Enhancement of chirality production from the vacuum by time-dependent electromagnetic fields

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Ref: HT, Phys.Rev.Res. 2 (2020) 2, 023257 [2003.08948]
Fukushima, Hidaka, Shimazaki, HT, in preparation

Advertisement:
NEW theoretical review on strong-field QED
Fedotov, Ilderton, Karbstein, King, Seipt, HT, Torgrimsson, 2203.00019

Summary

Background

Chirality production from the vacuum for massive fermions

⇒ well understood for slow EM fields

- Driven by the Schwinger mechanism
- Exponentially suppressed ⇒ Difficult to be observed in experiments

Problem

Enhancement mechanism for chirality production ?

Idea

Use time-dependent fast EM fields

Method

(1) Perturbation theory in the Furry picture

(2) Floquet approach (high-frequency expansion)

Result & Message

Chirality production is significantly enhanced if EM fields are fast

⇒ worthwhile to investigate chirality-related phenomena in strong-field QED

1. Introduction

2. Enhancement of chirality production
by dynamically assisted Schwinger mech.

3. Enhancement of chirality production
by perturbative mechanism with fast E

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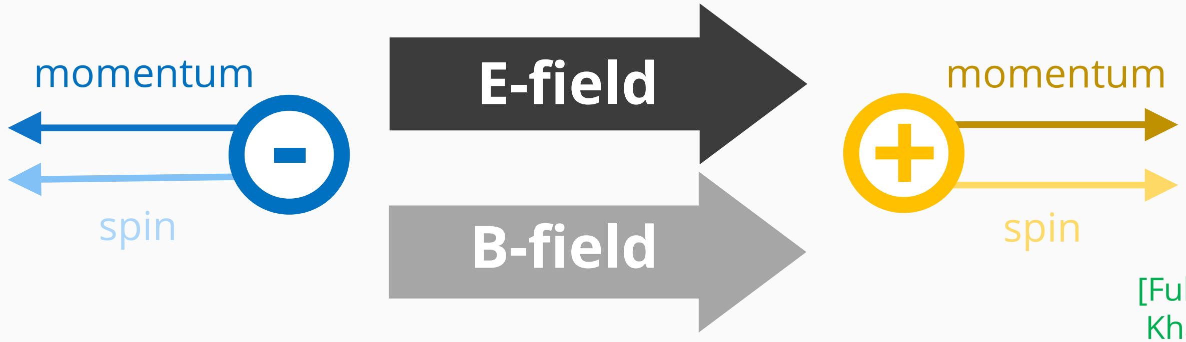
Chirality production

✔ **Chirality is produced through anomaly when $\boldsymbol{E} \cdot \boldsymbol{B} \neq 0$**

- Microscopically, the interplay b/w Schwinger mech. by E-field & Landau quant. by B-field



⇒ chirality ~ helicity $= +2 \times N_{\text{pair in LLL}}$

[Fukushima, Warringa, Kharzeev, Copinger, Shi, Ambjorn, Greensite, Peterson, HT, …]

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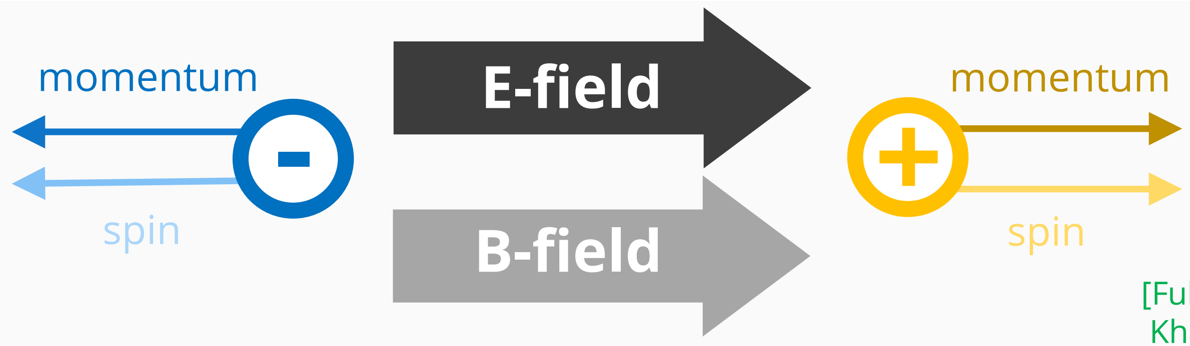

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- $\boldsymbol{E} \cdot \boldsymbol{B} \neq 0$ is realized in various physical situations and leads to interesting phenomena

  ex.1) In particle & cond-mat physics: Anomalous transport phenomena (e.g., Chiral Magnetic Effect, Chiral Separation Effect, Chiral Vortical Effect, ...)

  ex.2) In laser physics, $\boldsymbol{E} \cdot \boldsymbol{B} \neq 0$ can be realized by a collision of two counter propagating standing waves

$$\boldsymbol{A}_1 \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \sin\big(\omega(+x - t)\big), \boldsymbol{A}_2 \propto \begin{pmatrix} 0 \\ \sin\phi \\ \cos\phi \end{pmatrix} \sin\big(\omega(-x - t)\big) \Rightarrow \boldsymbol{E} \cdot \boldsymbol{B} = \frac{\cos\phi}{1 + \sin\phi}$$

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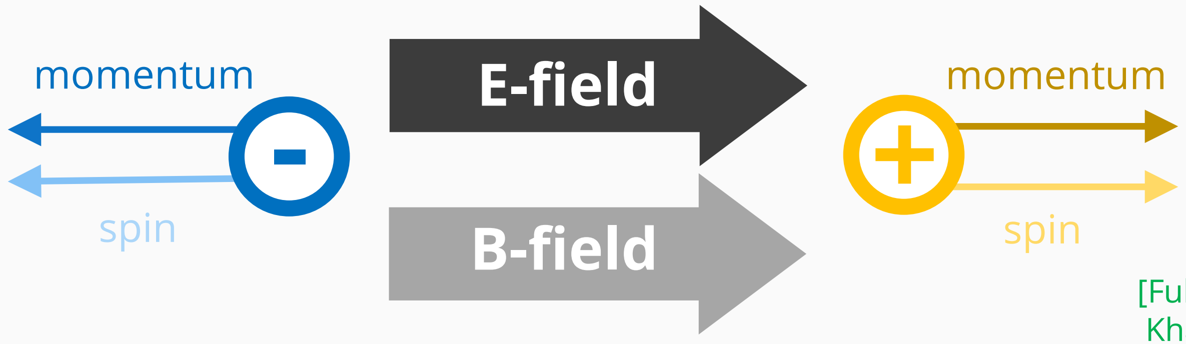

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✔ Chirality production is suppressed strongly by mass

$\because\ N_{\text{pair in LLL}} \propto \mathrm{e}^{-\#\, m^2/eE} \Rightarrow$ Tiny effects... Difficult to observe...

Q: Any way to avoid the mass suppression ?

How to enhance chirality production ?

Idea: Increase the number of pairs $N_{\mathrm{pair\ in\ LLL}}$ ⇒ Use fast E-field

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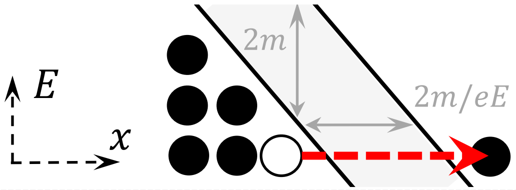
✔ **Physics of particle prod. changes depending on frequency Ω of E-field**

(1) Ω small ⇒ Non-perturbative tunneling (Schwinger mechanism)



$$N \sim \exp\left[-\# \times \begin{pmatrix}\text{gap}\\ \text{height}\end{pmatrix} \times \begin{pmatrix}\text{gap}\\ \text{length}\end{pmatrix}\right] \sim \exp\left[-\pi \frac{m^2}{eE}\right]$$

⇒ Strong exponential suppression

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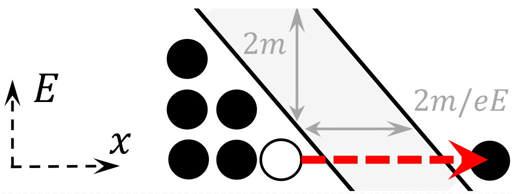

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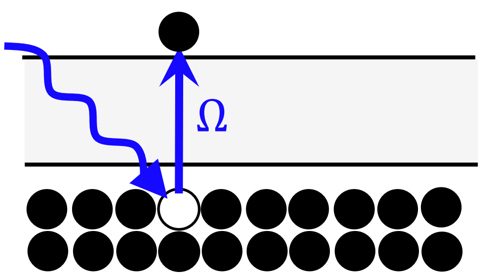
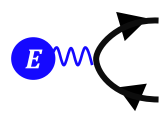
(2) Ω large ⇒ Perturbative one- (or multi-) photon pair production

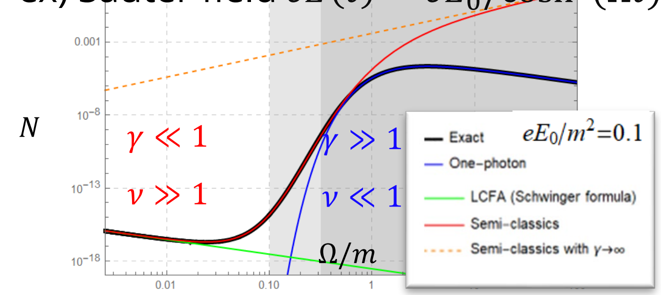


$$N \sim \left| \quad \right|^2 \propto \left(\frac{eE}{m^2}\right)^2$$



⇒ Weak power suppression

ex) Sauter field $eE(t) = eE_0/\cosh^2(\Omega t)$



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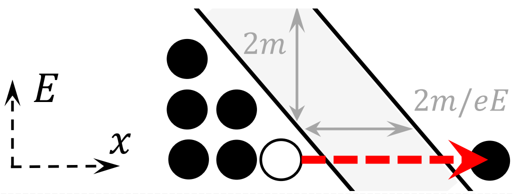

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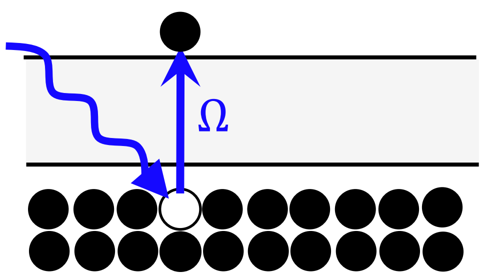
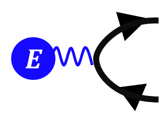


$N \sim \Bigg|$  $\Bigg|^2 \propto \left(\frac{eE}{m^2}\right)^2$

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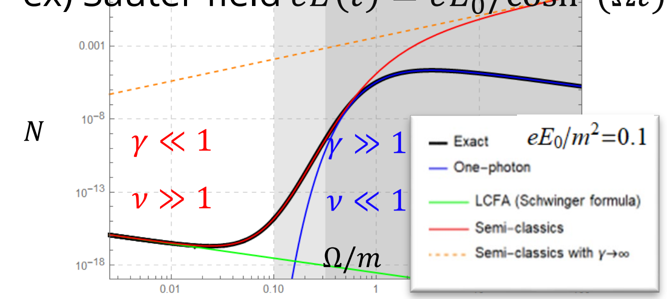
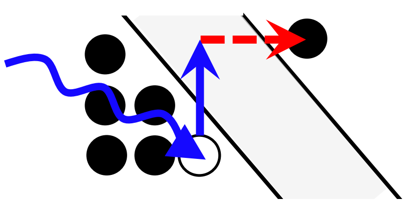


(3) Superposition of large & small Ω's ⇒ dynamically assisted Schwinger mech.



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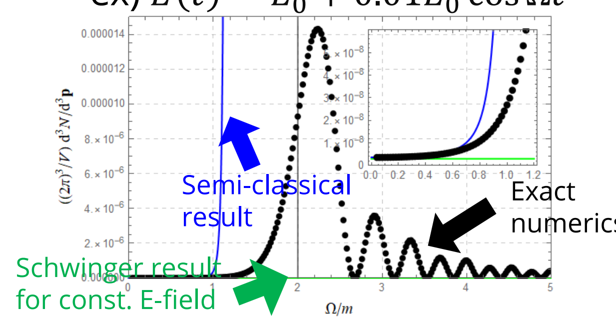
reduced by pert.

⇒ enhancement

ex) $E(t) = E_0 + 0.01E_0 \cos \Omega t$



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Method: Perturbation theory in the Furry picture

✔ **Reason:**

The most successful approach to the dynamically assisted Schwinger mech.

[Torgrimsson, Schneider, Oertel, Schutzhold (2017)]
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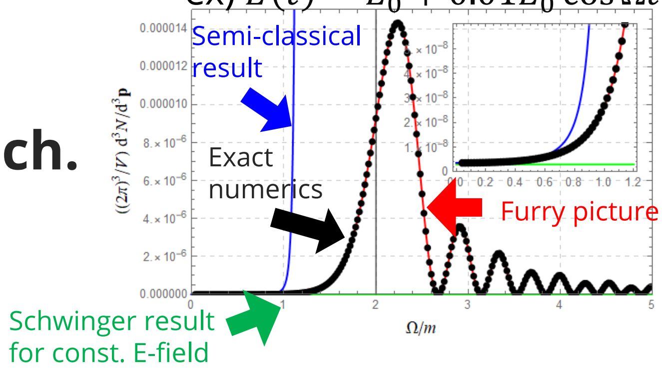

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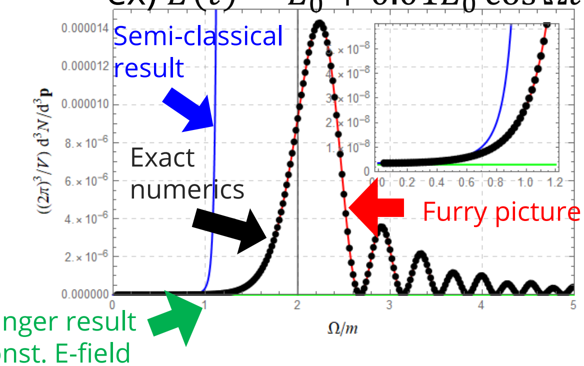


✔ **Extend the E-field case to $E||B$ case & compute chiral density**

SETUP: **Parallel strong slow $\bar{E}, \bar{B}$ with perturbative weak fast $\mathcal{E}$**

$$\boldsymbol{E}(t) = \bar{\boldsymbol{E}} + \boldsymbol{\mathcal{E}}(t)$$
$$\boldsymbol{B}(t) = \bar{\boldsymbol{B}}$$

STEP 1: **Solve Dirac eq. under $\bar{E}, \bar{B}$ non-perturbatively, and include effects of $\mathcal{E}$ perturbatively**

$$[i\partial\!\!\!/ - e\bar{A}\!\!\!/ - m]\hat{\psi} = e\mathcal{A}\!\!\!/\hat{\psi}$$

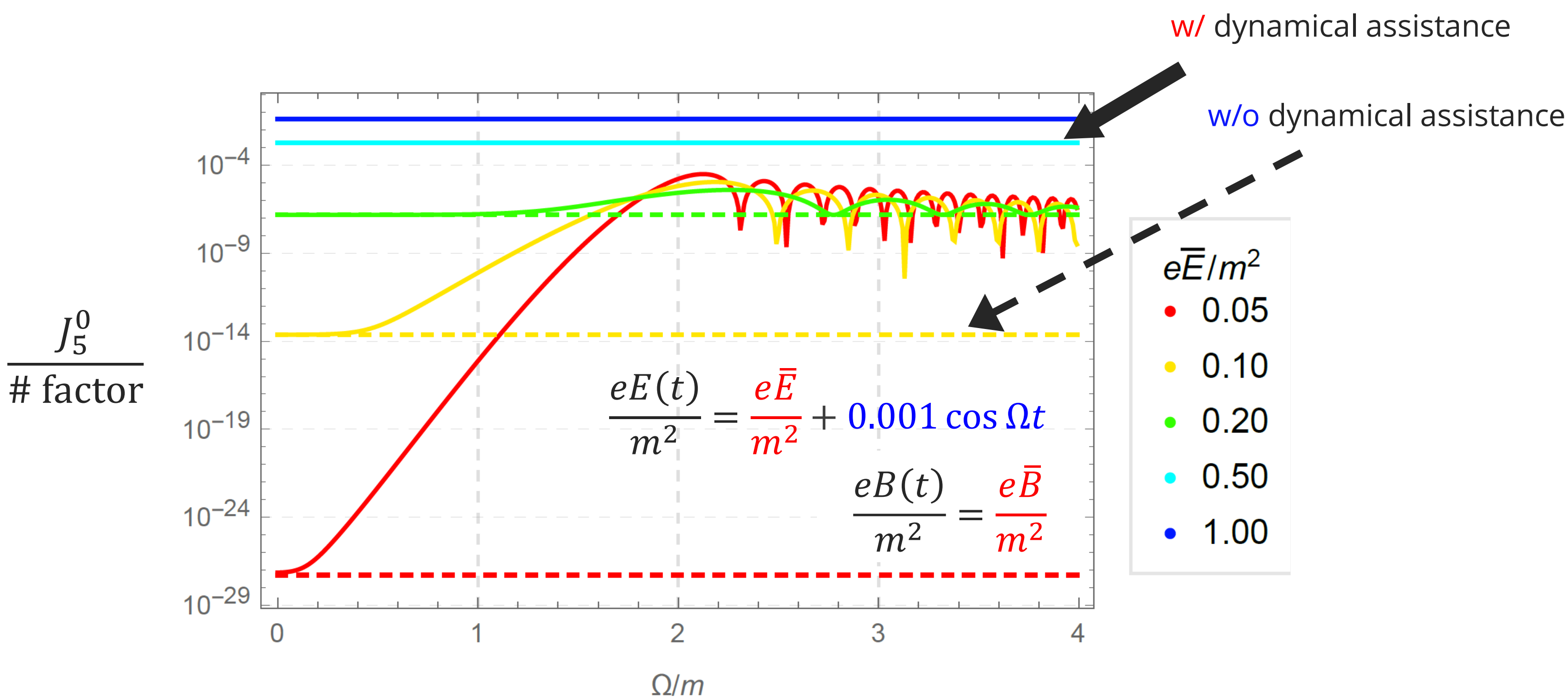
$$\Rightarrow \hat{\psi}(x) = \hat{\psi}^{(0)}(x) + \int_{-\infty}^{\infty} \mathrm{d}y^4 S_{\mathrm{R}}(x,y) e\mathcal{A}\!\!\!/(y)\hat{\psi}^{(0)}(y) + O(|e\mathcal{A}|^2)$$

STEP 2: **Compute VEV of chiral density operator**

$$J_5^0 \equiv \lim_{t\to\infty} \int \mathrm{d}\boldsymbol{x}^3 \left\langle \mathrm{vac;in} \left| \hat{\bar{\psi}}\gamma^5\hat{\psi} \right| \mathrm{vac;in} \right\rangle = O(1) + O(|e\mathcal{A}|^1) + O(|e\mathcal{A}|^2) + \cdots$$

Result: Enhancement in chirality production

[HT (2020)]



✔ Analytical formula: $\frac{J_5^0}{VT} = \frac{e\bar{E}e\bar{B}}{2\pi^2} e^{-\pi\frac{m^2}{e\bar{E}}} \times [1 + \frac{2\pi}{T}\left(\frac{m^2}{e\bar{E}}\right)^2 \int_0^\infty d\omega \left| \frac{\tilde{\mathcal{E}}(\omega)}{\bar{E}} \, {}_1\tilde{F}_1(1 - \frac{i}{2}\frac{m^2}{e\bar{E}}; 2; \frac{i}{2}\frac{\omega^2}{e\bar{E}}) \right|^2]$

⇒ Huge enhancement by the dynamical assistance !!

- chirality production becomes free from the exponential suppression
- enhancement becomes more significant for more massive case

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Quite successful in cond-mat community to describe high-frequency periodically driven systems

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✔ Floquet approach

Review: [Bukov, D'Alessio, Polkovnikov (2015)]

Assumption: Hamiltonian is periodic in time $H(t,x) = H(t+T,x)$

STEP 1: Use Floquet theorem to the time-translation unitary operator

$$U(t,t';x) = \mathrm{e}^{-\mathrm{i}K(t',x)}\mathrm{e}^{+\mathrm{i}H_{\mathrm{F}}(x)(t-t')}\,\mathrm{e}^{+\mathrm{i}K(t,x)}$$

where $K(t,x) = K(t+T,x)$ and $H_{\mathrm{F}}(x)$ is time independent

STEP 2: Use high-frequency expansion to perturbatively determine H_{F} & K

From $-\mathrm{i}\partial_t U(t,t';x) = H(t;x)U(t,t';x)$,

$$H_{\mathrm{F}}(x) = \mathrm{e}^{+\mathrm{i}K(t,x)}H(t,x)\mathrm{e}^{-\mathrm{i}K(t,x)} + \mathrm{i}\epsilon^{-1}\frac{\partial \mathrm{e}^{+\mathrm{i}K(t,x)}}{\partial t}\mathrm{e}^{-\mathrm{i}K(t,x)}$$

Formally assuming that the time derivative is large, i.e., $\partial_t \to \epsilon^{-1}\partial_t$ ($\epsilon \ll 1$)
and then determine H_{F}, K perturbatively in ϵ

STEP 3: Use obtained U to compute observables

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✔ In this work:

Apply Floquet approach to chirality prod. in QED1+1: $H(t,x) \coloneqq \begin{pmatrix} -\mathrm{i}\partial_x + eA^0 - eA^1 & m \\ m & +\mathrm{i}\partial_x + eA^0 + eA^1 \end{pmatrix}$

Result: Enhancement in chirality production

$$J_5^0(t,x) = \frac{1}{\pi}\left[\epsilon \int^t \mathrm{d}t'\, eE + \epsilon^3(\partial_x^2 - 2m^2)\int^t \mathrm{d}t' \int^{t'} \mathrm{d}t'' \int^{t''} \mathrm{d}t''' eE + O(\epsilon^5)\right]$$

✔ **Completely different parameter dependence compared to the slow E-field case**

slow case: $J_5^0(t,x) = \frac{1}{\pi}\int^t \mathrm{d}t' eE(t',x)\, \mathrm{e}^{-\pi\frac{m^2}{|eE(t,x)|}}$

⇒ No exponential suppression in the fast case

⇒ Enhancement by the fast E-field !!

(Even if E is weak, it produces huge chirality as long as it is fast)

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