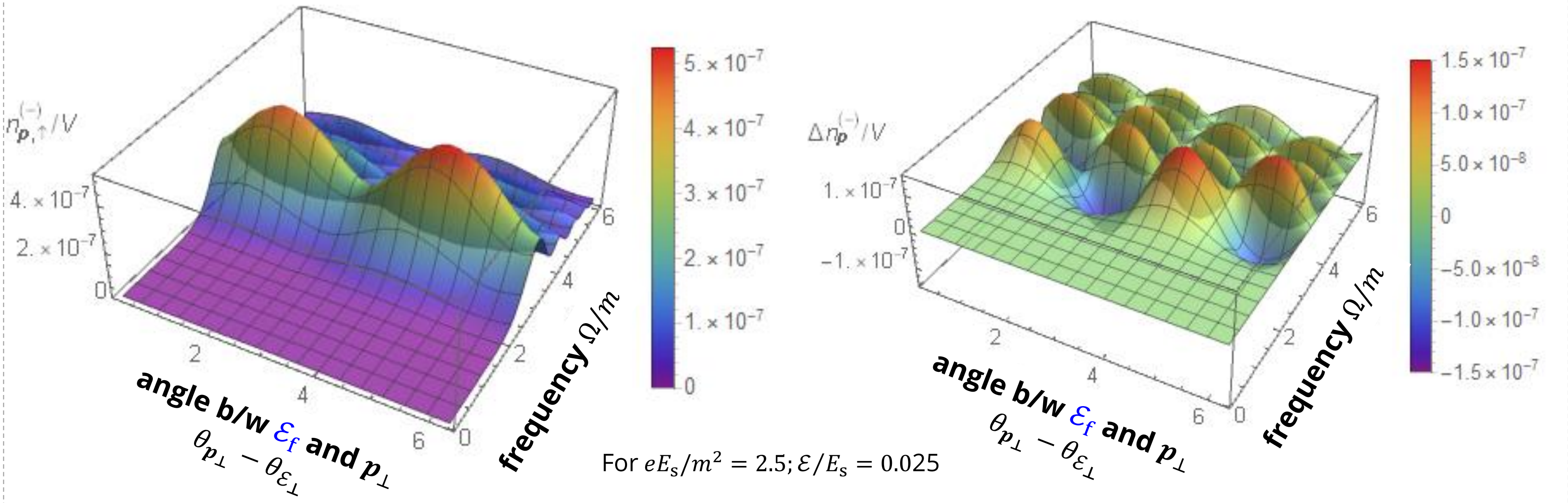
Numerical studies: [Blinne, Strobel (2015)] [Strobel, Xue (2015)] [Woller, Bauke, Keitel (2015)] [Kohlfurst (2019)]

Result (6/6): Spin-dependent production

✔ **Non-parallel superposition :** $\boldsymbol{E} = \begin{pmatrix} \mathcal{E}\cos\theta_{\mathcal{E}_\perp} \times \cos\Omega t \\ \mathcal{E}\sin\theta_{\mathcal{E}_\perp} \times \cos\Omega t \\ E_\mathrm{s} \end{pmatrix}$

momentum dist. of spin up $\frac{1}{V}\frac{\mathrm{d}^3 N_\uparrow}{\mathrm{d}\boldsymbol{p}^3}$

diff. b/w spin up and down $\frac{1}{V}\frac{\mathrm{d}^3 N_\uparrow}{\mathrm{d}\boldsymbol{p}^3} - \frac{1}{V}\frac{\mathrm{d}^3 N_\downarrow}{\mathrm{d}\boldsymbol{p}^3}$



For $eE_\mathrm{s}/m^2 = 2.5; \mathcal{E}/E_\mathrm{s} = 0.025$

- The dynamical assistance to N is basically the same as the parallel case
- Spin-dependence appears ⇒ O(10%) effect ⇒ non-negligible
- $\theta_{\boldsymbol{p}_\perp}$-dependent because of the spin-orbit interaction $\boldsymbol{s}\cdot(\boldsymbol{p}\times\boldsymbol{\mathcal{E}})$

Message of Part II

(1) The dynamically assisted Schwinger mechanism is an analogue of the Franz-Keldysh effect in cond-mat

(2) Perturbation theory in the Furry picture provides a very powerful analytical formula

(3) Not only the enhancement due to the quantum tunneling and one-photon assistance, but also oscillation appears due to the quantum reflection

(4) Spin-dependent production occurs when transverse E-field is time-dependent

[HT, PRD (2019)]
[Huang, HT, PRD (2019)]
[Huang, Matsuo, HT, PTEP (2019)]

Introduction

⇓

Part I:

Interplay b/w non-pert. & pert. production mechanisms

⇓

Part II:

Dynamically assisted Schwinger mechanism

⇓

Part III:

Dynamical assistance to chirality production

⇓

Summary

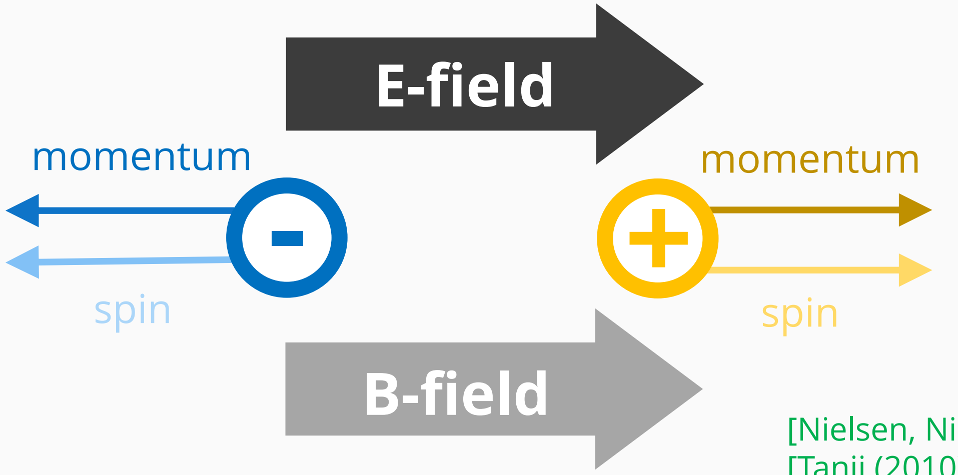
Background: Chirality production

- ✓ **Chirality is produced through anomaly when $\boldsymbol{E}\cdot\boldsymbol{B} \neq 0$**

Background: Chirality production

✔ Chirality is produced through anomaly when $\boldsymbol{E}\cdot\boldsymbol{B}\neq 0$

- Microscopically, caused by the interplay b/w Schwinger mech. by E-field & Landau quantization by B-field



⇒ chirality ~ helicity $= +2 \times N_{\textbf{pair in LLL}}$

[Nielsen, Ninomiya (1983)]
[Tanji (2010)]
[Fukushima, Kharzeev, Warringa (2010)]
[Copinger, Fukushima, Pu (2018)]
etc …

- $\boldsymbol{E}\cdot\boldsymbol{B}\neq 0$ may/can be realized at: heavy-ion collisions, early Universe, laser, …

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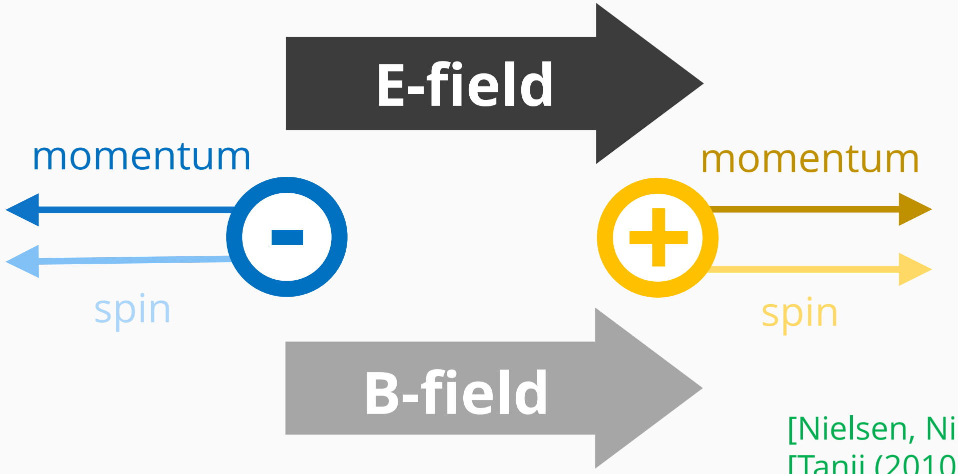

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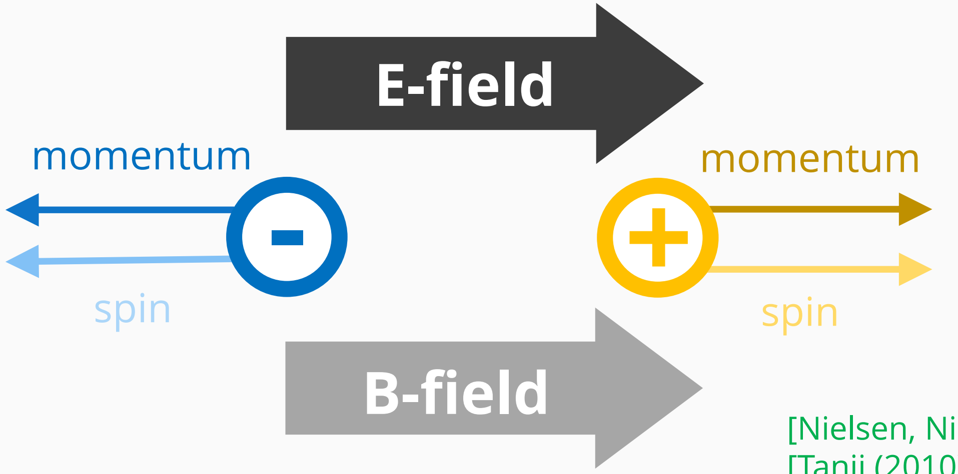

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Q: Any way to avoid the mass suppression ? ⇒ A: Dynamical assistance !

Theory: Perturbation theory in the Furry picture

✔ Extend the E-field case (Part II) to $E||B$ case

- Repeat essentially the same calculation, but with slight modifications:

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SETUP: **Parallel $\bar{\boldsymbol{E}}, \bar{\boldsymbol{B}}$ with perturbative weak fast E-field $\mathcal{E}$**

$$\boldsymbol{E}(t) = \bar{\boldsymbol{E}} + \mathcal{E}(t)$$
$$\boldsymbol{B}(t) = \bar{\boldsymbol{B}}$$

STEP 1: **Solve Dirac eq. under $\bar{\boldsymbol{E}}, \bar{\boldsymbol{B}}$ non-perturbatively, and include effects of $\mathcal{E}$ perturbatively**

$$[i\not{\partial} - \boldsymbol{e}\not{\bar{A}} - m]\hat{\psi} = \boldsymbol{e}\not{\mathcal{A}}\hat{\psi}$$

$$\Rightarrow \hat{\psi}(x) = \hat{\psi}^{(0)}(x) + \int_{-\infty}^{\infty} \mathrm{d}y^4 S(x,y)\boldsymbol{e}\not{\mathcal{A}}(y)\hat{\psi}^{(0)}(y) + O(|\boldsymbol{e}\mathcal{A}|^2)$$

STEP 2: **Compute VEV of chirality operator**

$$Q_5 \equiv \lim_{t\to\infty} \int \mathrm{d}\boldsymbol{x}^3 \left\langle \mathrm{vac;\,in} \middle| \hat{\bar{\psi}}\gamma^5\hat{\psi} \middle| \mathrm{vac;\,in} \right\rangle = O(1) + O(|\boldsymbol{e}\mathcal{A}|^1) + O(|\boldsymbol{e}\mathcal{A}|^2) + \cdots$$

Result (1/2): Analytical formula

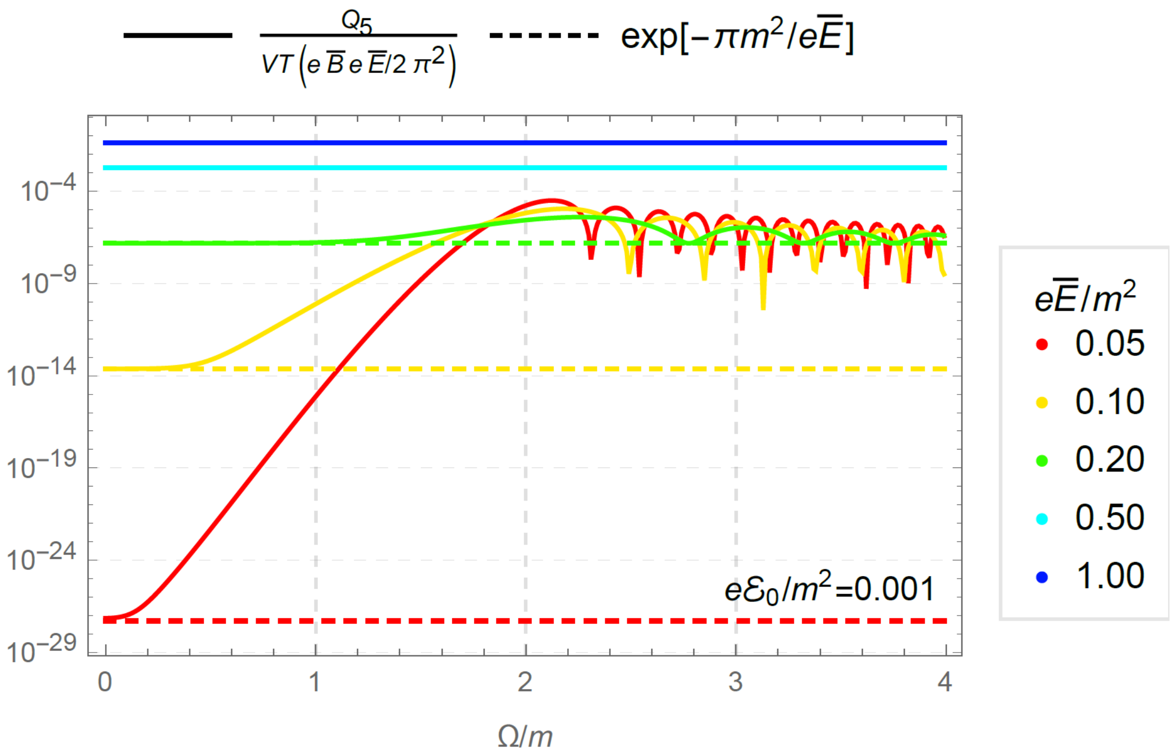
$$\frac{Q_5}{VT} = \frac{e\bar{E}\,e\bar{B}}{2\pi^2}\,\mathrm{e}^{-\pi\frac{m^2}{e\bar{E}}} \times \left[1 + \boxed{\frac{2\pi}{T}\left(\frac{m^2}{e\bar{E}}\right)^2 \int_0^\infty \mathrm{d}\,\omega \left|\frac{\tilde{\mathcal{E}}(\omega)}{\bar{E}}\,{}_1\tilde{F}_1\!\left(1 - \frac{\mathrm{i}}{2}\frac{m^2}{e\bar{E}}; 2; \frac{\mathrm{i}}{2}\frac{\omega^2}{e\bar{E}}\right)\right|^2}\,\right]$$

Dynamical assistance by fast E-field $\mathcal{E}$

- Dynamical assistance
  - is positive ⇒ **chirality is always enhanced**
  - goes away when $m \to 0$ ⇒ **important only for massive case**
  - is independent of B-field

    (∵ B-field does not supply energy, i.e., does not affect the prod. mech.)

Result (2/2): Plot for chirality production



- ✔ An oscillating pert. as demonstration: $\frac{eE(t)}{m^2} = \frac{e\bar{E}}{m^2} + 0.001\cos\Omega t \quad , \quad \frac{eB(t)}{m^2} = \frac{e\bar{B}}{m^2}$
- ✔ The same behavior as the dynamically assisted Schwinger mech.
  - Free from the exponential suppression due to the enhancement
  - Enhancement becomes largest around the mass gap
  - Oscillation above the mass gap

Message of Part III

Chirality production can be enhanced significantly via the dynamical assistance

[HT, PRR (2020)]

Introduction

⇓

Part I:

Interplay b/w non-pert. & pert. production mechanisms

⇓

Part II:

Dynamically assisted Schwinger mechanism

⇓

Part III:

Spin & chirality production

⇓

Summary

Summary

I discussed the Schwinger mechanism under time-depending E-field:

Part I: Interplay b/w non-pert. & pert. production mechanisms

- The interplay is controlled by $\gamma \equiv \frac{m\Omega}{eE}$ (Keldysh parameter) and also by $\nu \equiv \frac{eE}{\Omega^2}$
- Semi-classical methods are invalid when $\nu \gtrsim 1$, where one-photon process dominates
- One-photon production is very efficient, compared to non-pert. tunneling

Part II: Dynamically assisted Schwinger mechanism

- Point out relation to Franz-Keldysh effect in cond-mat.
- Analytical analysis based on perturbation theory in the Furry picture
- Not only enhancement due to quantum tunneling & one-photon assist,
  but also oscillation due to quantum reflection
- Spin-dependent production occurs when transverse E-field is time-dependent

Part III: Dynamical assistance to chirality production

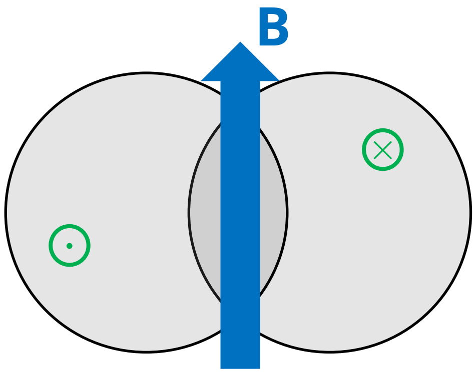
- Chirality production can be enhanced significantly via the dynamical assistance

BACKUP

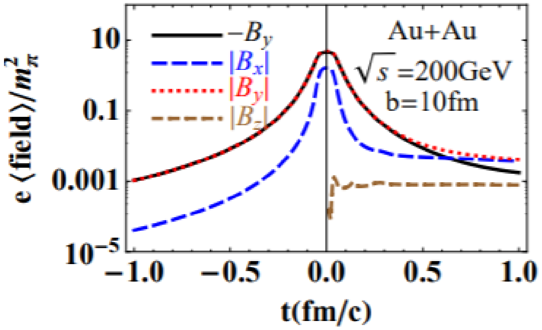
Strong EM fields in HIC

✔ **3 ways to produce (as far as I know)**

① (ultra)peripheral

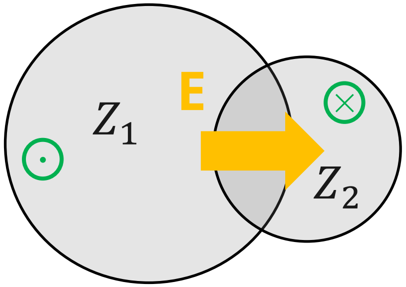


$$eB \sim \frac{\alpha Z v \gamma}{r^2} \sim \alpha Z \gamma \times m_\pi^2$$

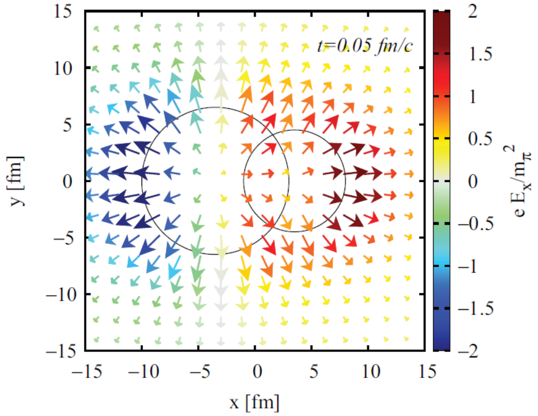


[Deng, Huang (2012)]

② asymmetric

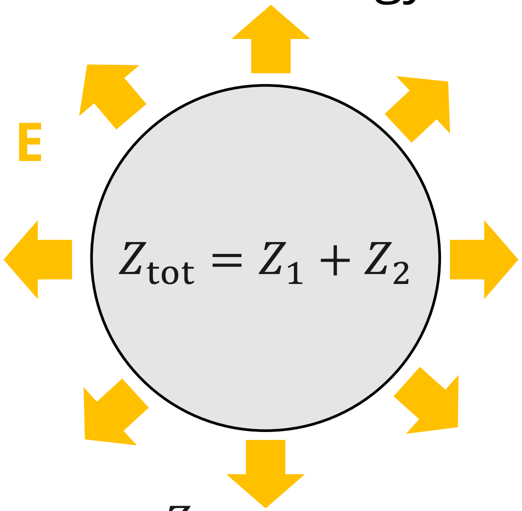


$$eE \sim \frac{\alpha(Z_1 - Z_2)\gamma}{r^2} \sim \alpha(Z_1 - Z_2)\gamma \times m_\pi^2$$

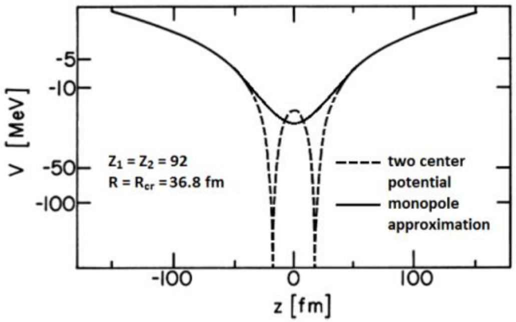


[Voronyuk, Toneev, Voloshin, Cassing (2014)]

③ low-energy



$$eE \sim \frac{\alpha Z_{\rm tot}}{r^2} \sim \alpha Z_{\rm tot} \times m_\pi^2$$



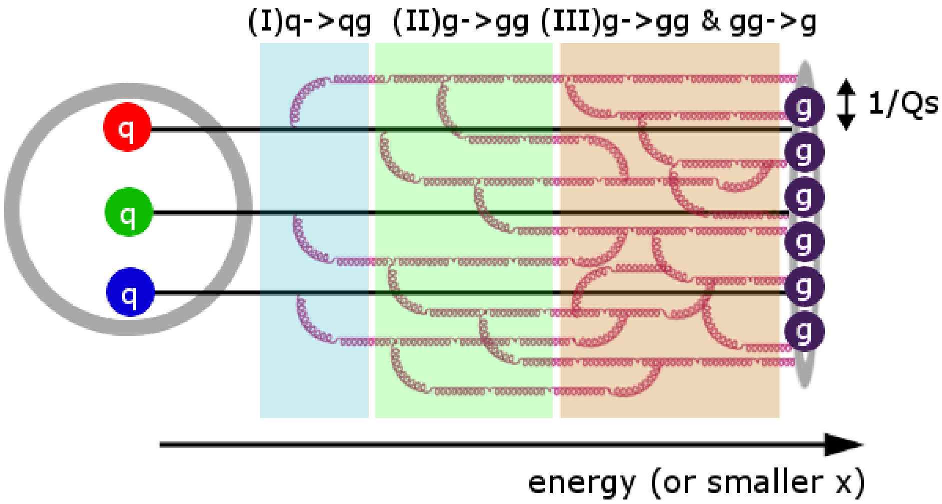
[Review: Rafelski, Kirsch, Muller, Reinhardt, Greiner (2014)]

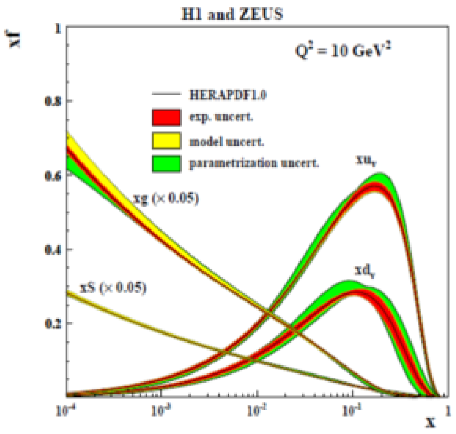
Glasma

✔ **Gluon saturation of ultra-relativistic nuclei**

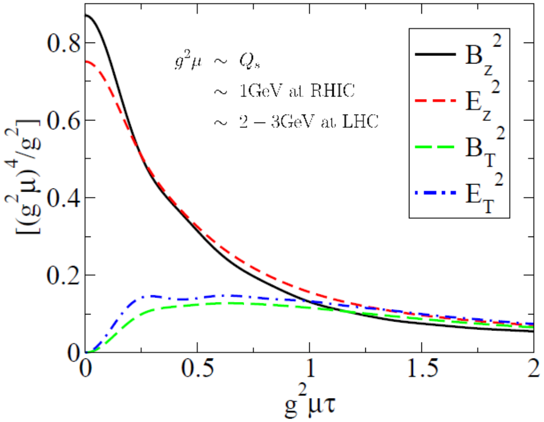
[McLerran, Venugopalan (1994)]

⇒ something like a “color capacitor”
w/ huge color charge density = $O(Q_s) = O(\text{a few GeV})$





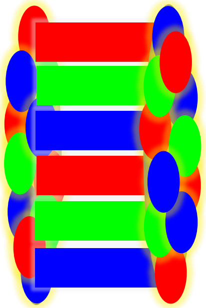
[Lappi, McLerran (2006)]



✔ **High-energy heavy-ion collisions**

≃ **formation of “color condenser”**

⇒ **strong color flux tubes**



Old ideas: [Low (1975)] [Nussinov (1975)] [Casher, Neuberger, Nussinov (1979)]