Introduction to strong-field QED

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(an SPDR in iTHEMS, studying high-energy particle & nuclear physics)

Plan

(QED = quantum electrodynamics)

Brief introduction to Strong-field QED

= an area to study what happens by a super strong light

(= electromagnetic field)

PART I: Basics of strong-field QED

- Why interesting ⇒ Unexplored non-perturbative regime of physics

  Nontrivial phenomena that we've never seen e.g. Vacuum decay, Birefringence, New phase of matter, ...

- Why timely ⇒ Recent availability of strong fields e.g. High-power laser, Magnetar, heavy-ion collisions, ...

PART II: Sauter-Schwinger effect and its connection to other areas

- An interdisciplinary topic

  ⇒ Exchange of ideas/techniques among physics is quite useful for better understandings

  ⇐ An example from my experience: High-harmonic generation from the vacuum

  ⇐ Math. & cond-mat. ideas helped me a lot

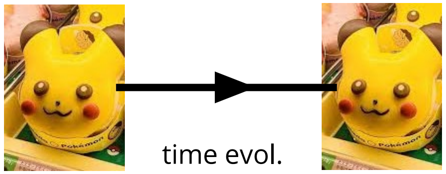
[HT, Fujimori, Misumi, Nitta, Sakai, (2021)] [HT, Hongo, Ikeda, (2021)]

PART I: Basics of strong-field physics

PART II: Sauter-Schwinger effect and its connection to other areas

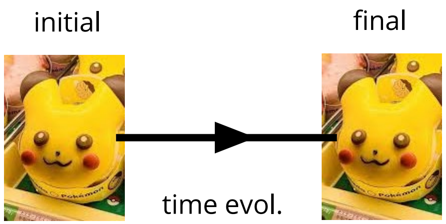
What if field becomes strong ?

initial final



No field

What if field becomes strong ?



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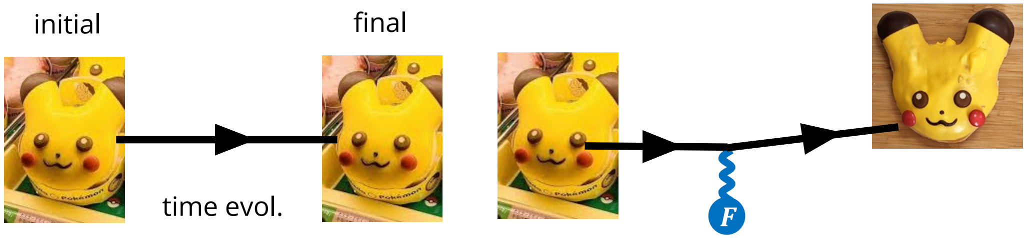

No field | **Weak field** | **Strong field**

Only minor changes

⇒ Perturbative

⇒ Very well understood
in both exp.& theor.

ex.) Electron (anomalous) magnetic moment $a := \frac{g-2}{2}$
≈ Electron energy shift in a weak magnetic field

$$a(\text{theor.}) = 1159652182.03 \ldots \times 10^{-12}$$
$$a(\text{exp.}) = 1159652180.73 \ldots \times 10^{-12}$$
[Aoyama, Kinoshita, Nio (2017)]

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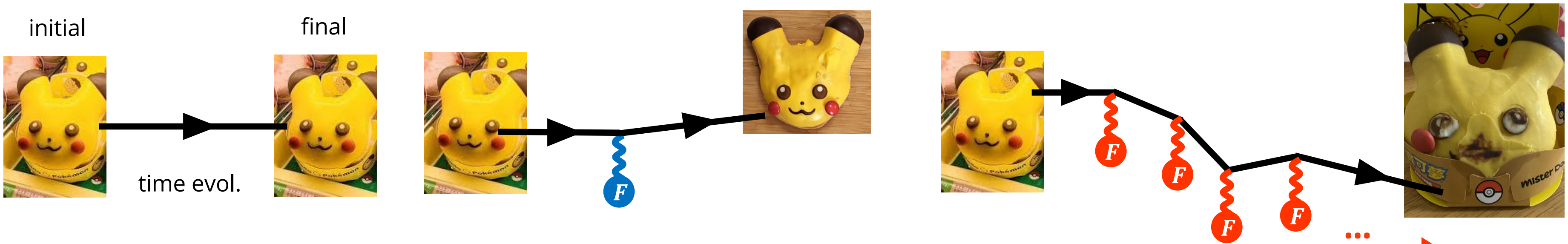

No field — **Weak field** — **Strong field**

Only minor changes

⇒ Perturbative

⇒ Very well understood in both exp.& theor.

Big change !

⇒ Non-Perturbative

⇒ Not understood well

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≈ Electron energy shift in a weak magnetic field

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[Aoyama, Kinoshita, Nio (2017)]

If field becomes strong, physics becomes totally different & nontrivial

When is field "strong" ?

Strong-field condition:

To significantly modify the original system with typical energy Δ,

the field must be more energetic than Δ

⇒ Strong-field condition: $\mathbf{\Delta < (energy\ scale\ of\ the\ field)}$

When is field “strong” ?

Strong-field condition:

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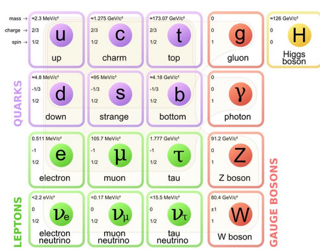
⇒ Strong-field condition: $\Delta < (\text{energy scale of the field})$

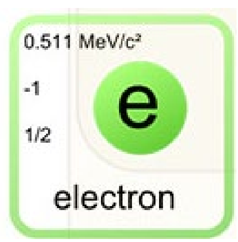
Estimate of the minimum field strength:

Standard Model



⇨

The (matter) particle having the minimum energy



- Charged ⇒ Couples to electromagnetic (EM) field
- $eE\ \mathrm{MeV}^2 > m_e^2 = (0.511\ \mathrm{MeV})^2 \approx O(10^{28}\ \mathrm{W/cm^2})$

∴ EXTREMELY strong ⇒ Impossible to realize within the current technology

LED ≈ $10^{-5}\ \mathrm{W/cm^2}$

Glasses ≈ $10^{3}\ \mathrm{W/cm^2}$

Laser beam machine < $10^{10}\ \mathrm{W/cm^2}$

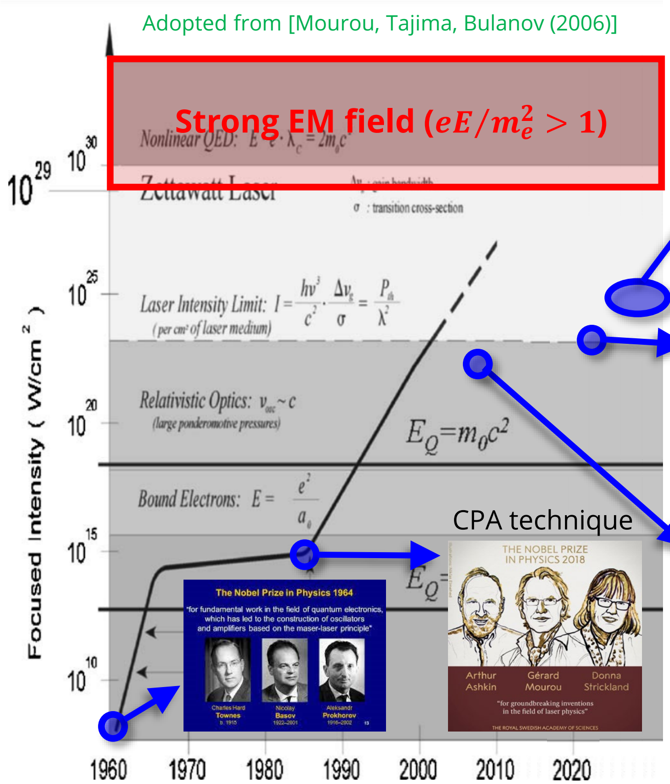
Recent availability of strong EM fields

The situation is changing:
Becoming able to create/observe strong EM fields

⇒ NOW is the best time to study physics of strong EM field
(=: strong-field QED)

High-power laser

Adopted from [Mourou, Tajima, Bulanov (2006)]



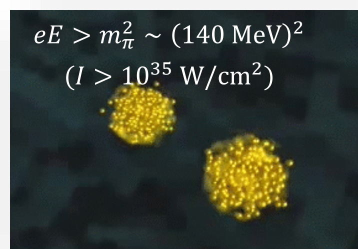
Starting soon
$I \gtrsim 10^{24}\mathrm{W/cm^2}$:
e.g., ELI @ Europe

Yoon et al., (2021)
Current strongest
$I \sim 10^{23}\mathrm{W/cm^2}$
CoReLS @ Korea

Yanovsky et al., (2008)
$I \sim 10^{22}\mathrm{W/cm^2}$
HERUCLES @ USA

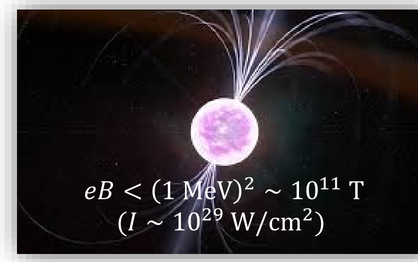
Systems with extreme conditions

- Heavy-ion collisions



At RHIC (2000~), LHC (2012~)

- Magnetar



Observation by IXPE, XL-Calibur (2022~)

Press release by Tamagawa lab. (2022)

What can happen with strong EM field ?

When $eE > m_e^2$, many non-trivial phenomena have been predicted to occur:

Review: [Fedotov, Ilderton, Karbstein, King, Seipt, HT, Torgrimsson, Phys. Rept. (2023)]

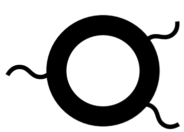
Sauter (1931), Schwinger (1951)

What I like the most: Production of particles from the vacuum (Sauter-Schwinger effect)

A few examples among many others (don't explain; ask later if interested)

Birefringence of photon in vacuum

Photon splitting



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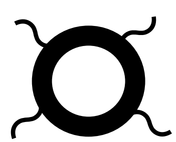
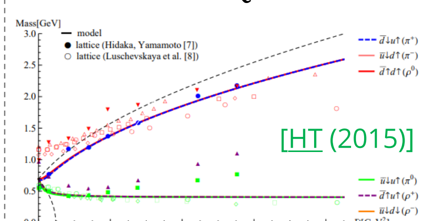
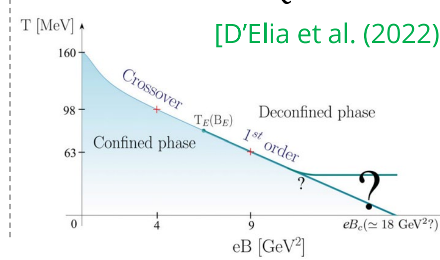


Hadron mass change (for $eB > \Lambda_{\rm QCD}^2$)



QCD phase diagram (for $eB > \Lambda_{\rm QCD}^2$)



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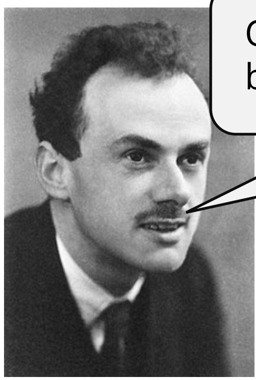
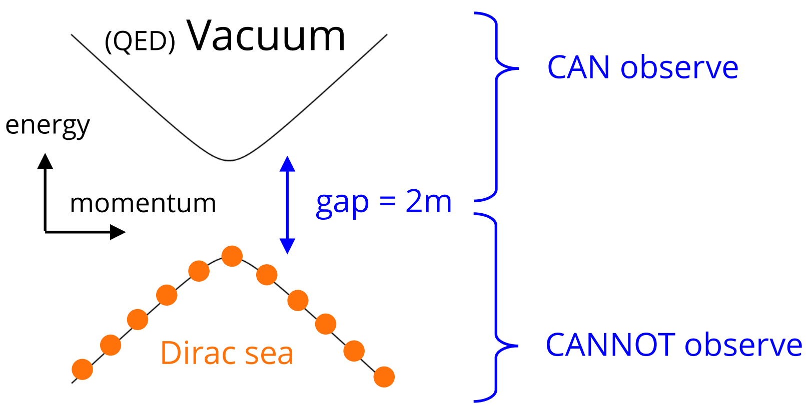


Dirac (1930)

Our vacuum is not an empty space,
but has a structure similar to semi-conductor



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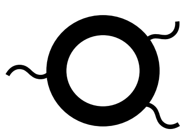

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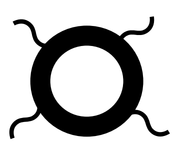

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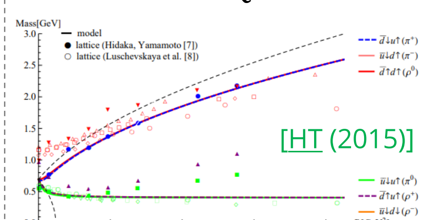

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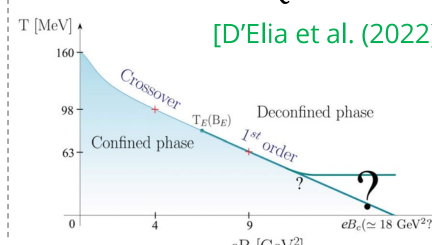

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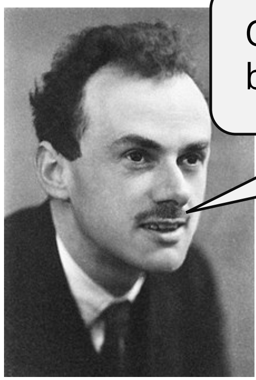

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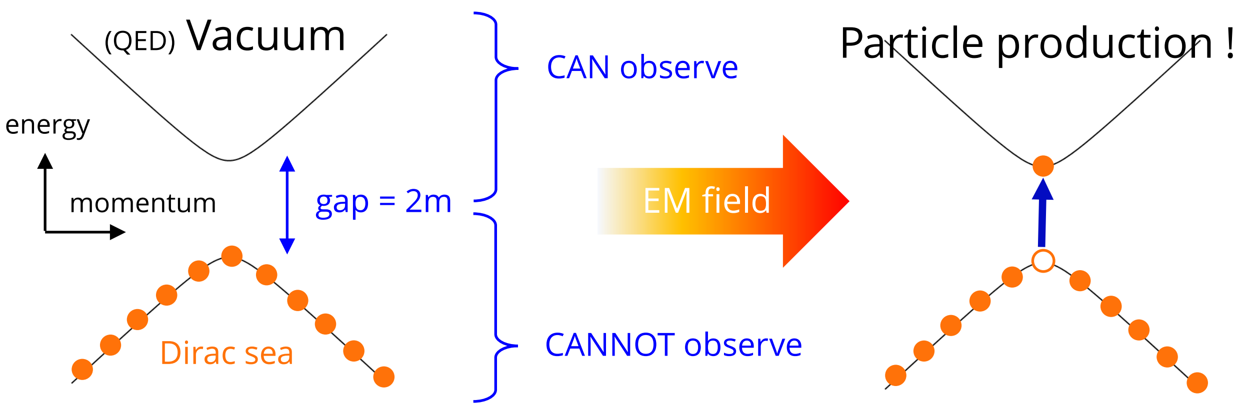
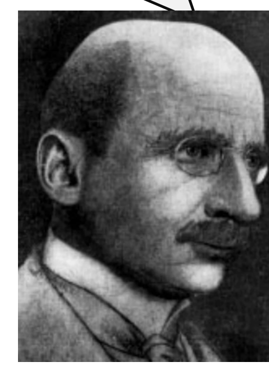
Then, real particles will be produced out of the vacuum by strong enough field





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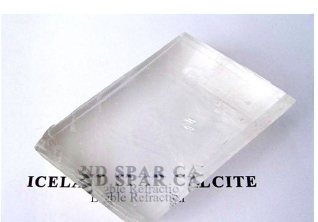

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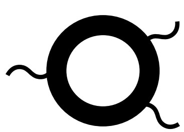

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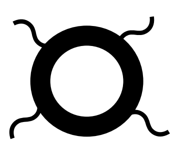

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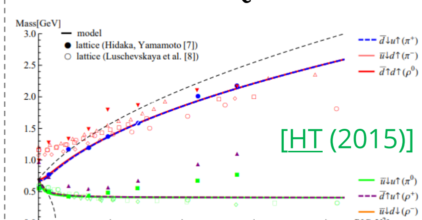

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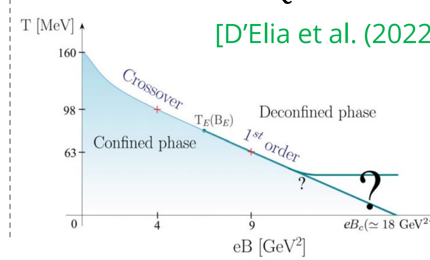

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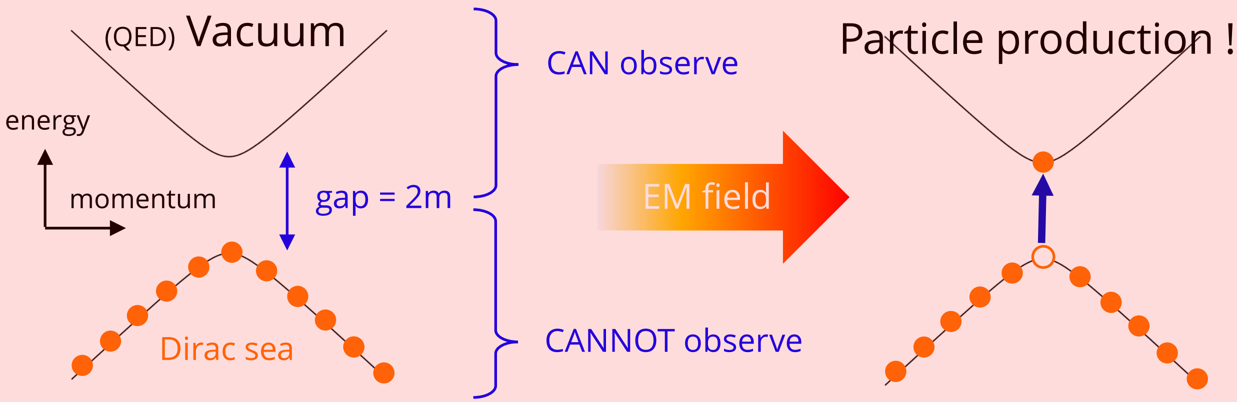

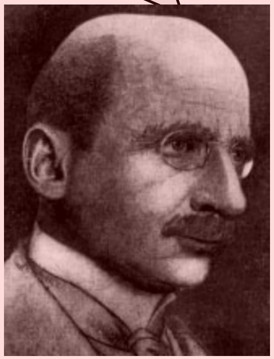

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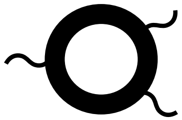

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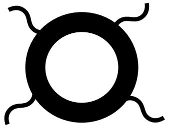

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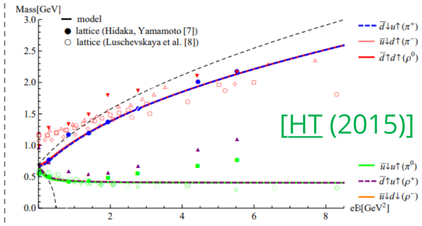

[HT (2015)]

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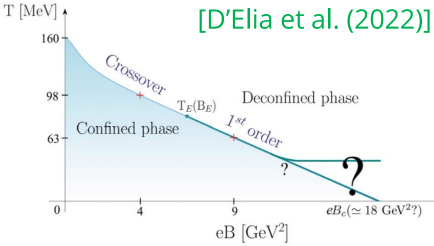

[D'Elia et al. (2022)]

PART I: Basics of strong-field physics

PART II: Sauter-Schwinger effect and its connection to other areas

Similar particle production mechanisms appear in many other areas of physics

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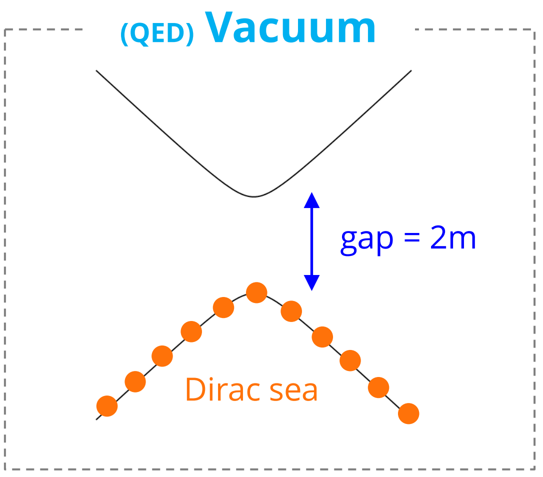
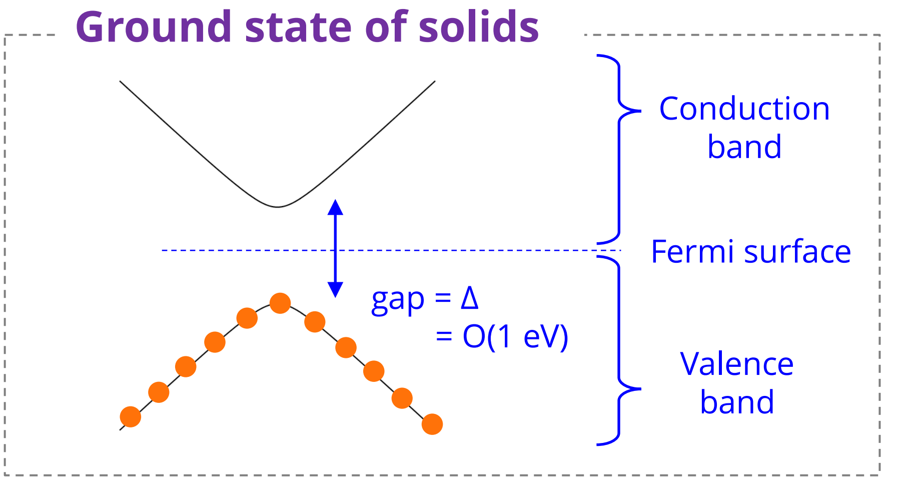
The most prominent example: Landau-Zener transition in solid

Landau (1932), Zener (1932)



v.s.



- The responses of the vacuum and ground state by an external EM field should be similar
- Indeed, the particle prediction rate and the excitation rate obey the same exponential formula

$$\Gamma_{\text{Sauter-Schwinger}} = \#\exp[\#\ m^2/_{eE}] \qquad \Gamma_{\text{Landau-Zener}} = \#\exp[\#\ \Delta^2/_{eE}]$$

Note: Similar exponential formula holds for particle production by other strong fields as well

Strong gravitational field ⇒ Hawking radiation, Strong inflaton field ⇒ (p)reheating of the early Universe, …

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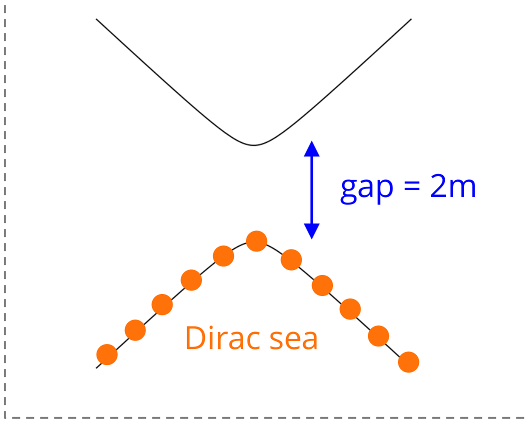
(QED) Vacuum



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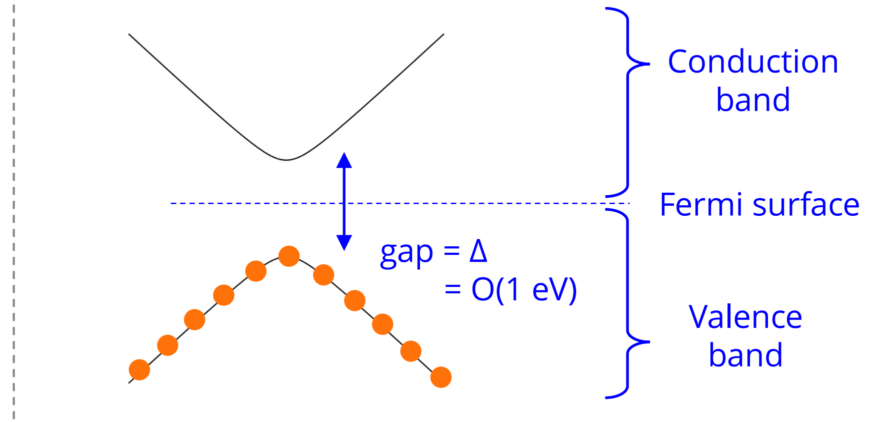
Ground state of solids



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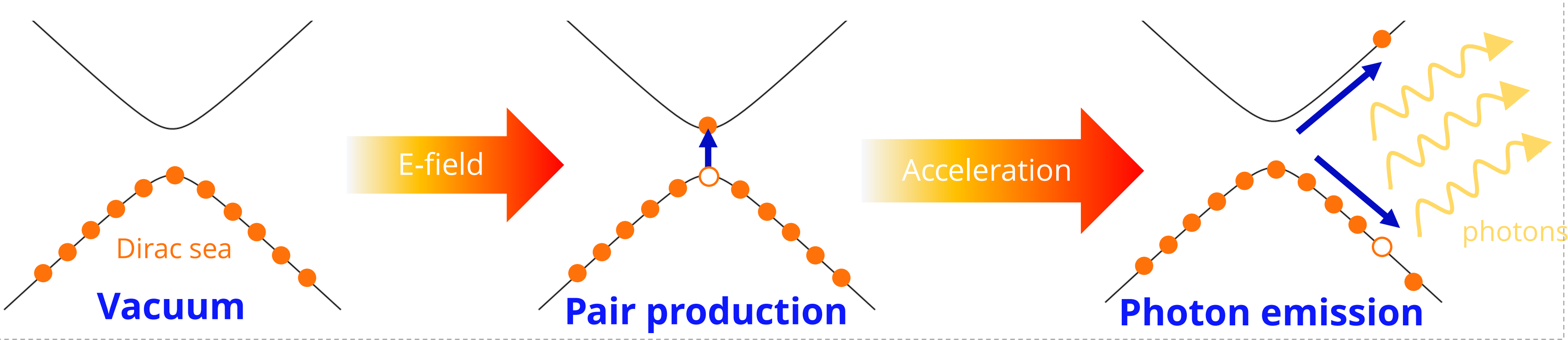
Strong gravitational field ⇒ Hawking radiation, Strong inflaton field ⇒ (p)reheating of the early Universe, ...

- Sauter-Schwinger effect has not been verified yet,
  but Landau-Zener transition has been observed and utilized, e.g., to design devices.

⇒ Importing (exporting) ideas from (to) other areas of physics is quite useful to better understand the Sauter-Schwinger effect (or strong-field QED in general)

Example: High-harmonic generation from the vacuum

Pair production by E-field leads to emission of photons



✔ **Consider AC field with frequency Ω:**

- Very naïve expectation: photon has frequency ω same to AC field ω=Ω

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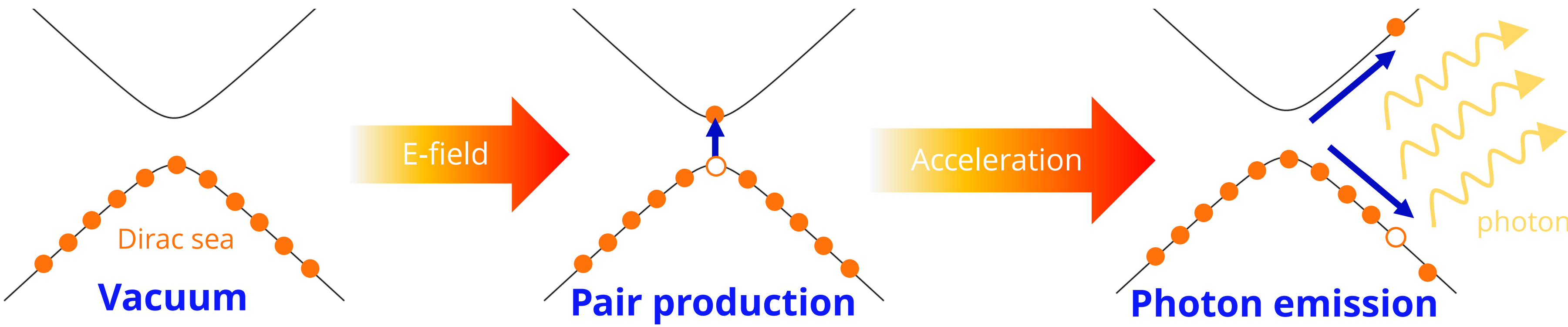

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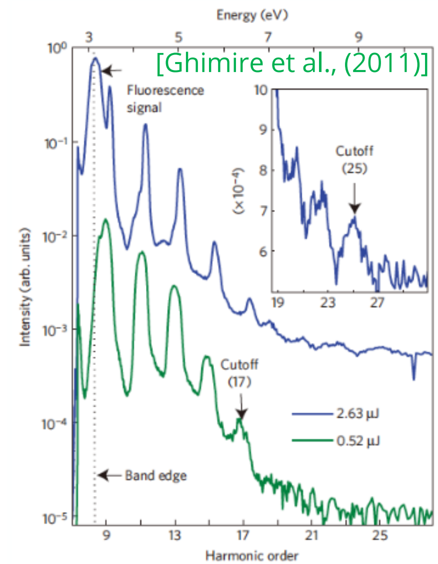
✔ Recent observation in semi-conductors

(also in many other materials)

- Naïve expectation is wrong
  ⇒ Exp. observation: Plateau structure
- Theoretical formulation is still immature even in cond.-mat
  ⇒ mostly numerics, and <u>analytical understanding is lacking</u>



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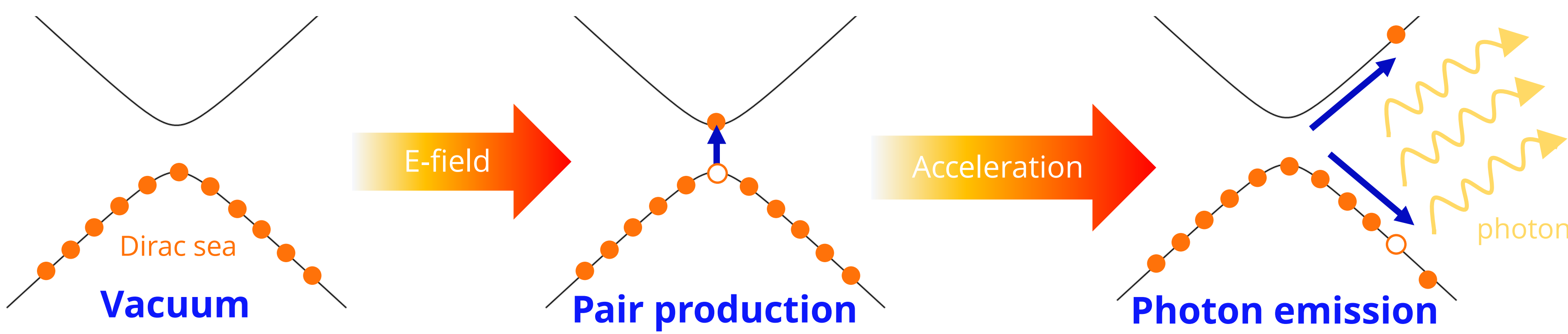

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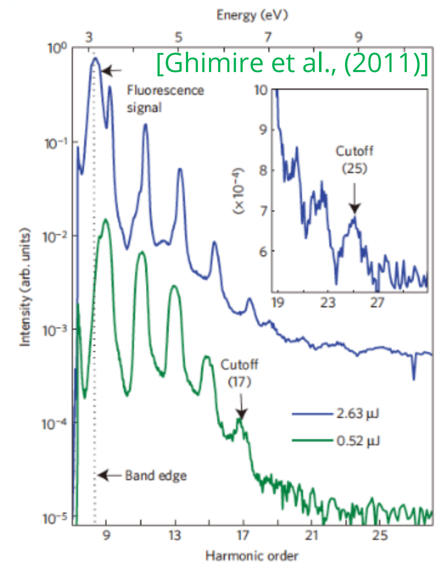

Q1: High-harmonic generation in QED from the vacuum ?

Q2: Is there any nice analytical method ?

A: Yes. Mathematics helped me to answer those !

How to formulate ? Typical math of strong-field QED

Roughly speaking, strong-field-QED calculations is reduced to the following:

Step 1: Solve a differential equation (to obtain wavefunction Φ ≈ )

The simplest example: Klein-Gordon equation with a time-dependent potential

$$0 = [\hbar^2 \partial_t^2 + Q(t)]\phi(t)$$

something small (Planck constant) — $\hbar$

Potential (= info. of strong field) — $Q(t)$

wavefunction — $\phi(t)$

decode

Step 2: Analyze the wavefunction (to predict sthg measurable O[Φ] ≈ )

- O depends on what you want to measure
- Typically, a bi-linear $\phi^\dagger X \phi$

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Solve a differential equation is purely a mathematical problem
⇒ Any good idea from mathematics
⇒ Exact WKB

Exact WKB method

Exact WKB = a nice method to solve ODE with a small parameter

[Voros (1983)]
[Pham, Dillinger, Delabaere,
Aoki, Koike, Takei, …]

= ”usual” WKB + Borel resum.

[Jeffery (1924)] [Wentzel (1926)]
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A perturbation theory w.r.t. $\hbar$ (or adiabatic approx.)

- Consider $0 = [\hbar^2 \partial_t^2 + Q(t)]\phi(t) \overset{t \equiv \hbar\tau}{\Leftrightarrow} [\partial_\tau^2 + Q(\hbar\tau)]\phi(\tau)$

$$\Rightarrow \phi_\pm(t;\hbar) := \exp\left[\mp \frac{\mathrm{i}}{\hbar}\int_{t_0}^{t} \mathrm{d}t' \sqrt{Q(t')}\right] \times \sum_{n=0}^{\infty} \psi_{\pm,n}(t)\hbar^n$$

0^{th} order = plane wave

$\sim \exp[\mp \frac{\mathrm{i}}{\hbar}\sqrt{Q}t]$

Perturbation w.r.t. $\hbar$

Exact WKB method

Exact WKB = a nice method to solve ODE with a small parameter

[Voros (1983)]
[Pham, Dillinger, Delabaere, Aoki, Koike, Takei, ...]

= "usual" WKB + Borel resum.

[Jeffery (1924)] [Wentzel (1926)]
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Perturbation w.r.t. $\hbar$

- WKB expansion makes sense if the perturbative part is convergent
- However, $\psi_{\pm,n} \sim n!$ in general (e.g., Airy function $Q(t) \propto t$)

⇒ WKB expansion has zero radius of convergence ⇒ ill-defined !!!

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A resummation scheme for factorially divergent ~n! series

- Consider the div. part of WKB expansion $\psi_\pm(t;\hbar) := \sum_{n=0}^{\infty} \psi_{\pm,n}(t)\hbar^n$

① Construct "Borel transformation": $B[\psi_\pm](t;\eta) := \sum_{n}^{\infty} \frac{\psi_{\pm,n}(t)}{n!}\eta^n$

② Laplace trans. gives "Borel sum": $\Psi_\pm(t;\hbar) := \int_0^\infty \frac{\mathrm{d}\eta}{\hbar}\, \mathrm{e}^{-\eta/\hbar} B[\psi_\pm](t;\eta)$

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② Laplace trans. gives "Borel sum": $\Psi_\pm(t;\hbar) := \int_0^\infty \frac{d\eta}{\hbar}\, e^{-\eta/\hbar} B[\psi_\pm](t;\eta)$

- $\Psi_\pm$ is well-defined and is a natural analytic continuation of $\psi_\pm$

$\Leftarrow$ Indeed, reduces to the original result if you adimit $\sum\int = \int\sum$

$$\Psi_\pm = \int_0^\infty \frac{d\eta}{\hbar}\, e^{-\eta/\hbar} \sum_n^{\infty} \frac{\psi_{\pm,n}(t)}{n!}\eta^n \sim \sum_n^{\infty} \frac{\psi_{\pm,n}(t)}{n!} \int_0^\infty \frac{d\eta}{\hbar}\, e^{-\eta/\hbar}\eta^n = \sum_{n=0}^{\infty} \psi_{\pm,n}(t)\hbar^n$$

$\Rightarrow$ **$\Psi_\pm$ gives a well-defined version of the WKB solution !**

- Note: in practice, some approximations shall be used in Borel resum...

How to formulate ? Typical math of strong-field QED

Step 1: Solve a differential equation (to obtain wavefunction Φ ≈

The simplest example: Klein-Gordon equ… …me-dependent potential

$$0 = [\hbar^2 \partial_t^2 + Q(t)]\phi(t)$$

something small (Planck constant) — potential — wave… …ion

DONE

decode

Step 2: Analyze the wavefunction (to predict sthg measurable O[Φ] ≈)

- O depends on what you want to measure
- Typically, a bi-linear $\phi^\dagger X \phi$ ⇐ For HHG, current $J \sim e\phi^\dagger \hat{p}\phi$ or $J \sim \bar{\psi}\gamma\psi$ (for fermions)

Application to high-harmonic generation (1/2)

Setup: A monochromatic E-field $E(t) = E_0 \cos(\Omega t)$ applied onto QED vacuum

What I did: Compute harmonic spectrum **numerically** and **analytically with WKB**

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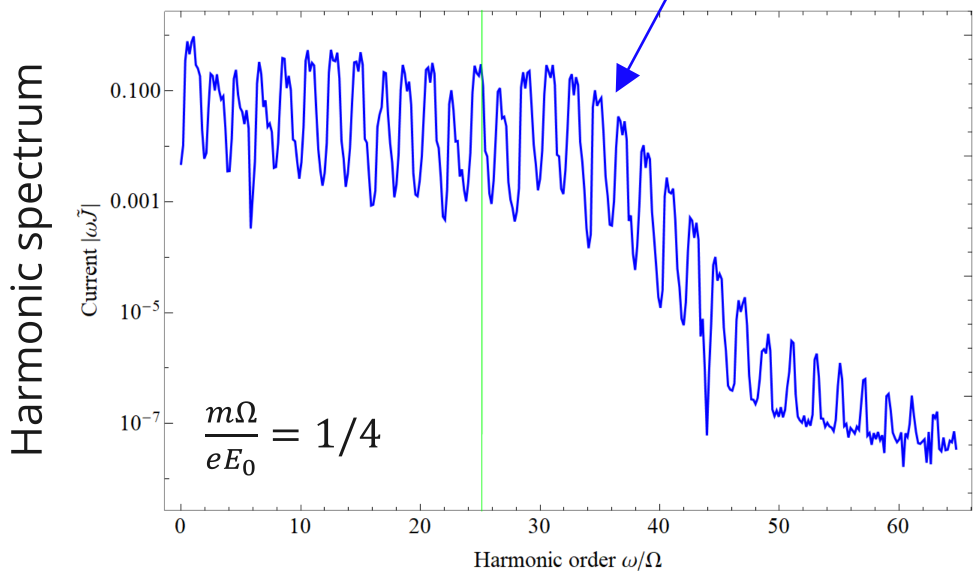

- High harmonics appears also in QED when field becomes strong !

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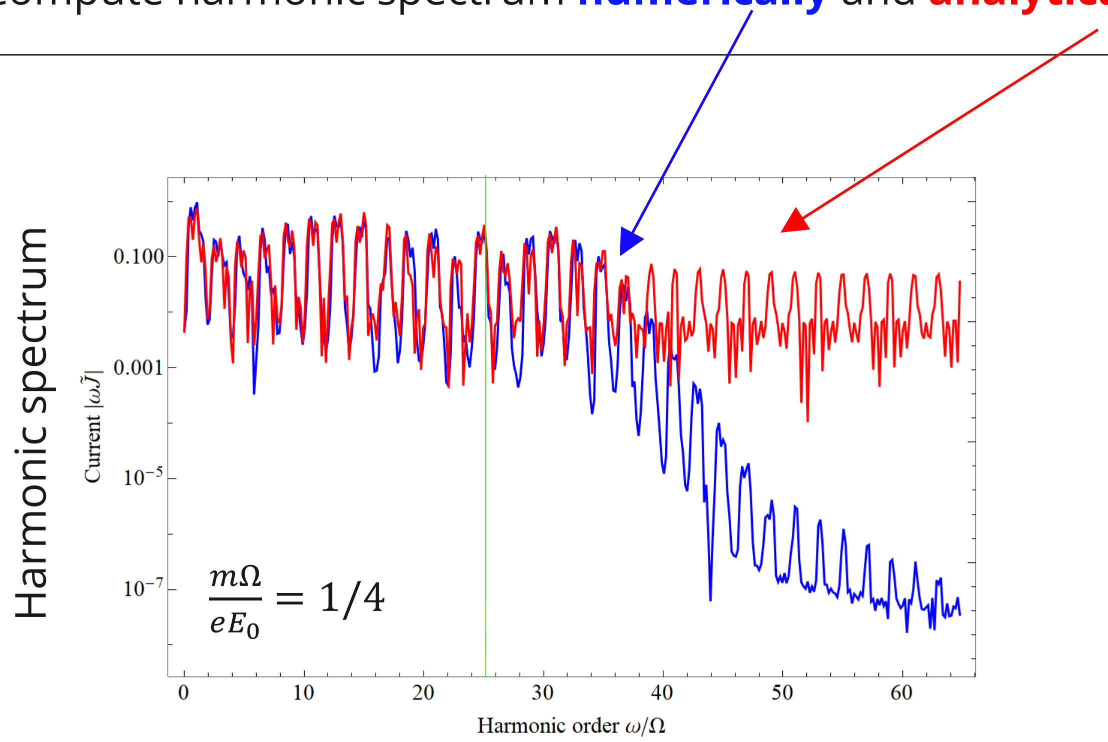
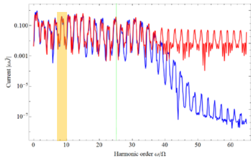


- High harmonics appears also in QED when field becomes strong !
- WKB is good in the plateau regime (i.e., before the spectrum drops)

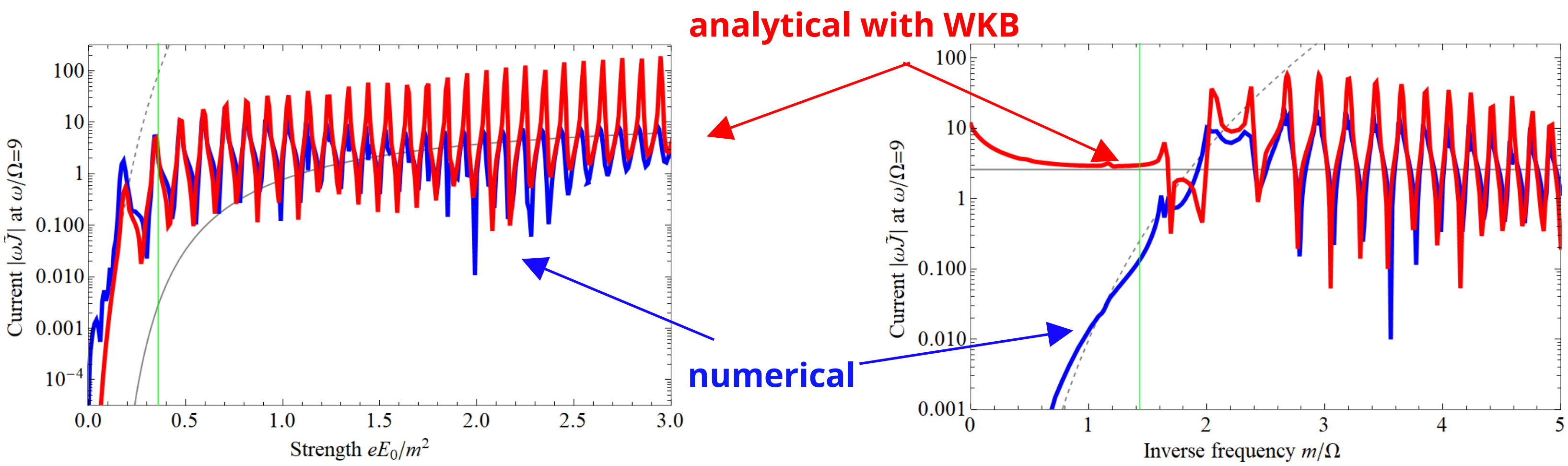
Application to high-harmonic generation (2/2)

- **WKB works more in the deep non-perturbative regime $E_0 \to \text{large}, \Omega \to \text{small}$**

- **Demonstration: magnitude of the harmonic peak at $\omega/\Omega = 9$**



Dependence on strength E_0 Dependence on frequency Ω



- **Lessons:**

  (1) WKB makes it easier to analyze the non-perturbative regime

  (2) Saturation & oscillation of the harmonic intensity

  ⇒ consistent with recent semi-conductor exp. [Xia et al., (2020)]

  (but only E_0-dep. is measured and Ω-dep. is our prediction)

PART I: Basics of strong-field physics

PART II: Sauter-Schwinger effect and its connection to other areas

Summary

(QED = quantum electrodynamics)

Brief introduction to Strong-field QED
= an area to study what happens by a super strong light

(= electromagnetic field)

PART I: Basics of strong-field QED

- Why interesting ⇒ Unexplored non-perturbative regime of physics
  Nontrivial phenomena that we've never seen e.g. Vacuum decay, Birefringence, New phase of matter, …
- Why timely ⇒ Recent availability of strong fields e.g. High-power laser, Magnetar, heavy-ion collisions, …

PART II: Sauter-Schwinger effect and its connection to other areas

- An interdisciplinary topic: Analogues in many other areas of physics
  ⇒ Exchange of ideas/techniques among physics is quite useful for better understandings
  e.g. Landau-Zener (cond.-mat), Hawing radiation (gravity), reheating in the early Universe (cosmology), …
- Mathematically, the problem is essentially solving a differential equation
  ⇒ Mathematical techniques to solve differential equations are quite useful
- As such an example: High-harmonic generation from the vacuum
  ⇐ Based on (exact) WKB in mathematics and an idea in cond.-mat
  [HT, Fujimori, Misumi, Nitta, Sakai, (2021)] [HT, Hongo, Ikeda, (2021)]

Intuitive picture

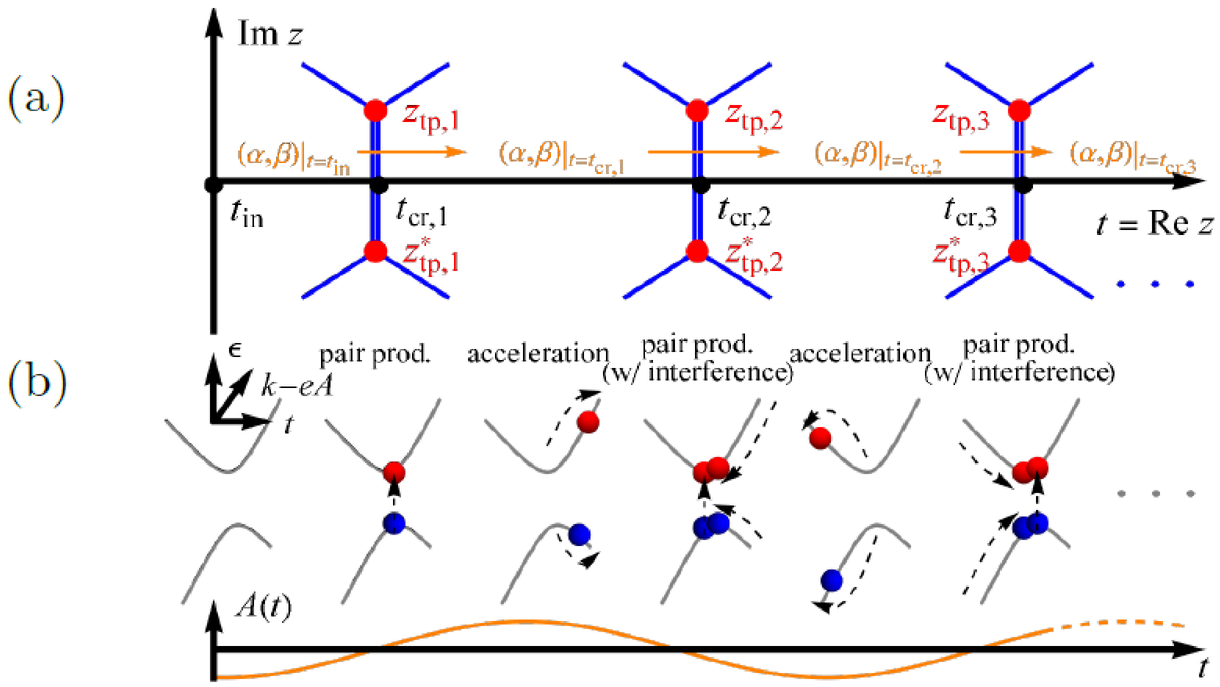


FIG. 1. (a) A typical Stokes graph, composed of Stokes lines (blue lines) and turning points (red points), and (b) the corresponding physical processes during the real-time evolution.