How time-dependent electric fields affect the Schwinger mechanism?

Hidetoshi TAYA

Keio U. → RIKEN (from this April)

What is the Schwinger mechanism?

[Sauter (1932)] [Heisenberg, Euler (1936)] [Schwinger (1951)]

✔ **Vacuum pair production occurs in the presence of strong fields**

Intuitive picture for a slow electric field = **quantum tunneling**

What is the Schwinger mechanism?

[Sauter (1932)] [Heisenberg, Euler (1936)] [Schwinger (1951)]

✔ Vacuum pair production occurs in the presence of strong fields

Intuitive picture for a slow electric field = **quantum tunneling**

✔ For constant strong E-field $E(t,x) = E$, it's understood well (at least, theoretically)

$$N_{e^\pm} = \frac{(eE)^2 VT}{(2\pi)^3} \times \exp\left[-\pi \frac{m^2}{eE}\right] \sim \exp[-\# \times (\text{gap height}) \times (\text{gap length})]$$

- Non-perturbative ⇒ Interesting, since it is the unexplored region of QED (or QFT)
- Vacuum process ⇒ Fundamental, since all the physical processes occur on top of vacuum
- However, exponentially suppressed i.t.o. mass ⇒ Not confirmed by experiments yet…

cf.) Guinness world record: [Yanovsky et al (2008)]

HERCULES laser $eE \sim \sqrt{10^{22}\ \mathrm{W/cm^2}} \sim (0.01\ \mathrm{MeV})^2 \ll m_e^2 \sim \sqrt{10^{29}\ \mathrm{W/cm^2}} \sim (0.5\ \mathrm{MeV})^2$

Timeliness

✔ **Now is the best time to study the Schwinger mechanism !**

Developments in experimental technologies ⇒ **Novel strong-field sources**

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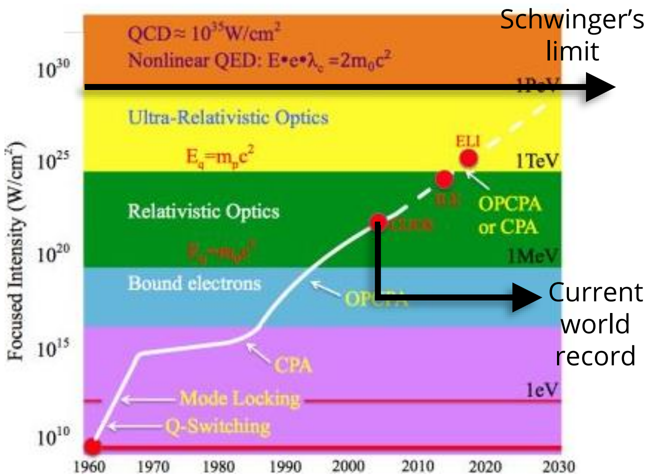
ex.1) Intense lasers



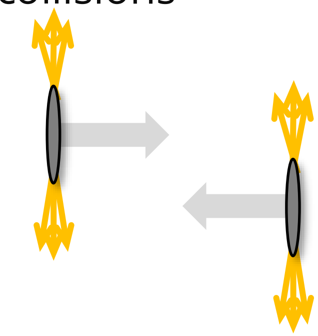
[Mourou, Tajima, Bulanov (2006)]

ex.2) Heavy-ion collisions

- (ultra)peripheral collisions

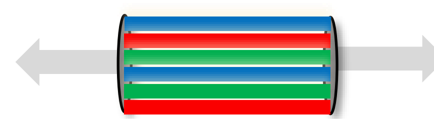
$$eE, eB \sim \frac{\alpha Z v \gamma}{r^2} \sim \alpha Z \gamma \times m_\pi^2$$

[Skokov, Illarionov, Toneev (2009)]
[Deng, Huang (2012)]



- glasma (color flux tube)

$$gE_{\text{color}}, gB_{\text{color}} \sim Q_s^2 \sim O(1)\ \text{GeV}^2$$



[Lappi, McLerran (2006)]

- Also in other geometries:
  - asymmetric coll. [Hirono, Hongo, Hirano (2014)] [Voronyuk, et al (2014)]
  - low-energy coll. [Review: Rafelski, et al (2014)] [Maltsev et al (2019)]

[Allor, Cohen, McGady (2008)] [Solinas, Amoretti, Giazotto (2021)]

ex.3) Cond-mat analogues: Graphene, Cold atom, Superconductor, Semiconductor, …

[Szpak, Shutzhold (2012)] [Thesis by Linder; 1807.08050]

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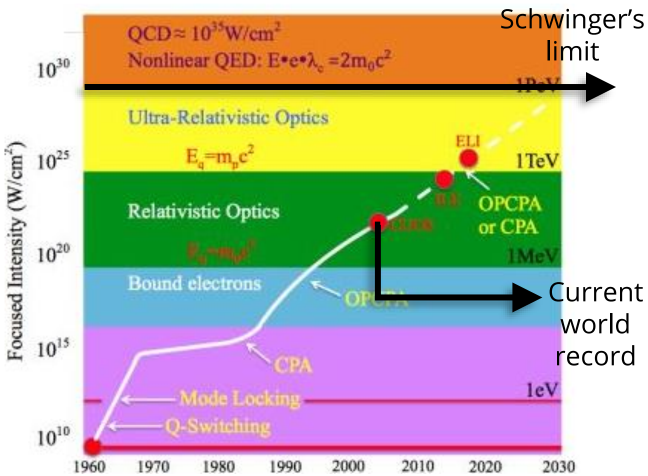

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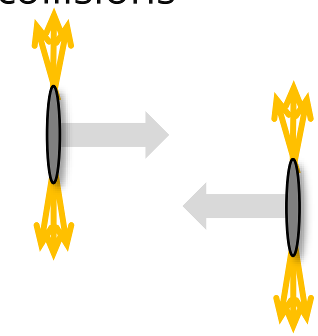

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✔ Schwinger's result is insufficient for actual problems ⇒ **Beyond Schwinger !**

- Inhomogeneous field (e.g., time- & space-dependence, more efficient field config., …)
- Realtime dynamics (e.g., backreaction, intermediate particle number, …)
- Higher loop effects (e.g., radiation, mass shift, …)
- Other observables (e.g., spin, chirality, high-harmonic generation…)
- …

Today's talk

Discuss the Schwinger mech. under time-depending E-field

Part I: Interplay b/w non-pert. & pert. production mechanisms

[HT, Fujii, Itakura, PRD (2014)]
[HT et al., JHEP (2021)]

Part II: Dynamically assisted Schwinger mechanism

[HT, PRD (2019)]
[Huang, HT, PRD (2019)]
[Huang, Matsuo, HT, PTEP (2019)]

Part III: Dynamical assistance to chirality production [HT, PRR (2020)]

Introduction

$\Downarrow$

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Part II:

Dynamically assisted Schwinger mechanism

$\Downarrow$

Part III:

Dynamical assistance to chirality production

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Summary

Background (1/2): Interplay b/w non-pert. & pert. mech.

✔ Consider time-dependent E-field, having strength eE_0 and frequency Ω

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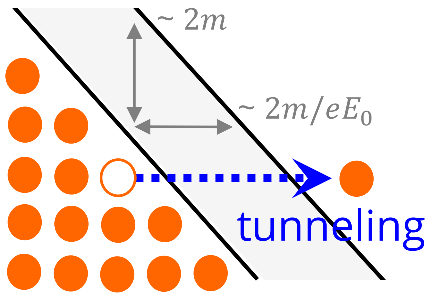
✔ **Consider time-dependent E-field, having strength eE_0 and frequency Ω**

- The tunneling picture should break down if the frequency Ω is large

Non-pert. tunneling $N \sim \exp[\#/eE_0]$



Tunneling time $\Delta t \sim \frac{2m}{eE_0}$

$\Rightarrow$ E-field should be slower than Δt

$\Rightarrow \Omega^{-1} \gtrsim \Delta t$

$\Rightarrow 1 \gtrsim \frac{\Delta t}{\Omega^{-1}} = \frac{\Omega m}{eE_0} \equiv \gamma$ (Keldysh parameter)

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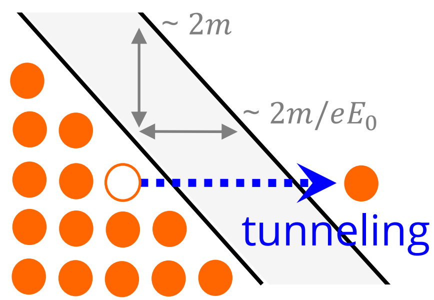

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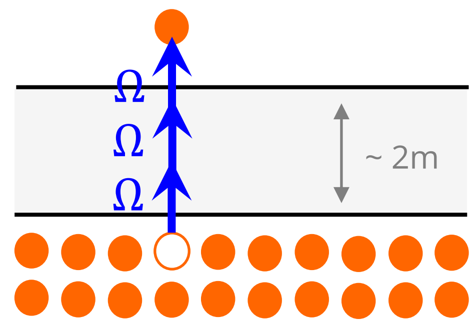
Pert. multi-photon $N \sim eE_0^{2n}$



Pair production when $n\Omega > 2m$

- For large Ω, E-field may behave like a photon and interact incoherently (perturbatively)

Background (2/2): Semi-classical analysis

✔ The interplay has been "confirmed" by (but only by) semi-classical analysis

- Trans-series expansion in terms of $\hbar$

$$N = \sum_{n,m} N_{n,m} \hbar^n \mathrm{e}^{-mS/\hbar} = (N_{0,1} + O(\hbar))\, \mathbf{e}^{-S/\hbar} + O(\mathrm{e}^{-2S/\hbar})$$

- Steepest descent analysis [Brezin, Itzykson (1970)]
- Imaginary-time method [Popov (1972)]
- Divergent asymptotic series method [Berry (1989)]
- Worldline instanton method [Dunne, Shubert (2005)]
- (exact) WKB [HT, Fujimori, Misumi, Nitta, Sakai (2020)]
- …

- Valid in the slow limit $\hbar \ll 1 \Leftrightarrow \Omega \ll 1$: $\because\ \mathrm{i}\hbar\partial_t\psi = H(\Omega t)\psi \xrightarrow[\tau \equiv t/\hbar]{} \mathrm{i}\partial_\tau\psi = H(\hbar\Omega\tau)\psi$

- Production number N is controlled solely by the Keldysh parameter $\gamma \equiv \dfrac{m\Omega}{eE_0}$ [Keldysh (1965)]

$$S = \pi \frac{m^2}{eE_0} g(\gamma) \xrightarrow[\text{Example: } eE(t) = eE_0\cos(\Omega t)]{} \begin{cases} \pi \dfrac{m^2}{eE_0} + O(\gamma^1) & \text{(slow limit } \gamma \ll 1) \\ \dfrac{2m}{\Omega} \ln\gamma^2 + \text{const.} + O(\gamma^{-1}) & \text{(fast limit } \gamma \gg 1) \end{cases}$$

See, e.g., [Dunne, Gies, Schubert, Wang (2006)] for an explicit expression for $g(\gamma)$

# of photons

pert. power-dep. after exponentiation

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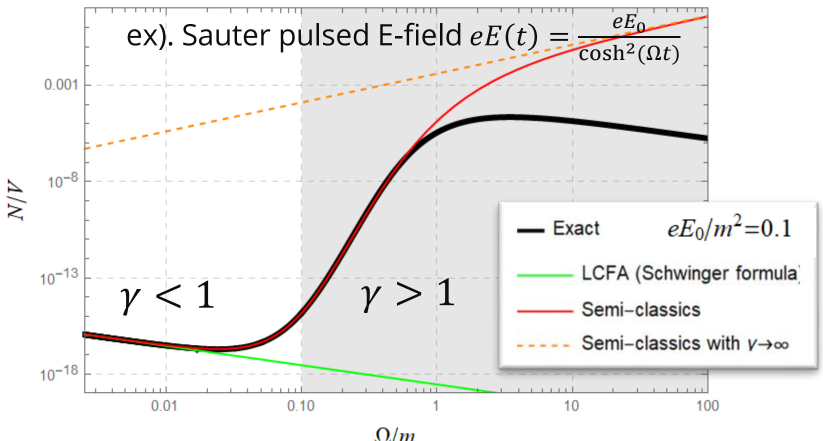
✔ However, you should not be convinced with the semi-classical argument

- Fast limit $\gamma \gg 1$ is dangerous

⇒ Q1: When are semi-classical methods really OK i.t.o. Ω ?



- Another dimensionless parameter should exist

$\because$ 3 dimensional**ful** parameters m, eE_0, Ω

⇒ 2 dimension**less** parameters **⇒ Q2: Why only γ ? The other has no role in the interplay ?**

Idea: One-photon process

✔ **One-photon process is the key to answer Q1 & Q2 !**

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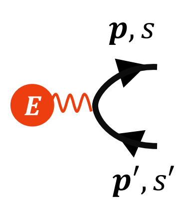
✔ **Larger Ω, more perturbative ⇒ one-photon dominates eventually**

- because there is no time to interact many times, no matter how strong eE_0 is

multiple scatterings — finite # of scatterings

- should cover different region, as photon # should be huge $\frac{2m}{\Omega} \to \infty$ in the semi-classics
- analytical formula is available (for any field config. !)

$$N_{1-\text{photon}} = \sum_{s,s'} \int \mathrm{d}^3\boldsymbol{p}\, \mathrm{d}^3\boldsymbol{p}' \left| \vphantom{\int} \right.$$

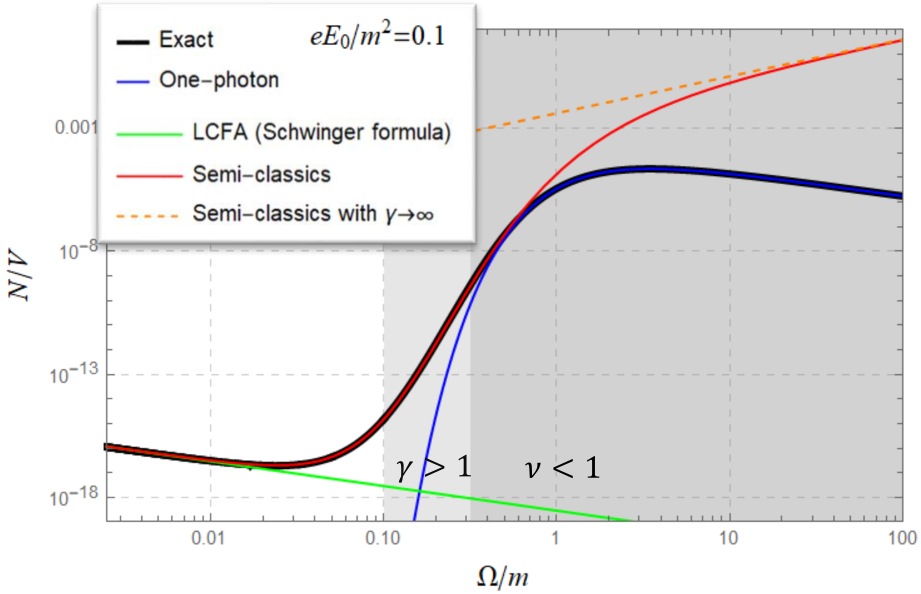


$$\left. \vphantom{\int} \right|^2 = \frac{V}{(4\pi)^2} \int_{2m}^{\infty} \mathrm{d}\omega \sqrt{1 - \frac{4m^2}{\omega^2}} \frac{1}{3} \left(2 + \frac{4m^2}{\omega^2}\right) \left| e\tilde{E}(\omega) \right|^2$$

[HT, Fujii, Itakura (2014)] [QFT textbook by Itzykson, Zuber]

Result: Interplay b/w tunneling, multi-, one-photon

✔ **An explicit demonstration for Sauter pulsed E-field** $eE(t) = \frac{eE_0}{\cosh^2(\Omega t)}$



- **One-photon dominates for large Ω**, where semi-classical methods fail,
- **One-photon is more efficient** than tunneling ; N is the largest at $\Omega \sim 2m$

✔ **Two dimensionless parameters** $\gamma = \frac{m\Omega}{eE_0}$, $\nu \equiv \frac{eE_0}{\Omega^2}$ **control the interplay**

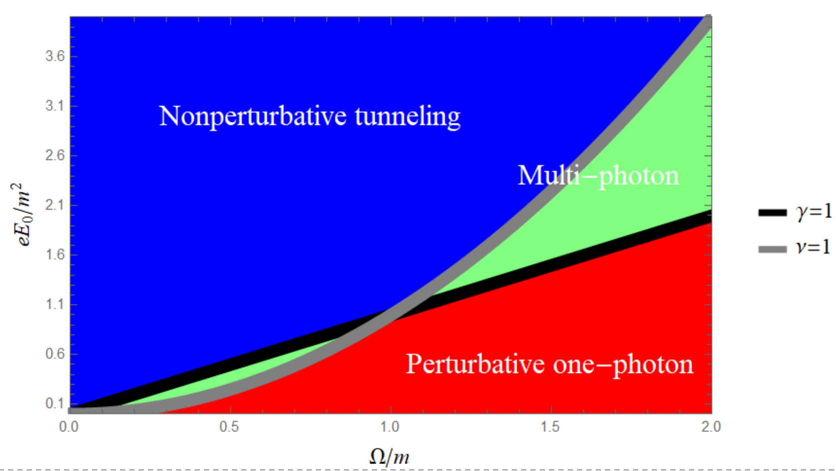
- Sauter field is solvable, which can be compared w/ Schwinger & one-photon

$\gamma \gg 1,\ \nu \ll 1 \Rightarrow$ pert. one−photon

$\gamma \ll 1,\ \nu \gg 1 \Rightarrow$ non−pert. tunneling

- $\nu = \frac{eE_0/\Omega}{\Omega} = \frac{\text{(work done by E-field)}}{\text{(photon energy)}}$

$= \text{(\# of photons involved)}$



Message of Part I

(1) The interplay b/w non-pert. & pert. production mechanisms is controlled

by $\gamma = \frac{m\Omega}{eE}$ (Keldysh parameter) and also by $\nu = \frac{eE}{\Omega^2}$ (~ # of photons involved)

(2) Semi-classical methods (e.g., worldline) are dangerous for $\gamma \gg 1$;

It breaks down for $\nu \gtrsim 1$, where one-photon process dominates

(3) One-photon production is very efficient, compared to non-pert. tunneling

[HT, Fujii, Itakura, PRD (2014)]
[HT, Fujimori, Misumi, Nitta, Sakai, JHEP (2021)]

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Summary

Background (1/2): Dynamically assisted Schwinger mechanism

- ✔ **In Part I, I discussed E-field having a single frequency mode Ω**
- ✔ **What if E-field is bi-frequent; superposition of slow strong + weak fast E-fields ?**

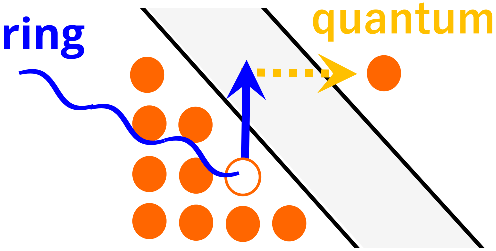
Background (1/2): Dynamically assisted Schwinger mechanism

✔ **In Part I, I discussed E-field having a single frequency mode Ω**

✔ **What if E-field is bi-frequent; superposition of slow strong + weak fast E-fields ?**

- Combinatorial effects of non-pert. & pert. mechs. ⇒ **dynamically assisted Schwinger mech.**

[Dunne, Gies, Schutzhold (2008), (2009)]
[Piazza et al (2009)] [Monin, Voloshin (2010)]



$$N \sim \exp[-\# \times (\text{gap height}) \times (\text{gap length})]$$ ⇒ **Enhancement in production**

(even though the fast field is very weak)

reduced by pert.

- Typically, analyzed within semi-classical methods [Dunne, Gies, Schutzhold (2008), (2009)]
- Phenomenological importance:

ex1) laser: available E-field is still weak ⇒ needs enhancement to observe the Schwinger mech

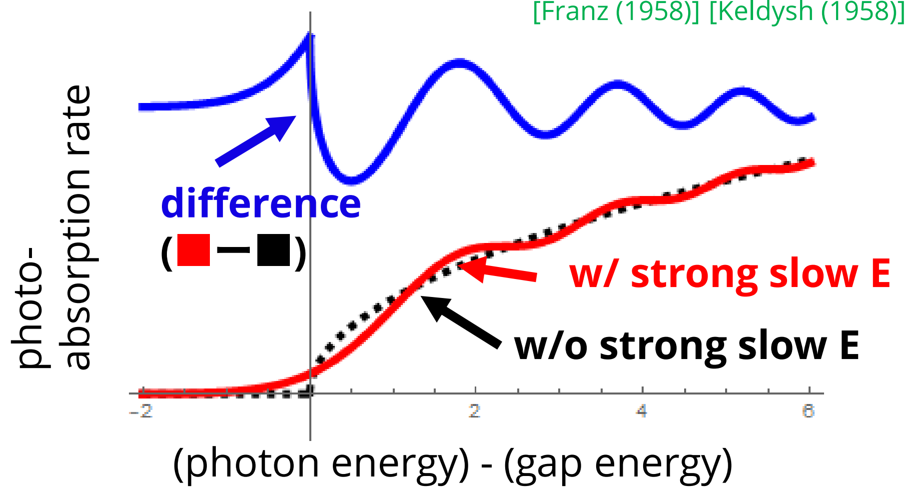
ex2) heavy-ion collisions: (Mini-)jets on top of glasma, Event generators (e.g., PYTHIA)

Background (2/2): Franz-Keldysh effect in cond-mat

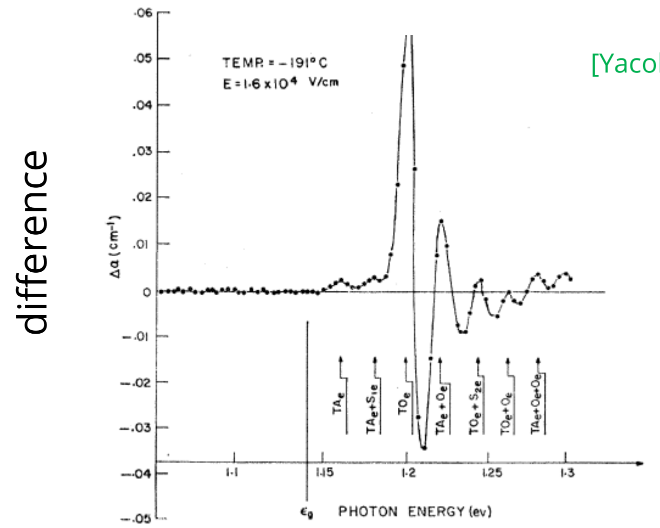
✔ **Apply strong slow E-field & a photon (~ weak fast E-field) onto a semi-conductor, and measure photo-absorption rate**

- photo-absorption rate ∼ Im[1-loop action] ∼ particle production rate

Theory [Franz (1958)] [Keldysh (1958)]



Experiment (w/ Si) [Yacoby (1966)]



- Enhancement below the mass gap

  ⇒ **Looks very similar to the dynamically assisted Schwinger mechanism (?)**

- Oscillation above the mass gap (Franz-Keldysh oscillation)
- Enhancement is maximized around the mass gap

What I am going to do

✔ **Get a better understanding of the dynamical assistance by …**

- Clarifying the relation to the Franz-Keldysh effect in cond-mat
- Establishing a novel analytical method, applicable for very fast E-field

$\Leftarrow$ Why such method needed ?

(1) Conventional semi-classical approaches are invalid for very fast E-field (result of Part I)

(2) Enhancement by pert. one-photon around $\Omega \sim 2m$ may be important (result of Part I)

(3) Such an enhancement is observed in Franz-Keldysh effect

(4) The Franz-Keldysh oscillation occurs for very fast E-field

- Revealing novel features, e.g., behavior at large Ω, spin generation, effective mass concept, …

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✔ Use "the perturbation theory in the Furry picture"

Goal: Evaluate $\frac{\mathrm{d}^3 N_s}{\mathrm{d}\boldsymbol{p}^3} = \langle \mathrm{vac} | \hat{a}^\dagger_{\boldsymbol{p},s}\, \hat{a}_{\boldsymbol{p},s} | \mathrm{vac} \rangle$ in the presence of strong slow E_s & weak fast $\mathcal{E}_\mathrm{f}$

Idea: Perturbative expansion $\hat{a}_{p,s}$ i.t.o. $\mathcal{E}_\mathrm{f}$, but no expansion i.t.o. E_s

[Furry (1951)] [Fradkin, Gitman, Shvartsman (1991)] [Torgrimsson, Schneider, Shutzhold (2017)]

Perturbation theory in Furry picture (1/3)

STEP 1 **Separate the total E into strong slow E_S & weak fast $\mathcal{E}_f$**

$$\boldsymbol{E} = \boldsymbol{E}_s + \boldsymbol{\mathcal{E}}_f$$

Perturbation theory in Furry picture (1/3)

STEP 1 **Separate the total E into strong slow E_{s} & weak fast $\mathcal{E}_{\mathrm{f}}$**

$$E = E_{\mathrm{s}} + \mathcal{E}_{\mathrm{f}}$$

STEP 2 **Solve Dirac eq. non-pert. w.r.t. E_{s}, but just pert. w.r.t. $\mathcal{E}_{\mathrm{f}}$**

$$[i\partial\!\!\!/ - eA\!\!\!/_{\mathrm{s}} - m]\hat{\psi} = e\mathcal{A}\!\!\!/_{\mathrm{f}}\hat{\psi}$$

$$\Rightarrow \hat{\psi}(x) = \hat{\psi}^{(0)}(x) + \int_{-\infty}^{\infty} \mathrm{d}y^4 S_{\mathrm{R}}(x,y) e\mathcal{A}\!\!\!/_{\mathrm{f}}(y)\hat{\psi}^{(0)}(y) + O(|e\mathcal{A}_{\mathrm{f}}|^2)$$

Here, $\hat{\psi}^{(0)}$ and S_{R} are **non-perturbatively dressed by E_{s}** as

$$[i\partial\!\!\!/ - eA\!\!\!/_{\mathrm{s}} - m]\hat{\psi}^{(0)} = 0$$

$$[i\partial\!\!\!/ - eA\!\!\!/_{\mathrm{s}} - m]S_{\mathrm{R}}(x,y) = \delta^4(x-y)$$

Perturbation theory in Furry picture (2/3)

STEP 3

Compute in/out annihilation operators $\hat{a}^{\text{in/out}}_{p,s}$, $\hat{b}^{\text{in/out}}_{p,s}$ from $\hat{\psi}$

$$\begin{pmatrix} \hat{a}^{\text{in/out}}_{\boldsymbol{p},s} \\ \hat{b}^{\text{in/out}\dagger}_{-\boldsymbol{p},s} \end{pmatrix} \equiv \lim_{t\to-\infty/+\infty} \int \mathrm{d}^3\boldsymbol{x} \begin{pmatrix} (u_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}})^\dagger \\ (v_{\boldsymbol{p},s}\mathrm{e}^{+i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}})^\dagger \end{pmatrix} \hat{\psi}(x)$$

Perturbation theory in Furry picture (2/3)

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$\Rightarrow$ $\hat{o}^{\text{in}}_{\boldsymbol{p},s}$, $\hat{o}^{\text{out}}_{\boldsymbol{p},s}$ **are inequivalent** $\hat{o}^{\text{in}}_{\boldsymbol{p},s} \neq \hat{o}^{\text{out}}_{\boldsymbol{p},s}$ and related with each other by a Bogoliubov transformation

$$\begin{pmatrix} \hat{a}^{\text{out}}_{\boldsymbol{p},s} \\ \hat{b}^{\text{out}\dagger}_{-\boldsymbol{p},s} \end{pmatrix} = \sum_{s'} \int \mathrm{d}^3\boldsymbol{p}' \begin{pmatrix} \alpha_{\boldsymbol{p},s;\boldsymbol{p}',s'} & \beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} \\ -\beta^*_{\boldsymbol{p},s;\boldsymbol{p}',s'} & \alpha^*_{\boldsymbol{p},s;\boldsymbol{p}',s'} \end{pmatrix} \begin{pmatrix} \hat{a}^{\text{in}}_{\boldsymbol{p}',s'} \\ \hat{b}^{\text{in}\dagger}_{-\boldsymbol{p}',s'} \end{pmatrix}$$

where, up to 1st order in $e\mathcal{A}_{\mathrm{f}}$,

$$\alpha_{\boldsymbol{p},s;\boldsymbol{p}',s'} = \int \mathrm{d}^3\boldsymbol{x}\, {}_{+}\psi^{(0)\text{out}\dagger}_{\boldsymbol{p},s}\, {}_{+}\psi^{(0)\text{in}}_{\boldsymbol{p}',s'} - i\int d^4x\, {}_{+}\bar{\psi}^{(0)\text{out}}_{\boldsymbol{p},s} e\not{\mathcal{A}}_{\mathrm{f}}\, {}_{+}\psi^{(0)\text{in}}_{\boldsymbol{p}',s'} + O(|e\mathcal{A}_{\mathrm{f}}|^2)$$

$$\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} = \int \mathrm{d}^3\boldsymbol{x}\, {}_{-}\psi^{(0)\text{out}\dagger}_{\boldsymbol{p},s}\, {}_{+}\psi^{(0)\text{in}}_{\boldsymbol{p}',s'} - i\int d^4x\, {}_{-}\bar{\psi}^{(0)\text{out}}_{\boldsymbol{p},s} e\not{\mathcal{A}}_{\mathrm{f}}\, {}_{+}\psi^{(0)\text{in}}_{\boldsymbol{p}',s'} + O(|e\mathcal{A}_{\mathrm{f}}|^2)$$

Here, ${}_{\pm}\psi^{(0)\text{in/out}}_{\boldsymbol{p},s}$ are sol. of Dirac eq. **dressed by eA_{s}** w/ different B.C.

$$[i\not\partial - e\not{A}_{\mathrm{s}} - m]\, {}_{\pm}\psi^{(0)\text{in/out}}_{\boldsymbol{p},s} = 0 \quad \text{w/} \lim_{t\to-\infty/+\infty} \begin{pmatrix} {}_{+}\psi^{(0)\text{in/out}}_{\boldsymbol{p},s} \\ {}_{-}\psi^{(0)\text{in/out}}_{\boldsymbol{p},s} \end{pmatrix} = \begin{pmatrix} u_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}} \\ v_{\boldsymbol{p},s}\mathrm{e}^{-i\omega_{\boldsymbol{p}}t}\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}} \end{pmatrix}$$

Perturbation theory in Furry picture (3/3)

STEP 4 **Evaluate the in-vacuum expectation value of # operator**

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} \equiv \left\langle \mathrm{vac;in} \middle| a^{\mathrm{out}\dagger}_{\boldsymbol{p},s} a^{\mathrm{out}}_{\boldsymbol{p},s} \middle| \mathrm{vac;in} \right\rangle = \sum_{s'} \int \mathrm{d}^3\boldsymbol{p}' \left| \beta_{\boldsymbol{p},s;\boldsymbol{p}',s'} \right|^2$$

Perturbation theory in Furry picture (3/3)

STEP 4 **Evaluate the in-vacuum expectation value of # operator**

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Assume $\boldsymbol{E}_{\mathbf{s}}$ is so slow that it can be approximated as a static E-field

$\Rightarrow$ analytical sol. of Dirac eq. ${}_{\pm}\psi^{(0)\mathrm{in/out}}_{\boldsymbol{p},s}$ is known

$\Rightarrow$ one can evaluate $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$ **exactly !**

Perturbation theory in Furry picture (3/3)

STEP 4 **Evaluate the in-vacuum expectation value of # operator**

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} \equiv \langle \mathrm{vac;in}| a^{\mathrm{out}\dagger}_{\boldsymbol{p},s} a^{\mathrm{out}}_{\boldsymbol{p},s} |\mathrm{vac;in}\rangle = \sum_{s'} \int \mathrm{d}^3\boldsymbol{p}' \left|\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}\right|^2$$

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$\Rightarrow$ one can evaluate $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$ **exactly !**

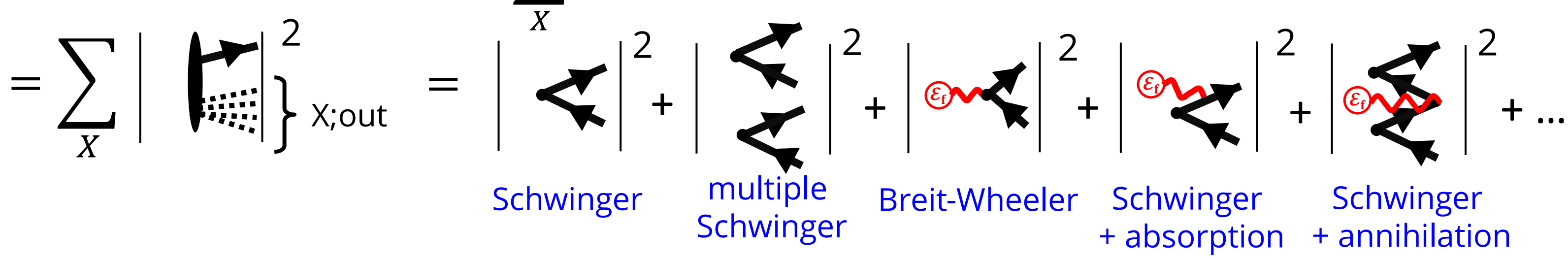
✔ Remarks

- **Directly computing VEV of # operator**

  $\Rightarrow$ **inclusive quantity** that includes all the processes up to 1st order in $\mathcal{E}_{\mathrm{f}}$

  [Baltz, McLerran (2001)]

  $$\langle \mathrm{vac;in}| a^{\mathrm{out}\dagger}_{\boldsymbol{p},s} a^{\mathrm{out}}_{\boldsymbol{p},s} |\mathrm{vac;in}\rangle = \sum_X \left|\langle e_{\boldsymbol{p},s} X;\mathrm{out}|\mathrm{vac;in}\rangle\right|^2$$



- **Production number ≠ Vacuum decay rate** [Cohen McGady (2008)]
- **No approximation in evaluating $\beta_{\boldsymbol{p},s;\boldsymbol{p}',s'}$** within 0-th order WKB [Torgrimsson et al (2017)]

Result (1/6): Analytical formula for parallel $E_{\rm s} \parallel \mathcal{E}_{\rm f}$

✔ **Analytical formula for $\boldsymbol{E} = (0{,}0, E_{\rm s} + \mathcal{E}_{\rm f}(t))$, with arbitrary time-dep.**

- is applicable even for very fast $\mathcal{E}_{\rm f}$ and reproduces numerics very well (show later)

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_{\rm s}}\right]$$

$$\times \left| 1 + \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}} \int_0^\infty d\omega \frac{\tilde{\mathcal{E}}_{\rm f}(\omega)}{E_{\rm s}} \exp\left[-\frac{i}{4}\frac{\omega^2+4\omega p_\parallel}{eE_{\rm s}}\right] {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}}; 2; \frac{i}{2}\frac{\omega^2}{eE_{\rm s}}\right)\right|^2$$

Result (1/6): Analytical formula for parallel $E_{\rm s} \,||\, \mathcal{E}_{\rm f}$

✔ **Analytical formula for $\boldsymbol{E} = (0{,}0, E_{\rm s} + \mathcal{E}_{\rm f}(t))$, with arbitrary time-dep.**

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Schwinger mech. by slow $E_{\rm s}$

Result (1/6): Analytical formula for parallel $E_s \parallel \mathcal{E}_f$

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- is applicable even for very fast $\mathcal{E}_f$ and reproduces numerics very well (show later)

$$\frac{d^3 N_e}{d\boldsymbol{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_s}\right]$$

$$\times \left| \boxed{1} + \boxed{\frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s}\int_0^\infty d\omega \frac{\tilde{\mathcal{E}}_f(\omega)}{E_s} \exp\left[-\frac{i}{4}\frac{\omega^2+4\omega p_\parallel}{eE_s}\right] {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s}; 2; \frac{i}{2}\frac{\omega^2}{eE_s}\right)} \right|^2$$

Schwinger mech. by slow E_s

Dynamical assistance by fast $\mathcal{E}_f$

- describes interplay b/w Schwinger & one-photon process smoothly:

  - Slow limit $\omega/\sqrt{eE_s} \ll 1$: ■ dominates ⇒ **Schwinger formula**

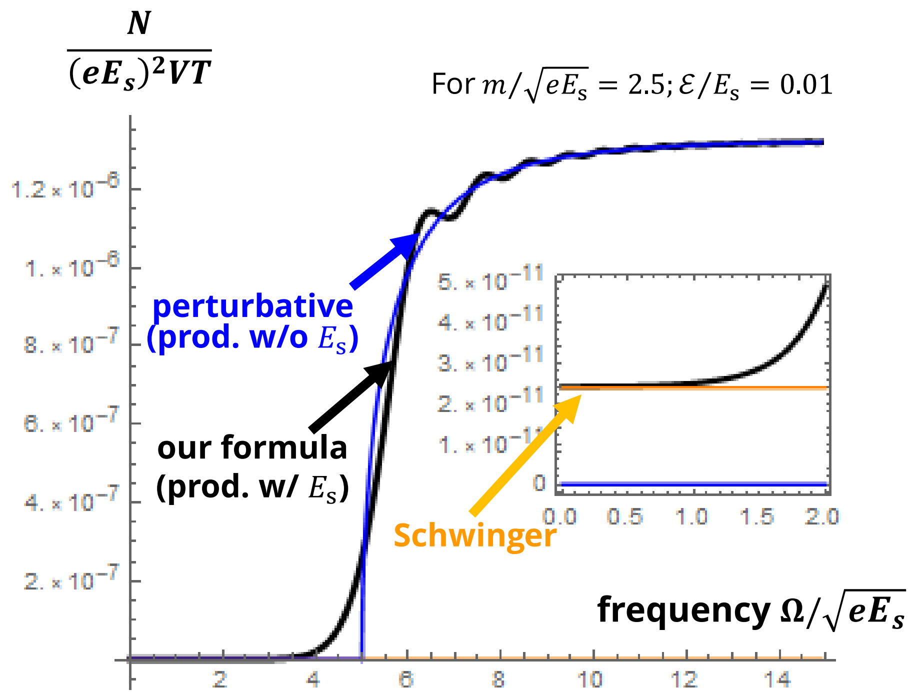
$$\frac{d^3 N_e}{d\boldsymbol{p}^3} \sim \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_s}\right]\left|1+\frac{\pi}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s}\frac{\mathcal{E}_f}{E_s}\right|^2 \sim \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{e(E_s+\mathcal{E}_f)}\right]$$

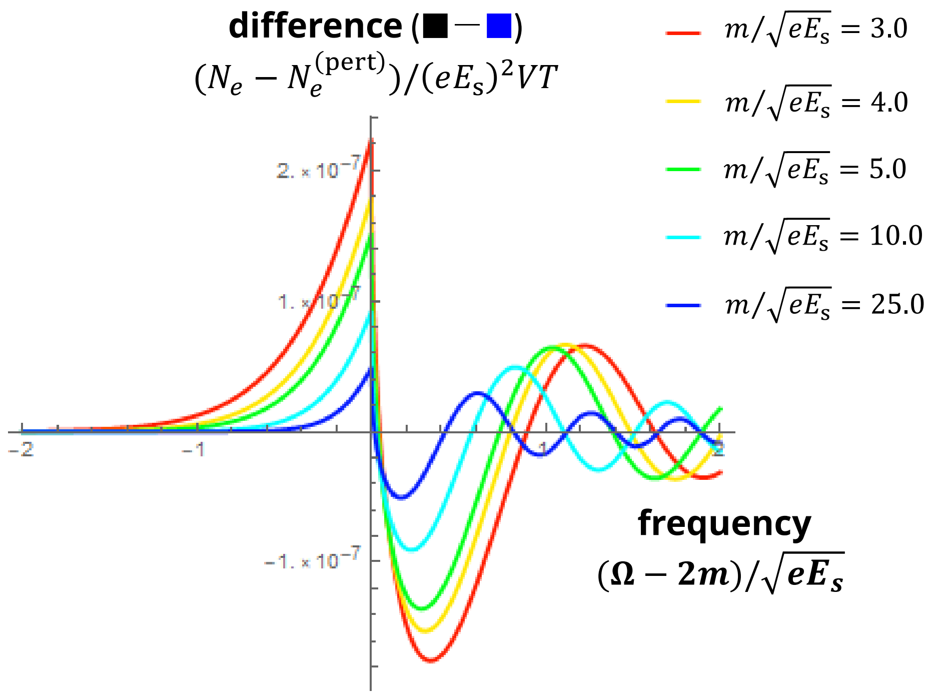
  - Fast limit $\omega/\sqrt{eE_s} \gg 1$: ■ dominates ⇒ **one-photon process**

$$\frac{d^3 N_e}{d\boldsymbol{p}^3} \sim \frac{V}{(2\pi)^3}\frac{1}{4}\frac{m^2+\boldsymbol{p}_\perp^2}{\omega_{\boldsymbol{p}}^2}\frac{|e\tilde{\mathcal{E}}_f(2\omega_{\boldsymbol{p}})|^2}{\omega_{\boldsymbol{p}}^2}$$

Result (2/6): Total N & Relation to Franz-Keldysh effect

✔ **For an oscillating perturbation $\mathcal{E}_\mathrm{f}(t) = \mathcal{E}\cos(\Omega t)$ as a demonstration**



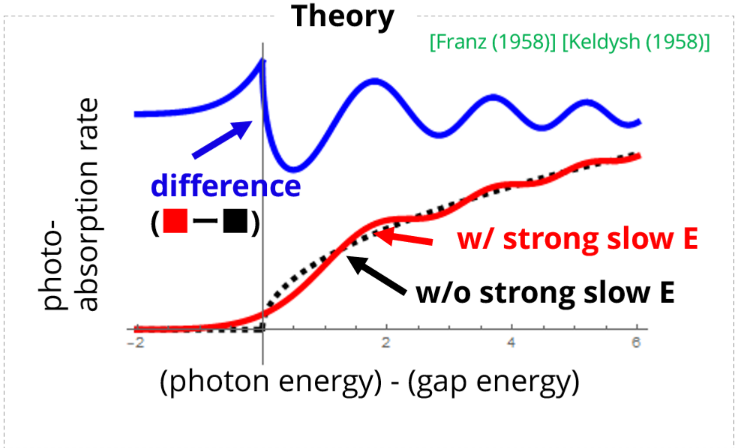


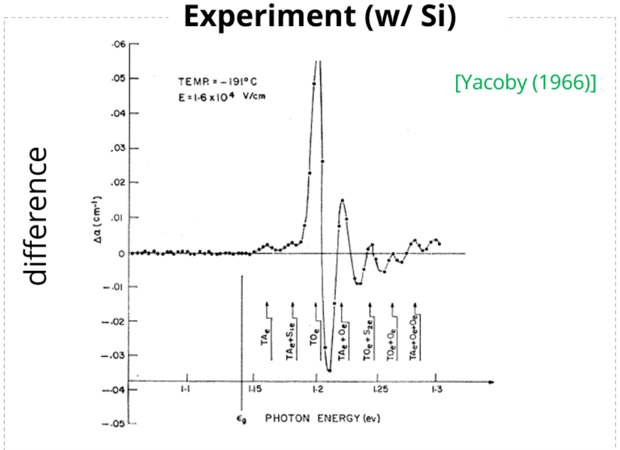
Result (2/6): Total N & Relation to Franz-Keldysh effect

Background (2/2): Franz-Keldysh effect in cond-mat

✔ **<u>Apply strong slow E-field & a photon (~ weak fast E-field)</u> onto a semi-conductor, and <u>measure photo-absorption rate</u>**

- photo-absorption rate ∼ Im[1-loop action] ∼ particle production rate





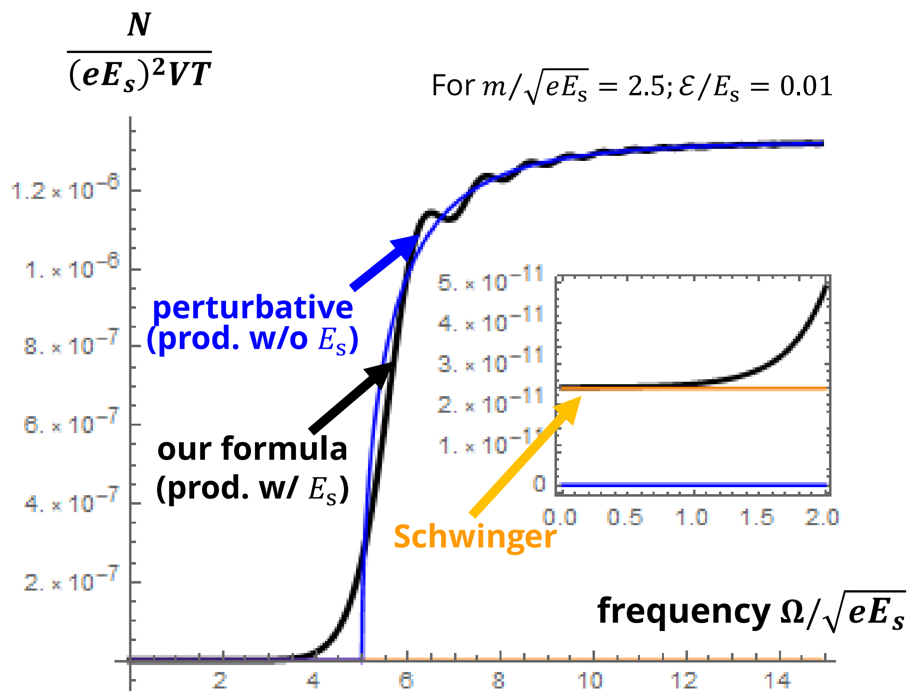
- Enhancement below the mass gap

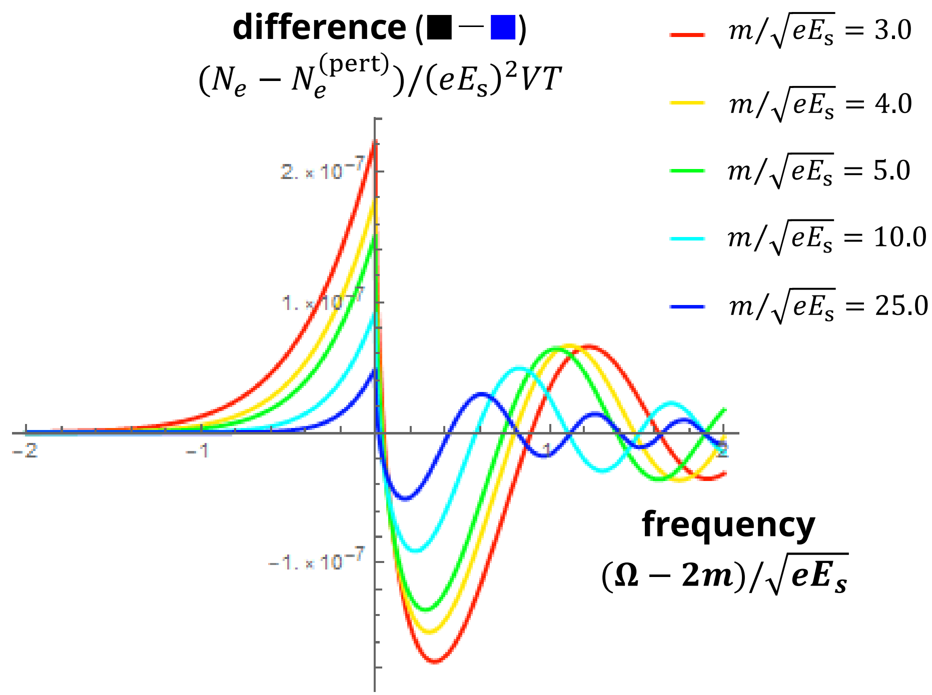
  ⇒ Looks very similar to the dynamically assisted Schwinger mechanism (?)

- Oscillation above the mass gap (Franz-Keldysh oscillation)
- Enhancement is maximized around the mass gap

Result (2/6): Total N & Relation to Franz-Keldysh effect

✔ **For an oscillating perturbation $\mathcal{E}_f(t) = \mathcal{E}\cos(\Omega t)$ as a demonstration**





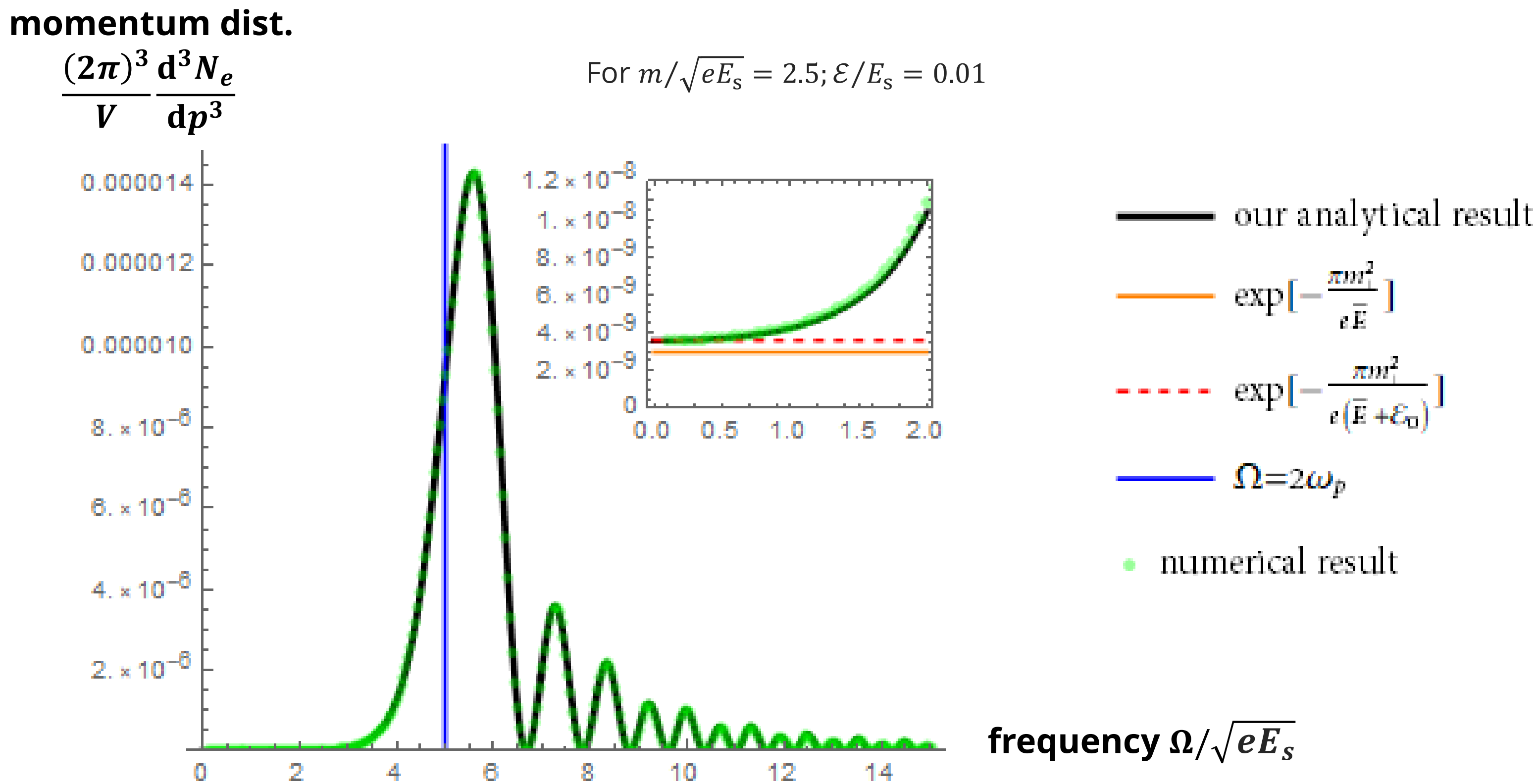
✔ **Completely the same as the Franz-Keldysh effect !**

- enhancement below the mass gap
- oscillation above the mass gap
- enhancement becomes the maximum at around the mass gap

⇒ dynamically assisted Schwinger mechanism = Franz-Keldysh effect

Result (3/6): Momentum dist. & comp. w/ numerics

✔ **Similar enhancement/oscillating behavior for the momentum dist.**



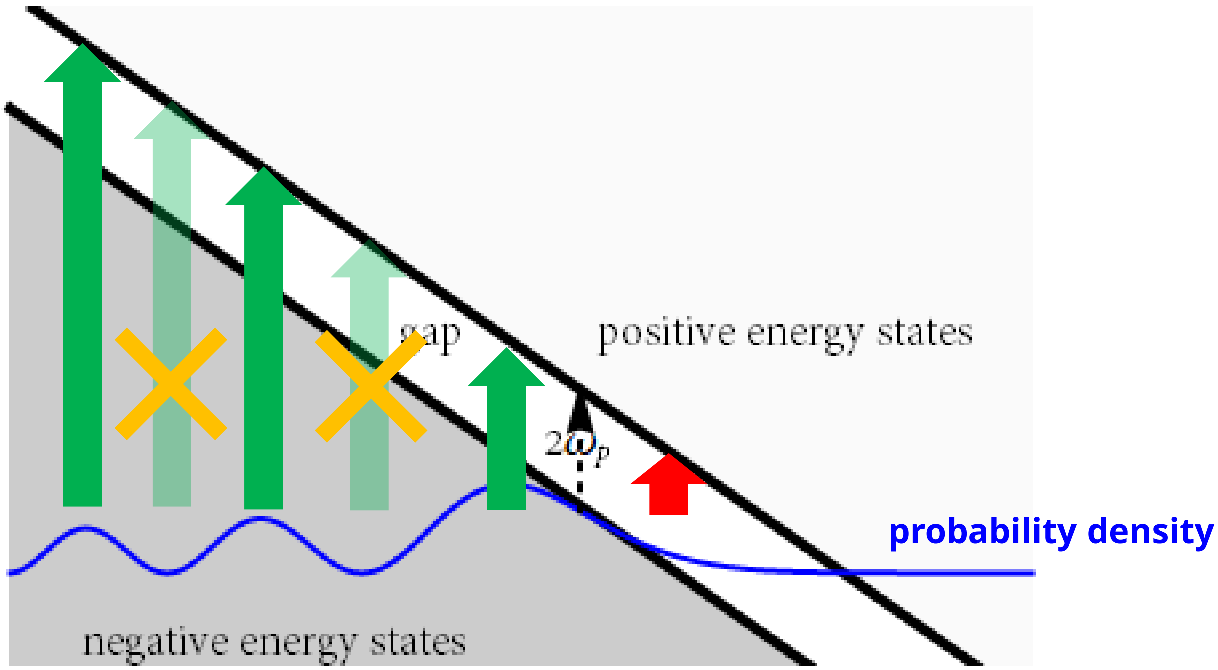
- **excellent agreement** b/w our analytical formula and the numerics

  ⇒ One-photon process it the essence of the dynamical assistance

- Another notable feature: the biggest peak (~ 1-photon peak) is above the gap $\Omega > 2\omega_p$

cf) effective mass shift in the multi-photon regime
[Kohlfurst, Gies, Alkofer (2014)]

Result (4/6): Physical interpretation



- **quantum tunneling** ⇒ **enhancement**
- **quantum reflection** ⇒ **oscillation**

⇒ production occurs most efficiently at the maxima

Result (5/6): Analytical formula for general $E_\mathrm{s} \nparallel \mathcal{E}_\mathrm{f}$

✔ Generalization of the analytical formula for $E_\mathrm{s} \parallel \mathcal{E}_\mathrm{f} \to E_\mathrm{s} \nparallel \mathcal{E}_\mathrm{f}$

$$
\begin{aligned}
\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} &= \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_\mathrm{s}}\right] \\
&\times\Bigg[\Bigg|\, 1 + \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_\mathrm{s}}}\, \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}}\frac{\tilde{\boldsymbol{\mathcal{E}}}_\mathrm{f}(\omega)\cdot\boldsymbol{E}_\mathrm{s}}{E_\mathrm{s}^2}\, \mathrm{e}^{-i\frac{\omega^2}{4eE_\mathrm{s}}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}};2;\frac{i}{2}\frac{\omega^2}{eE_\mathrm{s}}\right) \\
&\qquad + i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_\mathrm{s}}}\, \frac{\tilde{\boldsymbol{\mathcal{E}}}_\mathrm{f}(\omega)\cdot\boldsymbol{p}_\perp}{E_\mathrm{s}\omega}\,\mathrm{Re}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_\mathrm{s}}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}};1;\frac{i}{2}\frac{\omega^2}{eE_\mathrm{s}}\right)\right] \\
&\qquad + s\times i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_\mathrm{s}}}\, \frac{(\tilde{\boldsymbol{\mathcal{E}}}_\mathrm{f}(\omega)\times\boldsymbol{p}_\perp)\cdot\boldsymbol{E}_\mathrm{s}}{E_\mathrm{s}^2\omega}\,\mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_\mathrm{s}}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}};1;\frac{i}{2}\frac{\omega^2}{eE_\mathrm{s}}\right)\right]\Bigg|^2 \\
&\qquad + \Bigg|\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_\mathrm{s}}}\, \frac{m}{\omega}\frac{\tilde{\mathcal{E}}_\mathrm{f}^x(\omega)+is\tilde{\mathcal{E}}_\mathrm{f}^y(\omega)}{E_\mathrm{s}}\,\mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_\mathrm{s}}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_\mathrm{s}};1;\frac{i}{2}\frac{\omega^2}{eE_\mathrm{s}}\right)\right]\Bigg|^2\Bigg]
\end{aligned}
$$

- becomes complicated (green = new terms), but the basic structure is the same

Result (5/6): Analytical formula for general $E_{\mathrm{s}} \nparallel \mathcal{E}_{\mathrm{f}}$

✔ **Generalization of the analytical formula for $E_{\mathrm{s}} \parallel \mathcal{E}_{\mathrm{f}} \to E_{\mathrm{s}} \nparallel \mathcal{E}_{\mathrm{f}}$**

$$
\begin{aligned}
\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_{\mathrm{s}}}\right] \\
\times \Bigg[\Bigg| \boxed{1} + \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}} \frac{\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{f}}(\omega)\cdot \boldsymbol{E}_{\mathrm{s}}}{E_{\mathrm{s}}^2} \mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}; 2; \frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right) \\
+ i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{f}}(\omega)\cdot \boldsymbol{p}_\perp}{E_{\mathrm{s}}\omega} \mathrm{Re}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}; 1; \frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)\right] \\
+ s\times i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{(\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{f}}(\omega)\times \boldsymbol{p}_\perp)\cdot \boldsymbol{E}_{\mathrm{s}}}{E_{\mathrm{s}}^2\omega} \mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}; 1; \frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)\right] \Bigg|^2 \\
+ \Bigg| \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\mathrm{s}}}} \frac{m}{\omega} \frac{\tilde{\mathcal{E}}_{\mathrm{f}}^x(\omega) + is\tilde{\mathcal{E}}_{\mathrm{f}}^y(\omega)}{E_{\mathrm{s}}} \mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\mathrm{s}}}} {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\mathrm{s}}}; 1; \frac{i}{2}\frac{\omega^2}{eE_{\mathrm{s}}}\right)\right] \Bigg|^2 \Bigg]
\end{aligned}
$$

Schwinger mechanism by slow E_s

- becomes complicated (green = new terms), but the basic structure is the same

Result (5/6): Analytical formula for general $E_{\rm s} \not\parallel \mathcal{E}_{\rm f}$

✔ **Generalization of the analytical formula for $E_{\rm s} \parallel \mathcal{E}_{\rm f} \to E_{\rm s} \not\parallel \mathcal{E}_{\rm f}$**

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} = \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_{\rm s}}\right]$$

$$\times \left[\left| \boxed{1} + \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\rm s}}} \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}}\frac{\tilde{\boldsymbol{\mathcal{E}}}_{\rm f}(\omega)\cdot \boldsymbol{E}_{\rm s}}{E_{\rm s}^2}\, \mathrm{e}^{-i\frac{\omega^2}{4eE_{\rm s}}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}};2;\frac{i}{2}\frac{\omega^2}{eE_{\rm s}}\right)\right.\right.$$

$$+ i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\rm s}}} \frac{\tilde{\boldsymbol{\mathcal{E}}}_{\rm f}(\omega)\cdot \boldsymbol{p}_\perp}{E_{\rm s}\omega}\,\mathrm{Re}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\rm s}}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}};1;\frac{i}{2}\frac{\omega^2}{eE_{\rm s}}\right)\right]$$

$$\left. + s\times i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\rm s}}} \frac{(\tilde{\boldsymbol{\mathcal{E}}}_{\rm f}(\omega)\times \boldsymbol{p}_\perp)\cdot \boldsymbol{E}_{\rm s}}{E_{\rm s}^2\omega}\,\mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\rm s}}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}};1;\frac{i}{2}\frac{\omega^2}{eE_{\rm s}}\right)\right]\right|^2$$

$$\left. + \left|\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_{\rm s}}} \frac{m}{\omega}\frac{\tilde{\mathcal{E}}_{\rm f}^x(\omega)+is\tilde{\mathcal{E}}_{\rm f}^y(\omega)}{E_{\rm s}}\,\mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_{\rm s}}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_{\rm s}};1;\frac{i}{2}\frac{\omega^2}{eE_{\rm s}}\right)\right]\right|^2\right]$$

Dynamical assistance by fast $\mathcal{E}_{\rm f}$

Schwinger mechanism by slow $E_{\rm s}$

- becomes complicated (green = new terms), but the basic structure is the same

Result (5/6): Analytical formula for general $E_s \not\parallel \mathcal{E}_f$

✔ Generalization of the analytical formula for $E_s \parallel \mathcal{E}_f \;\to\; E_s \not\parallel \mathcal{E}_f$

Dynamical assistance by fast $\mathcal{E}_f$

Schwinger mechanism by slow E_s

$$\frac{\mathrm{d}^3 N_e}{\mathrm{d}\boldsymbol{p}^3} = \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+\boldsymbol{p}_\perp^2)}{eE_s}\right]$$

$$\times \Bigg[\Bigg| \boxed{1} + \int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_s}} \frac{1}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s}\frac{\tilde{\boldsymbol{\mathcal{E}}}_f(\omega)\cdot \boldsymbol{E}_s}{E_s^2}\, \mathrm{e}^{-i\frac{\omega^2}{4eE_s}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};2;\frac{i}{2}\frac{\omega^2}{eE_s}\right)$$

$$+i\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_s}} \frac{\tilde{\boldsymbol{\mathcal{E}}}_f(\omega)\cdot \boldsymbol{p}_\perp}{E_s\omega}\,\mathrm{Re}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_s}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};1;\frac{i}{2}\frac{\omega^2}{eE_s}\right)\right]$$

$$+s\times\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_s}} \frac{(\tilde{\boldsymbol{\mathcal{E}}}_f(\omega)\times \boldsymbol{p}_\perp)\cdot \boldsymbol{E}_s}{E_s^2\omega}\,\mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_s}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};1;\frac{i}{2}\frac{\omega^2}{eE_s}\right)\right]\Bigg|^2$$

$$+\left|\int_0^\infty d\omega\, \mathrm{e}^{-i\frac{\omega p_\parallel}{eE_s}} \frac{m}{\omega}\frac{\tilde{\mathcal{E}}_f^x(\omega) - is\tilde{\mathcal{E}}_f^y(\omega)}{E_s}\,\mathrm{Im}\left[\mathrm{e}^{-i\frac{\omega^2}{4eE_s}}\, {}_1F_1\left(1-\frac{i}{2}\frac{m^2+\boldsymbol{p}_\perp^2}{eE_s};1;\frac{i}{2}\frac{\omega^2}{eE_s}\right)\right]\right|^2\Bigg]$$

- becomes complicated (green = new terms), but the basic structure is the same
- **spin-dependence appears even without magnetic fields** [Takayoshi, Wu, Oka (2020)]

  ∵ Dirac particle has a spin-orbit coupling $\boldsymbol{s}\cdot(\boldsymbol{p}\times\boldsymbol{\mathcal{E}})$

  [Foldy, Wouthuysen (1950)] [Tani (1951)]

E_s $\quad$ $\mathcal{E}_f$

- can be applied to rotating E-fields: $E = E_0(\cos(\Omega t), \sin(\Omega t), 0) \sim (E_0, E_0\Omega t, 0)$

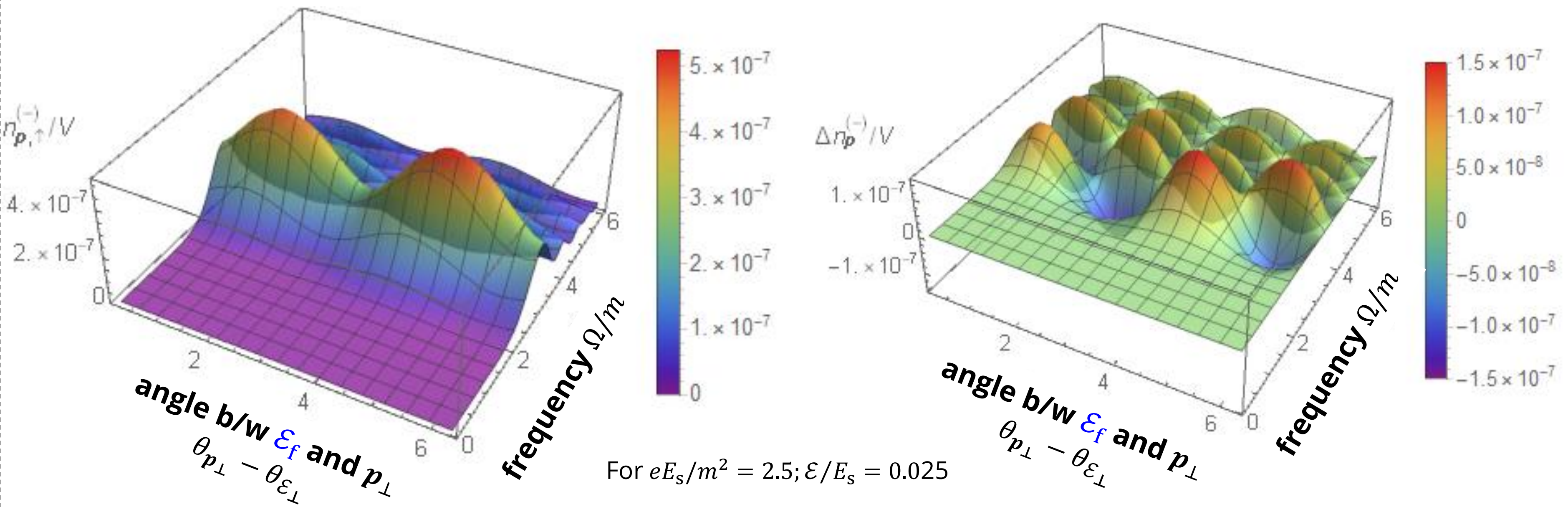
Numerical studies: [Blinne, Strobel (2015)] [Strobel, Xue (2015)] [Woller, Bauke, Keitel (2015)] [Kohlfurst (2019)]

Result (6/6): Spin-dependent production

✔ **Non-parallel superposition :** $\boldsymbol{E} = \begin{pmatrix} \mathcal{E}\cos\theta_{\mathcal{E}_\perp} \times \cos\Omega t \\ \mathcal{E}\sin\theta_{\mathcal{E}_\perp} \times \cos\Omega t \\ E_{\mathrm{s}} \end{pmatrix}$

momentum dist. of spin up $\frac{1}{V}\frac{\mathrm{d}^3 N_\uparrow}{\mathrm{d}\boldsymbol{p}^3}$

diff. b/w spin up and down $\frac{1}{V}\frac{\mathrm{d}^3 N_\uparrow}{\mathrm{d}\boldsymbol{p}^3} - \frac{1}{V}\frac{\mathrm{d}^3 N_\downarrow}{\mathrm{d}\boldsymbol{p}^3}$



For $eE_{\mathrm{s}}/m^2 = 2.5; \mathcal{E}/E_{\mathrm{s}} = 0.025$

- The dynamical assistance to N is basically the same as the parallel case
- Spin-dependence appears ⇒ O(10%) effect ⇒ non-negligible
- $\theta_{\boldsymbol{p}_\perp}$-dependent because of the spin-orbit interaction $\boldsymbol{s}\cdot(\boldsymbol{p}\times\boldsymbol{\mathcal{E}})$

Message of Part II

(1) The dynamically assisted Schwinger mechanism is an analogue of the Franz-Keldysh effect in cond-mat

(2) Perturbation theory in the Furry picture provides a very powerful analytical formula

(3) Not only the enhancement due to the quantum tunneling and one-photon assistance, but also oscillation appears due to the quantum reflection

(4) Spin-dependent production occurs when transverse E-field is time-dependent

[HT, PRD (2019)]
[Huang, HT, PRD (2019)]
[Huang, Matsuo, HT, PTEP (2019)]

Introduction

⇓

Part I:

Interplay b/w non-pert. & pert. production mechanisms

⇓

Part II:

Dynamically assisted Schwinger mechanism

⇓

Part III:

Dynamical assistance to chirality production

⇓

Summary

Background: Chirality production

- ✔ **Chirality is produced through anomaly when** $\boldsymbol{E} \cdot \boldsymbol{B} \neq 0$

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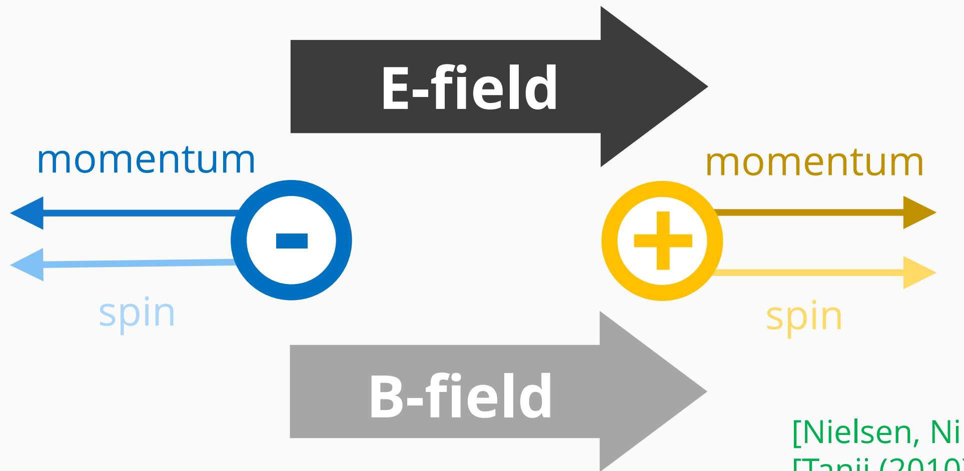
- Microscopically, caused by the interplay b/w Schwinger mech. by E-field & Landau quantization by B-field



⇒ **chirality ~ helicity** $= +2 \times N_{\textbf{pair in LLL}}$

[Nielsen, Ninomiya (1983)]
[Tanji (2010)]
[Fukushima, Kharzeev, Warringa (2010)]
[Copinger, Fukushima, Pu (2018)]
etc …

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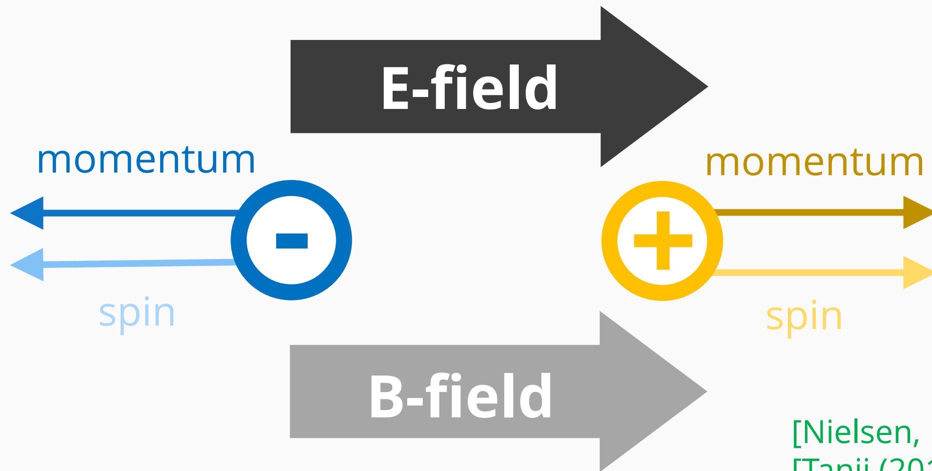

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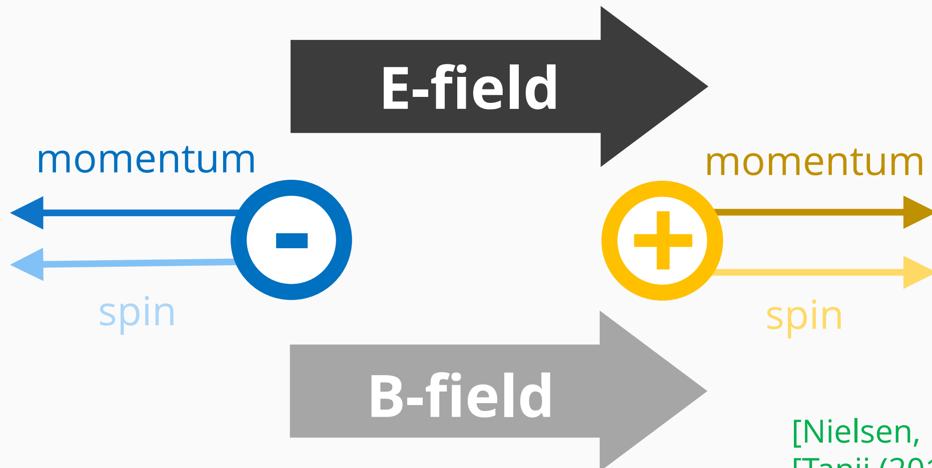

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Q: Any way to avoid the mass suppression ? ⇒ A: Dynamical assistance !

Theory: Perturbation theory in the Furry picture

✔ **Extend the E-field case (Part II) to $E||B$ case**

Theory: Perturbation theory in the Furry picture

✔ Extend the E-field case (Part II) to $E||B$ case

- Repeat essentially the same calculation:

SETUP: Parallel $\bar{\boldsymbol{E}}, \bar{\boldsymbol{B}}$ with perturbative weak fast E-field $\mathcal{E}$

$$\boldsymbol{E}(t) = \bar{\boldsymbol{E}} + \mathcal{E}(t)$$
$$\boldsymbol{B}(t) = \bar{\boldsymbol{B}}$$

STEP 1: Solve Dirac eq. under $\bar{\boldsymbol{E}}, \bar{\boldsymbol{B}}$ non-perturbatively, and include effects of $\mathcal{E}$ perturbatively

$$[i\partial\!\!\!/ - e\bar{A\!\!\!/} - m]\hat{\psi} = e\mathcal{A}\!\!\!/\hat{\psi}$$
$$\Rightarrow \hat{\psi}(x) = \hat{\psi}^{(0)}(x) + \int_{-\infty}^{\infty} \mathrm{d}y^4 S_\mathrm{R}(x,y) e\mathcal{A}\!\!\!/(y)\hat{\psi}^{(0)}(y) + O(|e\mathcal{A}|^2)$$

STEP 2: Compute VEV of chirality operator

$$Q_5 \equiv \lim_{t\to\infty} \int \mathrm{d}\boldsymbol{x}^3 \left\langle \mathrm{vac;in} \left| \hat{\bar{\psi}}\gamma^5\hat{\psi} \right| \mathrm{vac;in} \right\rangle = O(1) + O(|e\mathcal{A}|^1) + O(|e\mathcal{A}|^2) + \cdots$$

Result (1/2): Analytical formula

$$\frac{Q_5}{VT} = \frac{e\bar{E}e\bar{B}}{2\pi^2} \mathrm{e}^{-\pi\frac{m^2}{e\bar{E}}} \times \left[1 + \boxed{\frac{2\pi}{T}\left(\frac{m^2}{e\bar{E}}\right)^2 \int_0^\infty \mathrm{d}\,\omega \left| \frac{\tilde{\mathcal{E}}(\omega)}{\bar{E}} \, {}_1\tilde{F}_1\!\left(1 - \frac{\mathrm{i}}{2}\frac{m^2}{e\bar{E}}; 2; \frac{\mathrm{i}}{2}\frac{\omega^2}{e\bar{E}}\right) \right|^2}\right]$$

Dynamical assistance by fast E-field $\mathcal{E}$

- Dynamical assistance

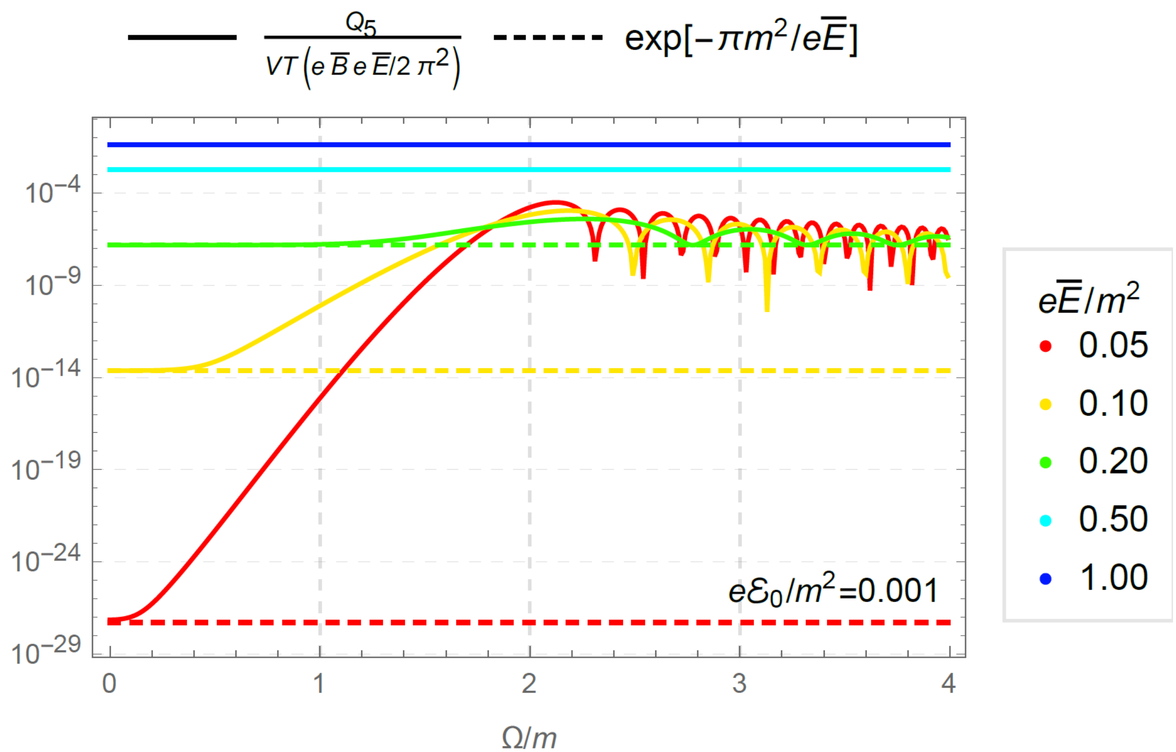
  - is positive ⇒ **chirality is always enhanced**

  - goes away when $m \to 0$ ⇒ **important only for massive case**

  - is independent of B-field

    (∵ B-field does not supply energy, i.e., does not affect the prod. mech.)

Result (2/2): Plot for chirality production



- ✔ An oscillating pert. as demonstration: $\frac{eE(t)}{m^2} = \frac{e\bar{E}}{m^2} + 0.001 \cos \Omega t$, $\frac{eB(t)}{m^2} = \frac{e\bar{B}}{m^2}$
- ✔ The same behavior as the dynamically assisted Schwinger mech.
  - Free from the exponential suppression due to the enhancement
  - Enhancement becomes largest around the mass gap
  - Oscillation above the mass gap

Message of Part III

Chirality production can be enhanced significantly via the dynamical assistance

[HT, PRR (2020)]

Introduction

$\Downarrow$

Part I:

Interplay b/w non-pert. & pert. production mechanisms

$\Downarrow$

Part II:

Dynamically assisted Schwinger mechanism

$\Downarrow$

Part III:

Spin & chirality production

$\Downarrow$

Summary

Summary

I discussed the Schwinger mechanism under time-depending E-field:

Part I: Interplay b/w non-pert. & pert. production mechanisms

- The interplay is controlled by $\gamma \equiv \frac{m\Omega}{eE}$ (Keldysh parameter) and also by $\nu \equiv \frac{eE}{\Omega^2}$
- Semi-classical methods are invalid when $\nu \gtrsim 1$, where one-photon process dominates
- One-photon production is very efficient, compared to non-pert. tunneling

Part II: Dynamically assisted Schwinger mechanism

- Point out relation to Franz-Keldysh effect in cond-mat.
- Analytical analysis based on perturbation theory in the Furry picture
- Not only enhancement due to quantum tunneling & one-photon assist, but also oscillation due to quantum reflection
- Spin-dependent production occurs when transverse E-field is time-dependent

Part III: Dynamical assistance to chirality production

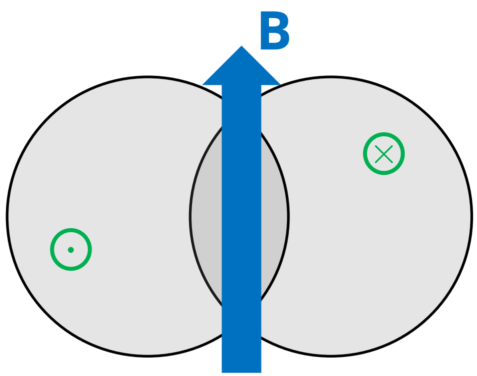
- Chirality production can be enhanced significantly via the dynamical assistance

BACKUP

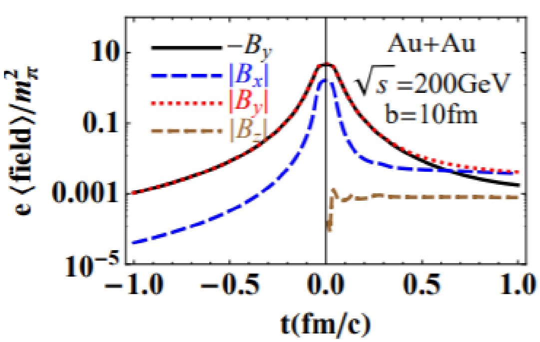
Strong EM fields in HIC

✔ **3 ways to produce (as far as I know)**

① (ultra)peripheral

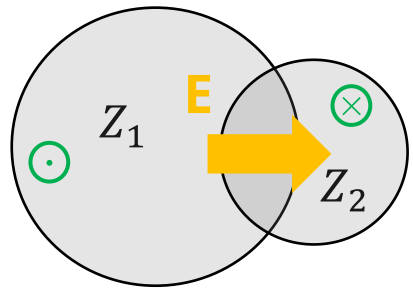


$$eB \sim \frac{\alpha Z v \gamma}{r^2} \sim \alpha Z \gamma \times m_\pi^2$$

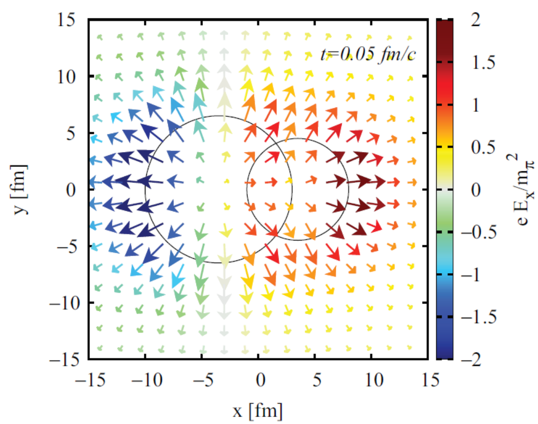


[Deng, Huang (2012)]

② asymmetric

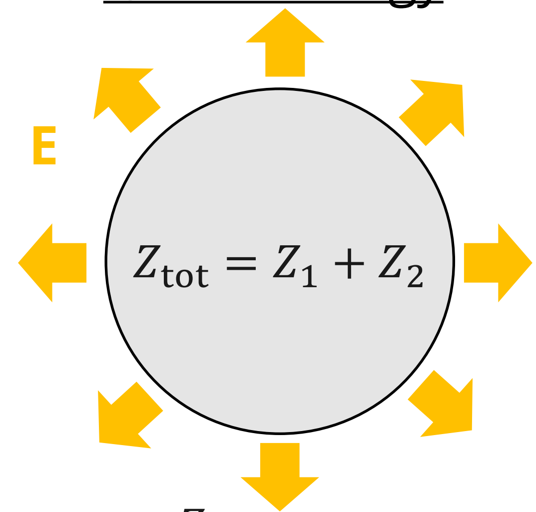


$$eE \sim \frac{\alpha (Z_1 - Z_2)\gamma}{r^2}$$
$$\sim \alpha (Z_1 - Z_2)\gamma \times m_\pi^2$$

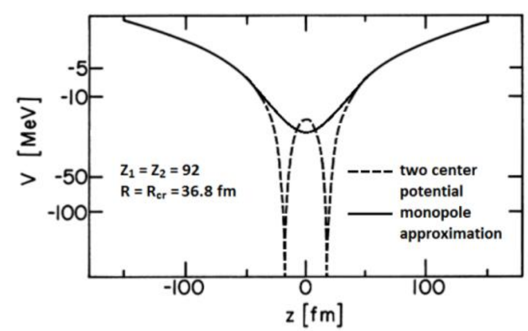


[Voronyuk, Toneev, Voloshin, Cassing (2014)]

③ low-energy



$$eE \sim \frac{\alpha Z_{tot}}{r^2} \sim \alpha Z_{tot} \times m_\pi^2$$

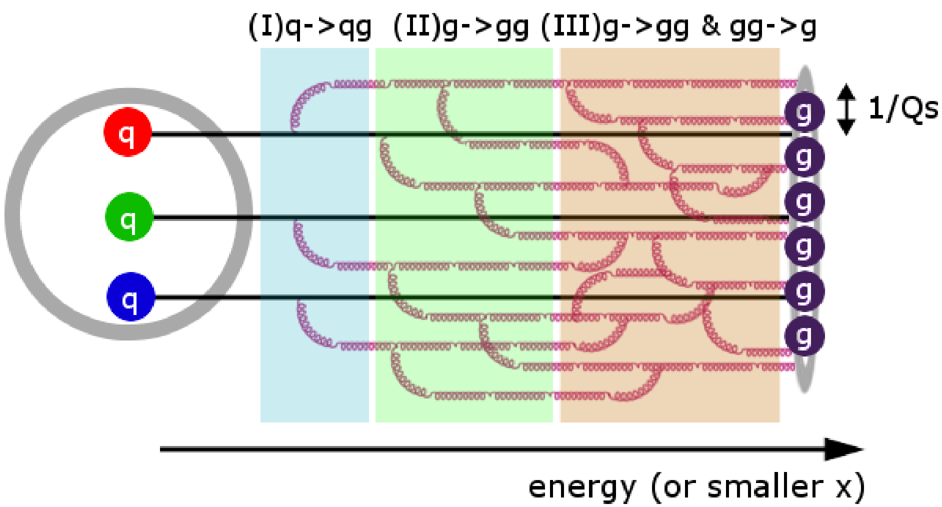


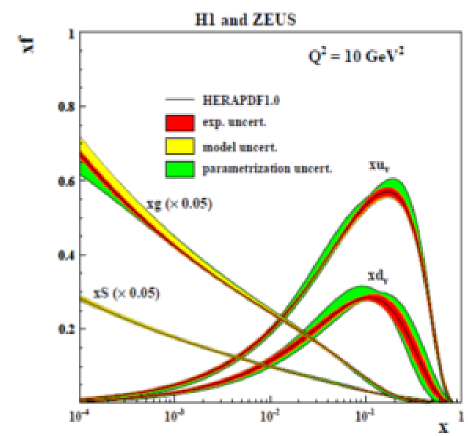
[Review: Rafelski, Kirsch, Muller, Reinhardt, Greiner (2014)]

Glasma

✔ **Gluon saturation of ultra-relativistic nuclei** [McLerran, Venugopalan (1994)]

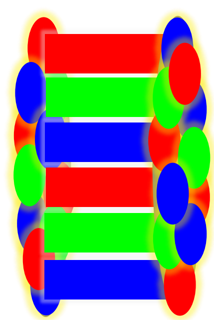
⇒ something like a "color capacitor"
w/ huge color charge density $= O(Q_s) = O(\text{a few GeV})$

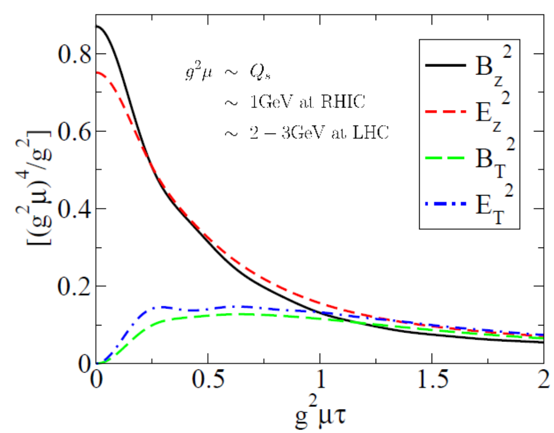




[Lappi, McLerran (2006)]

✔ **High-energy heavy-ion collisions**
≃ formation of "color condenser"
⇒ strong color flux tubes





Old ideas: [Low (1975)] [Nussinov (1975)] [Casher, Neuberger, Nussinov (1979)]