

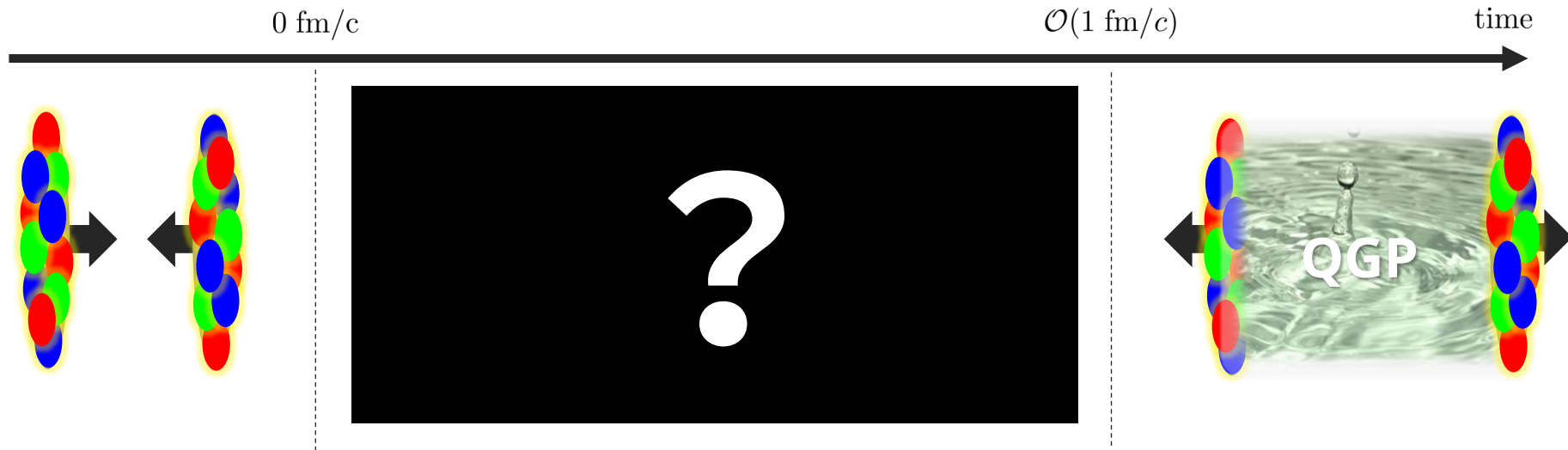
# **Initial state and thermal equilibrium**

**(Theory)**

**Hidetoshi Taya**  
**(RIKEN iTHEMS)**



# What I am going to talk



**This talk:** Review of what is happening in  (focus on the past 10-year developments)

**Short summary:** No longer a black box, thanks to

- Significant progress around 2015  
⇐ a weak coupling scenario & hydrodynamization
- Sophistication after ~2015 ⇐
  - ~2015's idea is becoming more robust
  - gradually making connection to exp.
  - making rich interactions to other fields

**Caveats:**

- Don't talk about CGC; please see [Talk by Fujii, Wed. 15:05~](#)
- This is a review talk, i.e., don't talk about my own works

**I. Progress ~2015**

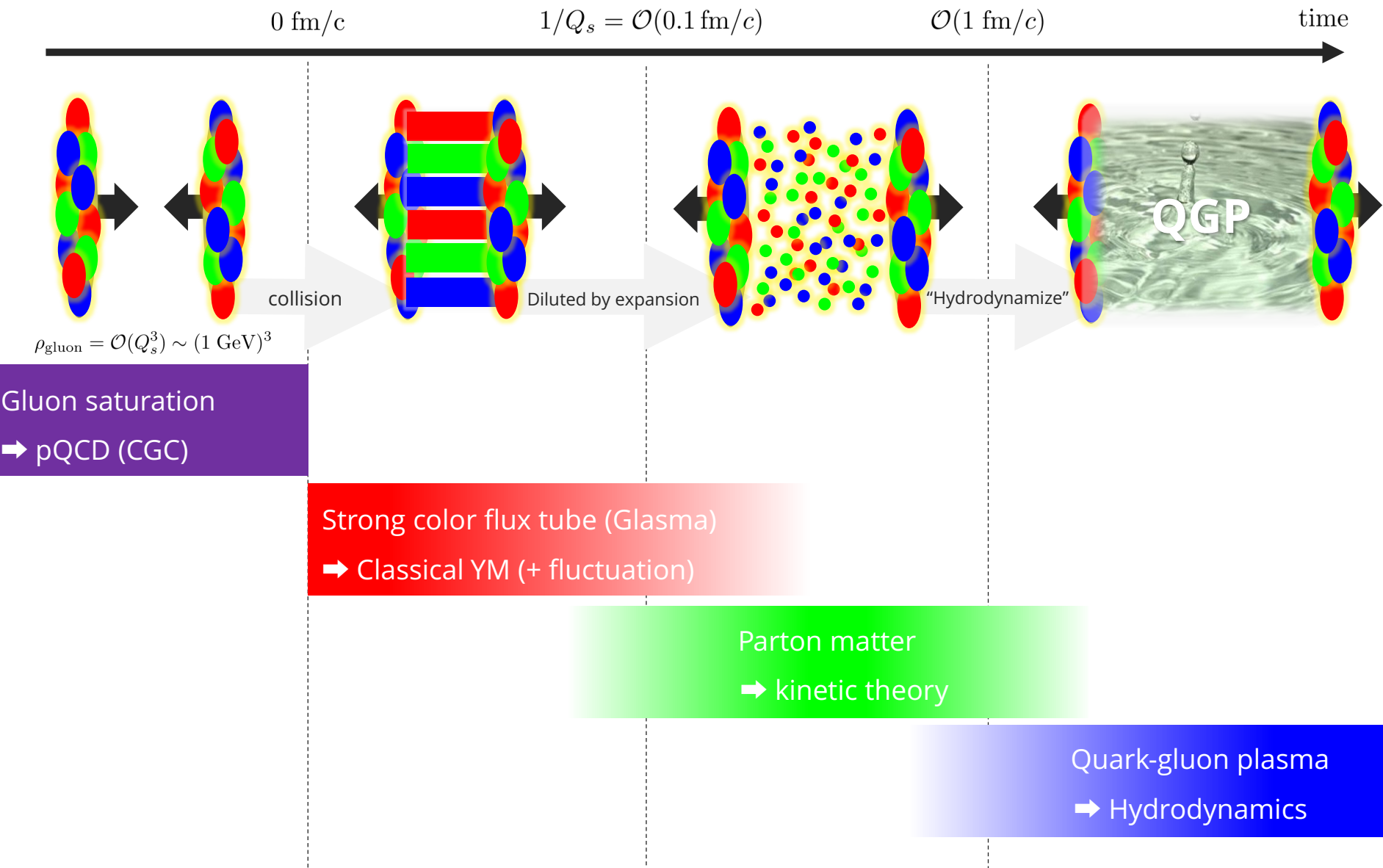
II. Progress after 2015

III. Summary

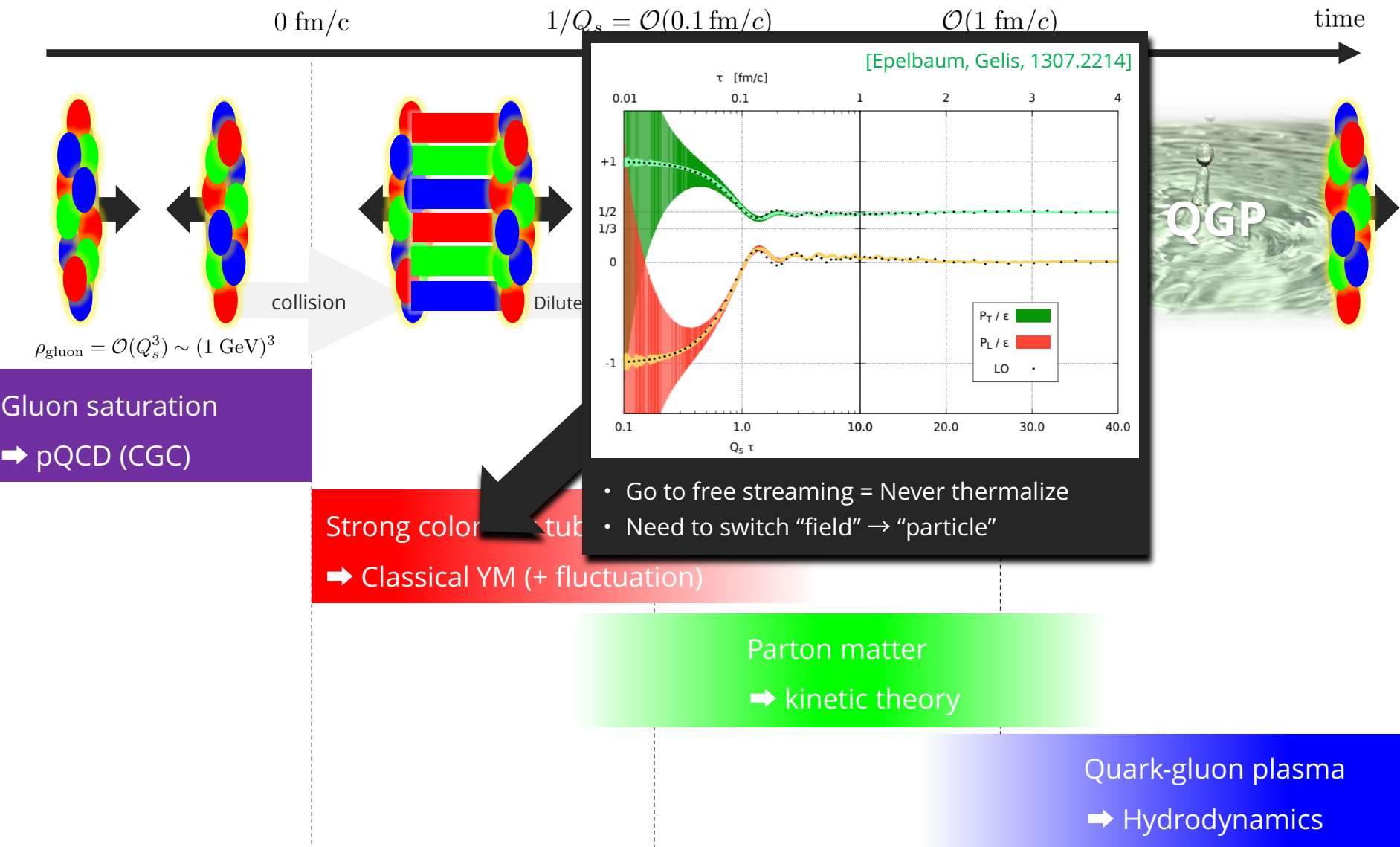
# Progress ~2015: weak coupling scenario & hydrodynamization



# Progress ~2015: weak coupling scenario & hydrodynamization

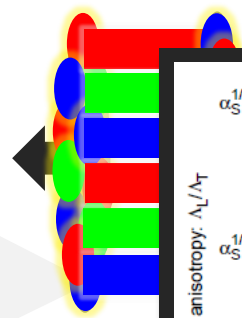
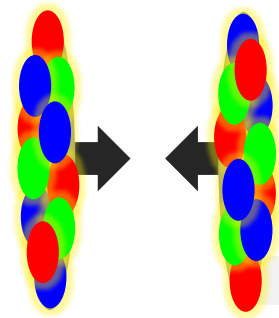


# Progress ~2015: weak coupling scenario & hydrodynamization



# Progress ~2015: weak coupling scenario & hydrodynamization

0 fm/c                       $1/Q_s = \mathcal{O}(0.1 \text{ fm}/c)$                        $\mathcal{O}(1 \text{ fm}/c)$                       time



collision

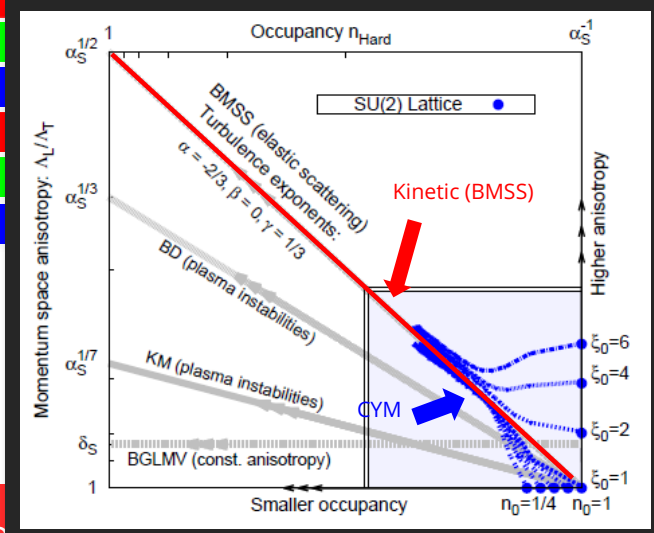
$\rho_{\text{gluon}} = \mathcal{O}(Q_s^3) \sim (1 \text{ GeV})^3$

Gluon saturation  
 → pQCD (CGC)

Strong color fields  
 → Classical YM (+ fluctuations)

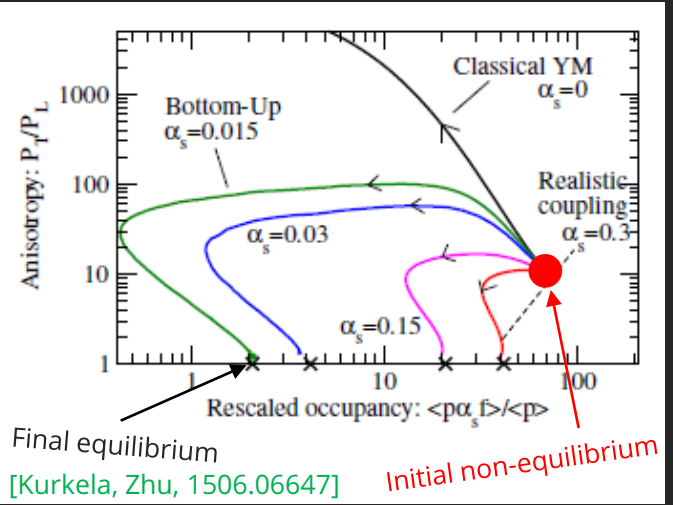
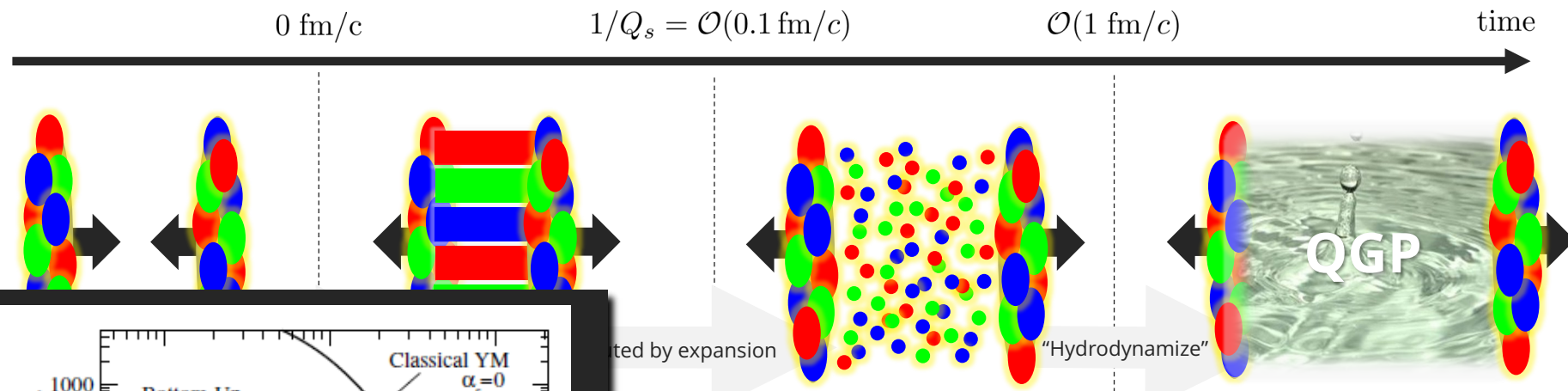
Parton matter  
 → kinetic theory

Quark-gluon plasma  
 → Hydrodynamics



- CYM and kinetics are smoothly connected with each other
  - overlap at  $1/g^2 \gg f \gg 1$ 
    - [Mueller, Son, hep-ph/0212198]
    - [Jeon, hep-ph/0412121]
  - consistent with bottom-up scenario
    - [Baier, Mueller, Schiff, Son, hep-ph/0009237]
  - non-thermal attractor
    - [Berges, Boguslavski, Schlichting, Venugopalan, 1303.5650]

# Progress ~2015: weak coupling scenario & hydrodynamization



- Kinetic theory of QCD (AMY) [Arnold, Moore, Yaffe, hep-ph/0209353] does describe isotropization & thermalization
- BUT, the time scale is too slow  $\tau \sim 10 \text{ fm}/c$

driven by expansion

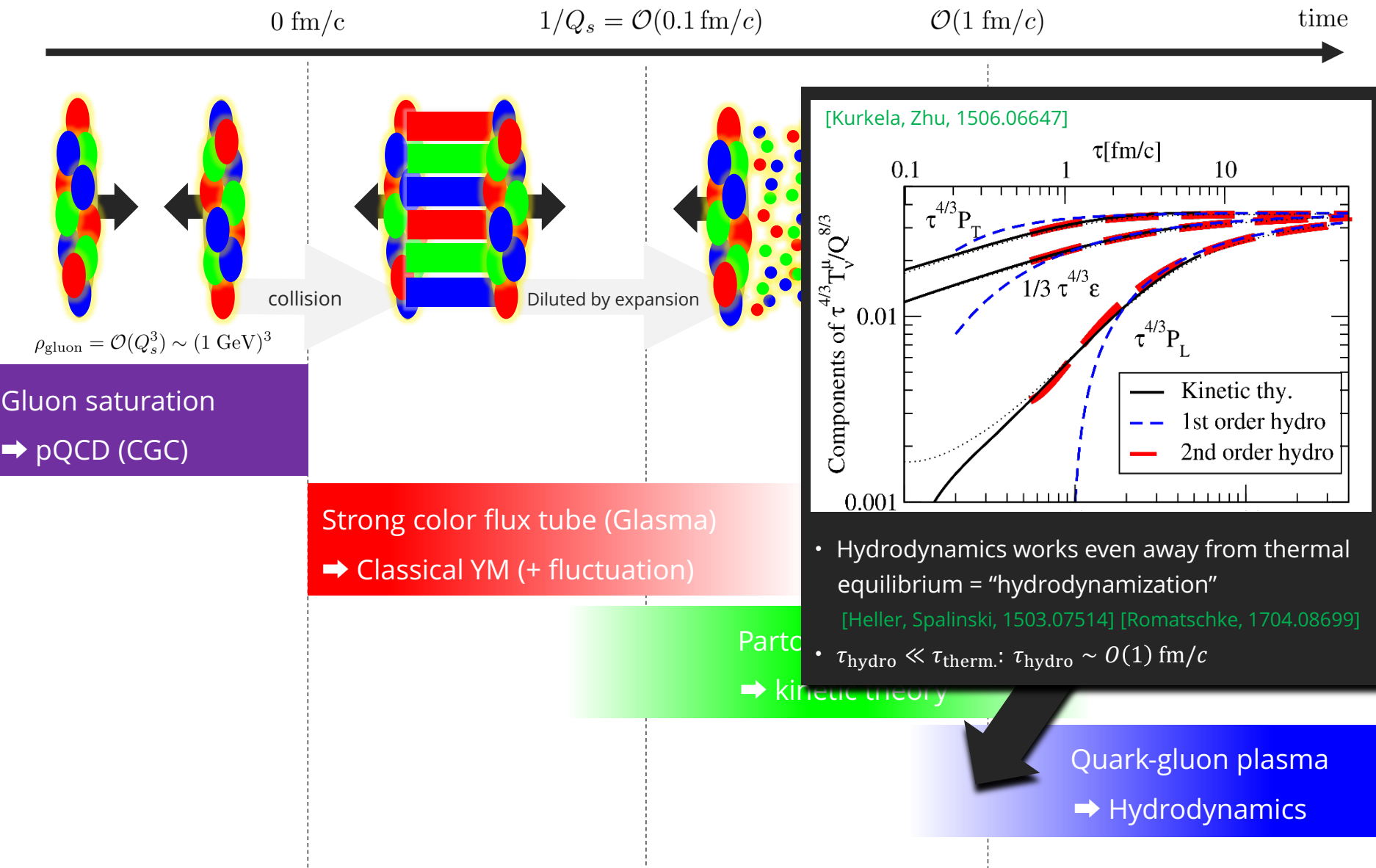
"Hydrodynamize"

fluctuation, ...  
 Parton matter  
 → kinetic theory

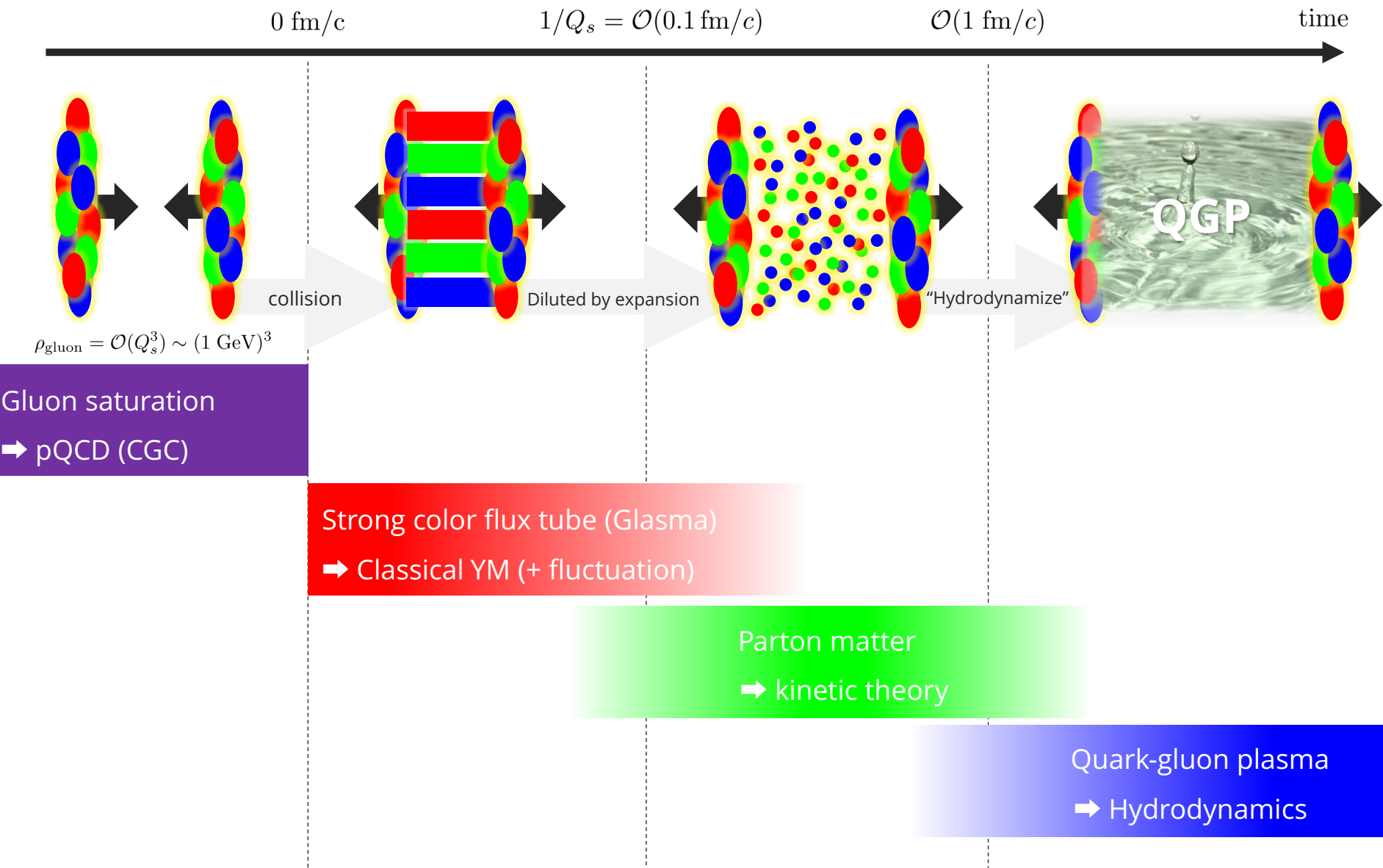
Quark-gluon plasma  
 → Hydrodynamics



# Progress ~2015: weak coupling scenario & hydrodynamization



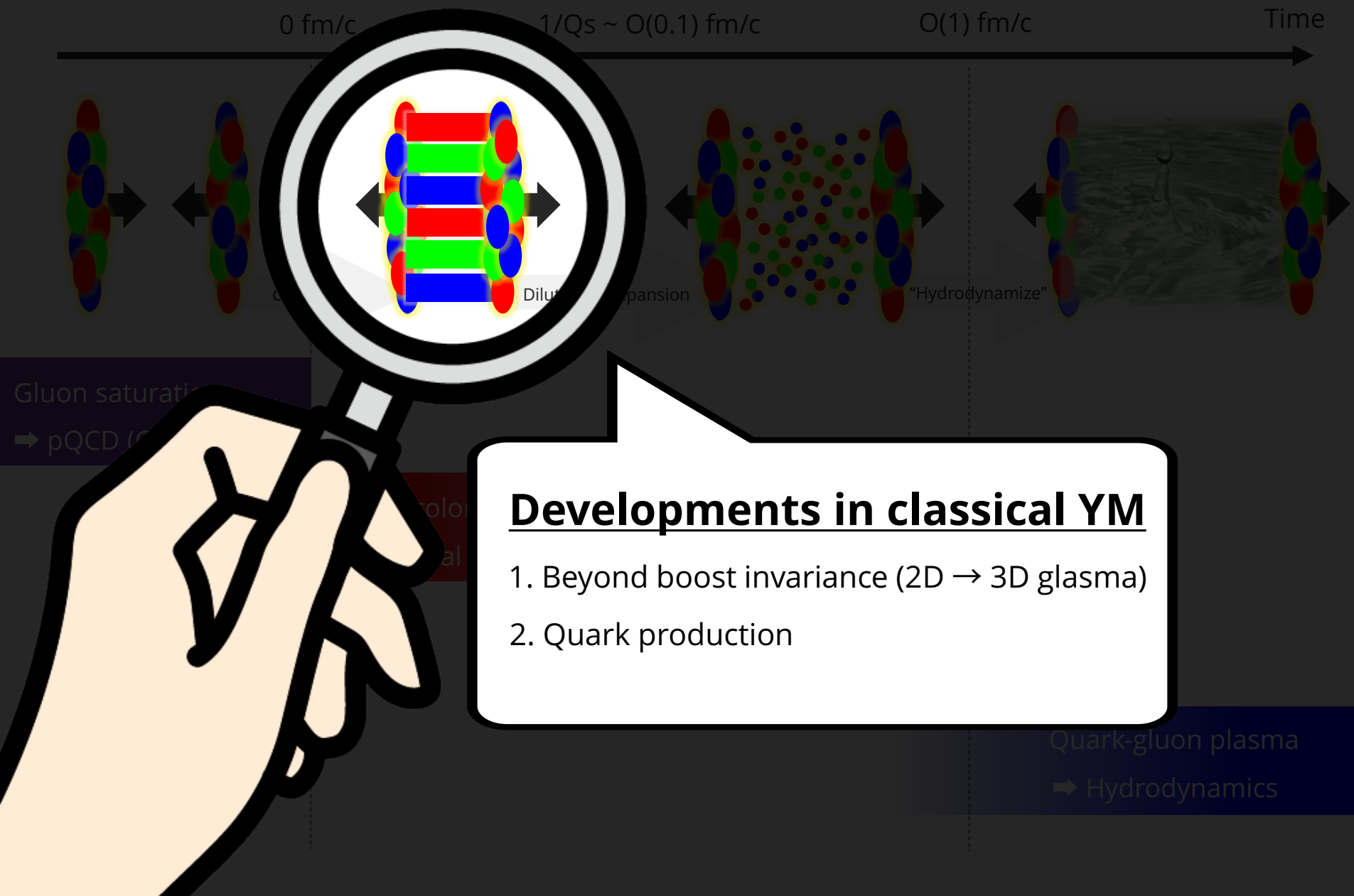
# Progress ~2015: weak coupling scenario & hydrodynamization



I. Progress ~2015

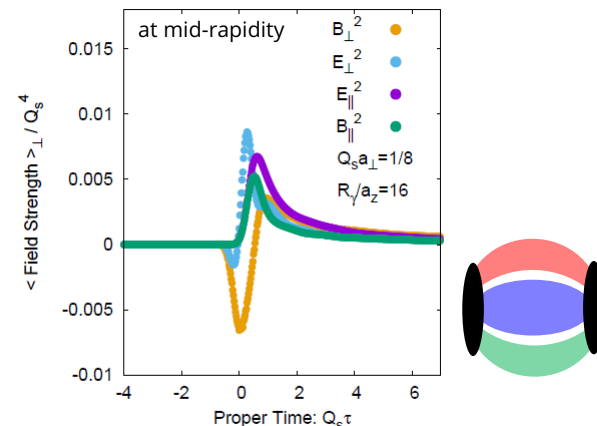
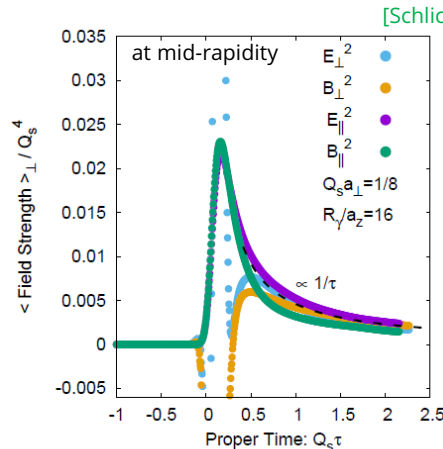
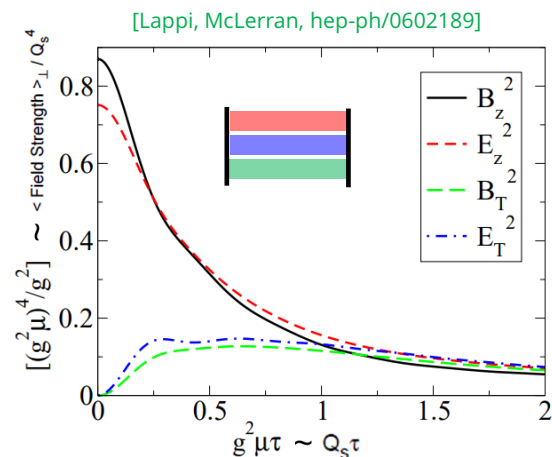
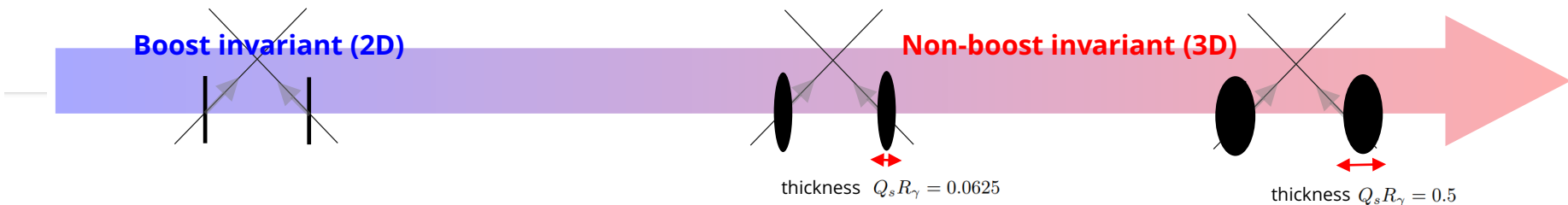
**II. Progress after 2015**

III. Summary



# 1. Beyond boost invariance (2D → 3D glasma)

**Idea:** Solve classical Yang-Mills eq.  $D_\mu F^{\mu\nu} = J^\nu$  with finite-thickness source  $J^\mu \not\propto \delta(x^\pm)$



- Early times: purely longitudinal
- Late times: free streaming

• Two ways to introduce thickness:

- JIMWLK [Schenke, Schlichting, 1605.07158]  
[McDonald, Jeon, Gale, 2001.08636 (proceeding)]
- Gaussian smearing [Ipp, Muller, 1703.00017]  
[Schlichting, Singh, 2010.11172]

Talk by Matsuda, Mon. 13:50~

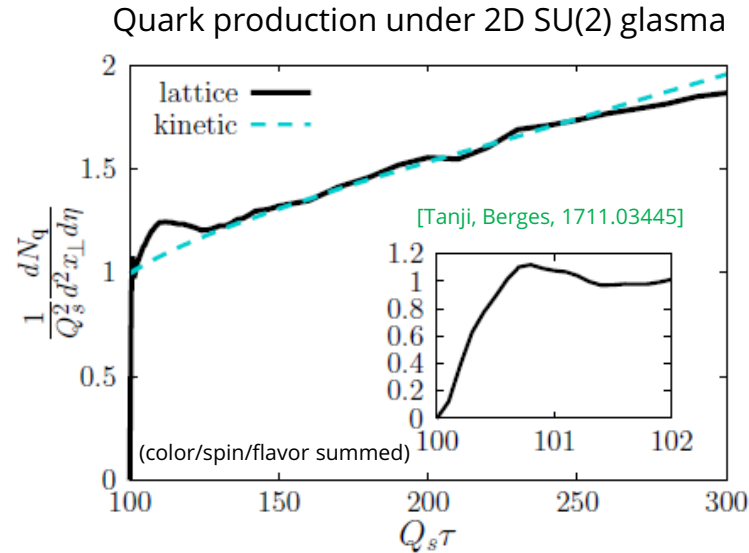
- Early times: Not purely longitudinal
- Late times: free streaming

**Lessons:**

- Non-boost invariance does affect the physics at early times
- Theorists have started to make a better modelling for longitudinal observables  
cf. reproduces longitudinal dependencies of multiplicity and flow [McDonald, Jeon, Gale, 2001.08636 (proceeding)]

# 2. Quark production

**Idea:** Solve Dirac eq. in the presence of glasma field  $0 = [i\partial - g\mathcal{A} - m]\psi$



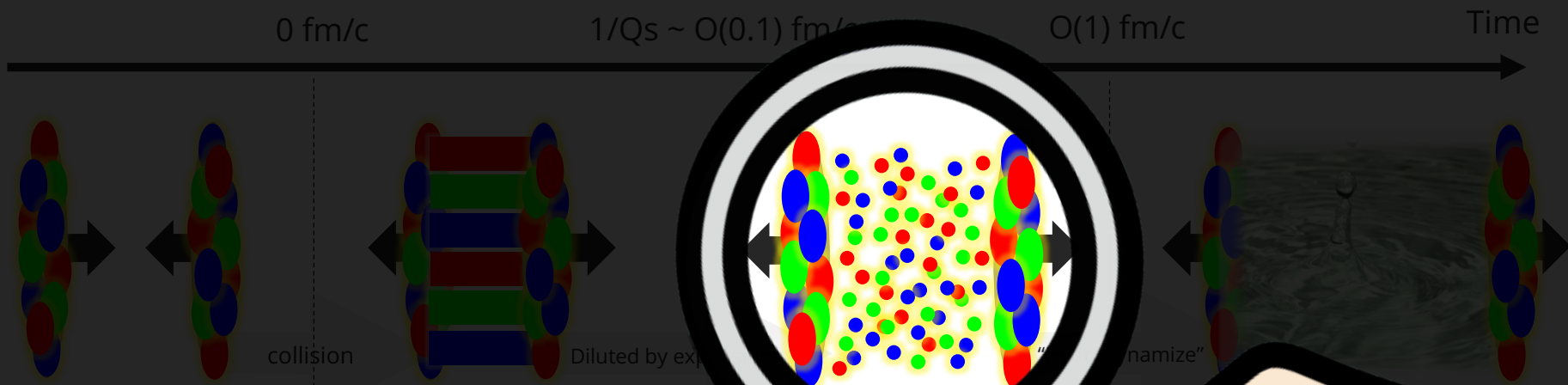
- Quark production is very fast: [Gelis, Kajantie, Lappi, hep-th/049508 & 0508229] [Gelfand, Hebenstreit, Berges, 1601.03576] [HT, 1609.06189] [Tanji, Berges, 1711.03445]  
 $Q_s^2 S_\perp = \mathcal{O}(1000) \Rightarrow \mathcal{O}(1000)$  quarks (per rapidity) are produced within  $\tau \lesssim 1/Q_s = \mathcal{O}(0.1 \text{ fm}/c)$
- Q: Why so fast?  
 A: Schwinger effect  $\Rightarrow$  occurs when energy supply by the field  $W = gE \times \tau \sim Q_s^2 \tau$  exceeds the mass gap  $W \gtrsim m$   
 $\Rightarrow \tau = m/Q_s^2 \ll 1/Q_s$

**Lessons:** • Fast & abundant quark production

- Can leave experimental imprints, e.g., charged flow in asymmetric collisions

Exp: [STAR, 1608.04100]

Thr. [Hirono, Hongo, Hirano, 1211.1114] [V. Voronyuk et al., 1410.1402]



0 fm/c

$1/Q_s \sim O(0.1) \text{ fm/c}$

$O(1) \text{ fm/c}$

Time

collision

Diluted by expansion

"thermalize"

Gluon saturation

→ pQCD (CGC)

matter  
kinetic theory

Quark-gluon plasma

→ Hydrodynamics

## Developments in kinetic theory

1. Inclusion of quarks
2. Numerical code: K $\phi$ MP $\phi$ ST

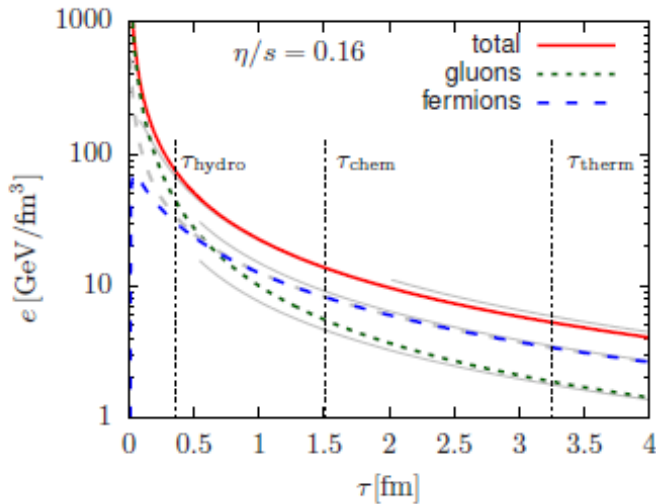
# 1. Inclusion of quarks

**Idea:** Add quark D.o.G. to relativistic kinetic eq.  $p^\mu \partial_\mu f = \mathcal{C}$

- 2 nice developments

## (1) Quark chemical equilibration

[Kurkela, Mazeliauskas, 1811.03040 & 1811.03068]

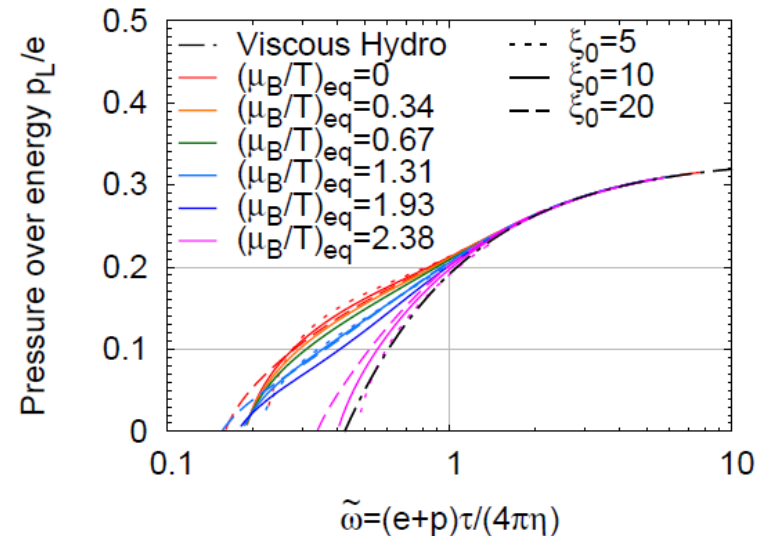


- Hierarchy of time scales  $\tau_{\text{hydro}} < \tau_{\text{chem}} < \tau_{\text{therm}}$
- Consistent with the two-stage equilibration picture: Gluons equilibrate first and then quarks [Shuryak, PRL (1992)]

Poster by Abdi, Tue. 18:00~

## (2) Impacts on isotropization/hydrodynamization

[Du, Schlichting, 2012.09068 & 2012.09079]



- Quark D.o.G lower the longitudinal pressure  $\Rightarrow$  Slower isotropization
- Finite  $\mu$  significantly affects the effective constitutive relation

## Lessons:

- Theorists are now able to discuss the initial quark dynamics in the kinetic phase  $\Rightarrow$  Will provide an input for, e.g., photon production at early times
- Quark D.o.G can affect the early-time behaviors
- ( • No glasma contr. in both. Including both of glasma & kinetic contr. is interesting. )

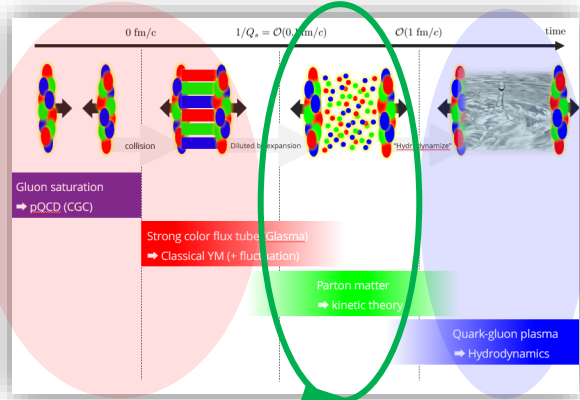


# 2. Numerical code: $K\phi MP\phi ST$ (1/2)

**Idea:** New hydro input accounting for the pre-equilibrium kinetic phase

Initial-state models:  
Glauber, MC-KLN, IP-Glasma, ...

Hydro models:  
MUSIC, VISUNU, ...

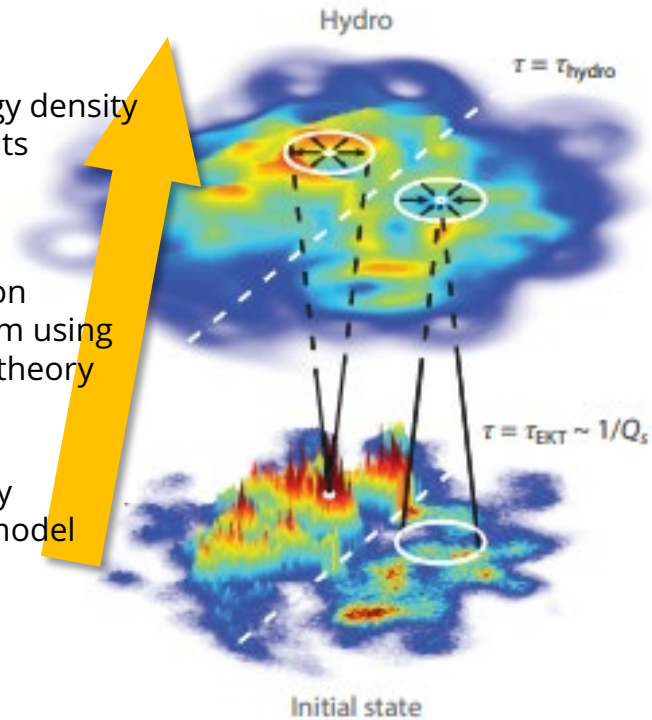


[Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, 1805.00961 & 1805.01604]

STEP 3: Use the energy density as hydro inputs

STEP 2:  $K\phi MP\phi ST$  evolution  
= evolve the system using linear response theory

STEP 1: Get initial energy density with some initial-state model



- Model of pre-equilibrium kinetic dynamics:  $K\phi MP\phi ST$

Basic assumptions: linear response, conformality, (extrapolation of) weak coupling

- Initially energy density only, recently extended to include conserved ( $U(1)$ , strangeness, baryon) charges (ICING)

[Carzon, Martinez, Noronha-Hostler, Plaschke, Schlichting, Sievert, 2301.04572]

**Lesson:** Now we have a numerical code to include the pre-equilibrium dynamics !

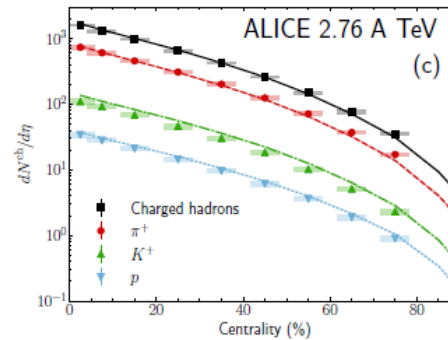
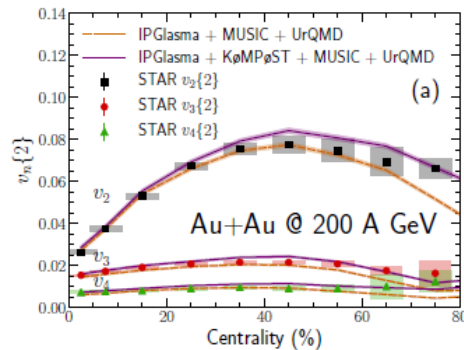
# 2. Numerical code: K $\phi$ MP $\phi$ ST (2/2)

**Idea:** Use K $\phi$ MP $\phi$ ST as an actual hydro input, and test how it works (or not)

- Can reproduce some observables like flow and multiplicity well

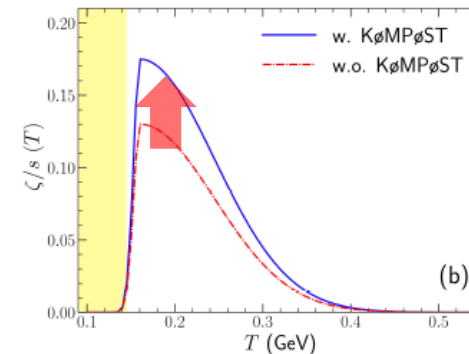
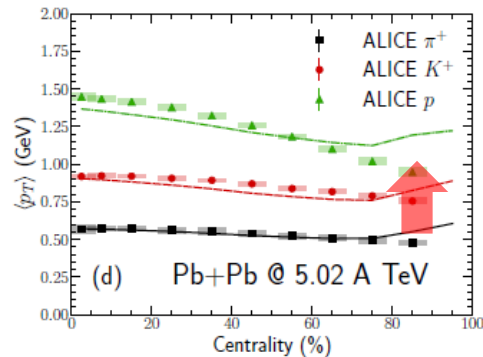
[Gale, Paquet, Schenke, Shen, 2106.11216]  
= IP glasma + K $\phi$ MP $\phi$ st + MUSIC + UrQMD

[Nunes da Silva, Chinellato, Hippert, Serenone, Takahashi, Denicol, Luzum, Noronha, 2211.10561]  
= Trento + K $\phi$ MP $\phi$ st + MUSIC + UrQMD



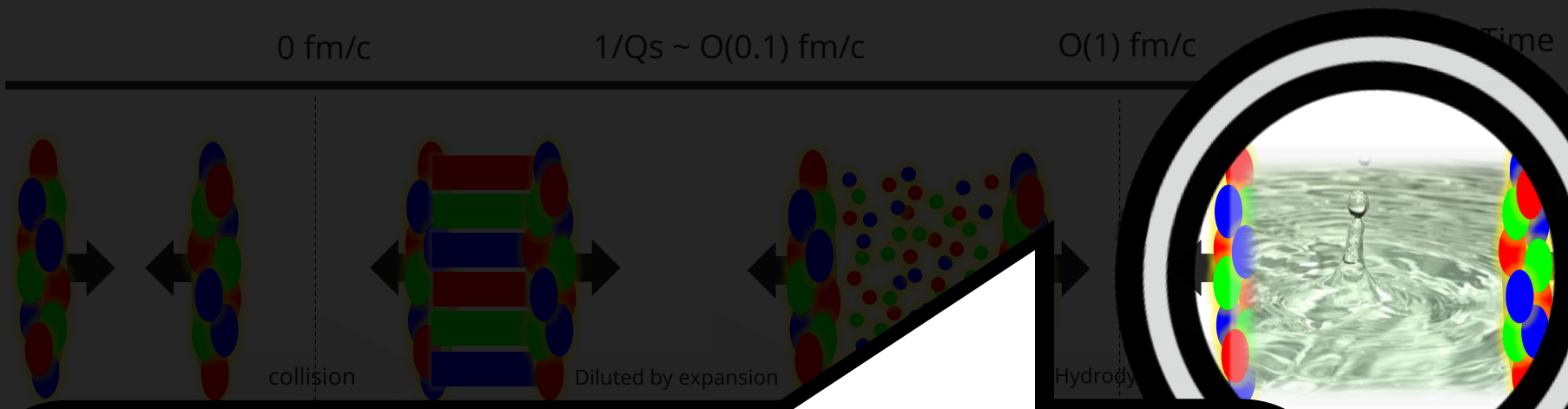
- Significant changes in some observables due to conformality

⇒ No bulk stress in K $\phi$ MP $\phi$ ST phase ⇒ More radial flow & less entropy production



**Lessons:** • Conformality seems a problem ⇒ Needs further sophistication

- Even so, clearly demonstrating that the pre-equilib. dynamics does affect when extracting the QGP properties



## Developments in hydrodynamization/attractor

1. Weak vs strong coupling
2. Two origins of attractor: early/late time attractors
3. Attractors beyond  $T_{\mu\nu}$
4. Lower symmetries (e.g. non-conformal case)
5. Implications to QCD-phase-diagram study in HIC

# 1. Weak vs strong couplings

**Idea:** Compare with the strong-coupling result of AdS/CFT

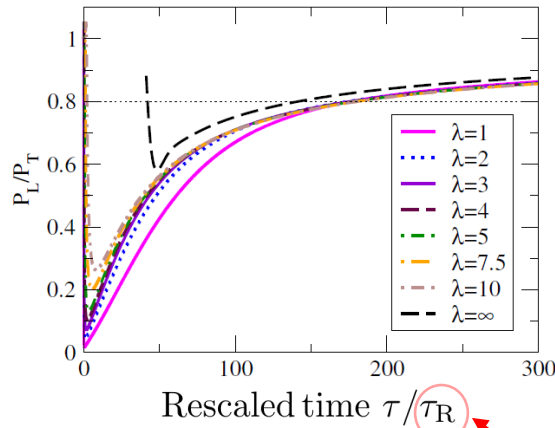
- Actual QCD coupling is not so small  $\alpha = \frac{g^2}{4\pi} \sim 0.3$  ( $\lambda = g^2 N_c \sim 10$ )

- Q: Are there any differences ?

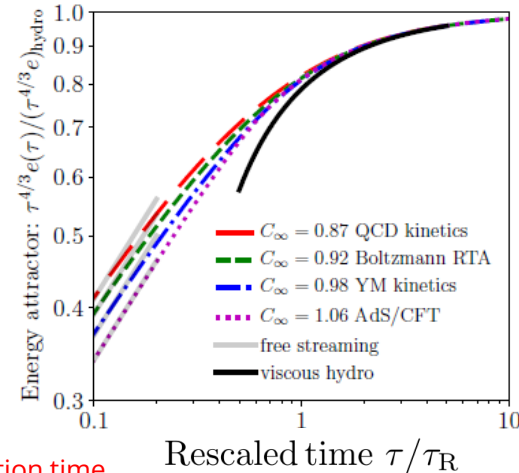
A: Basically consistent with the weak-coupling result after rescaling of time (though there do exist diffs.)  
(I will mention later)

Bjorken expansion with conformal symmetry

[Keegan, Kurkela Romatschke, van der Schee, Zhu, 1512.05347]  
Pressure anisotropy



[Giacalone, Mazeliauskas, Schlichting, 1908.02866]



**Lessons:**

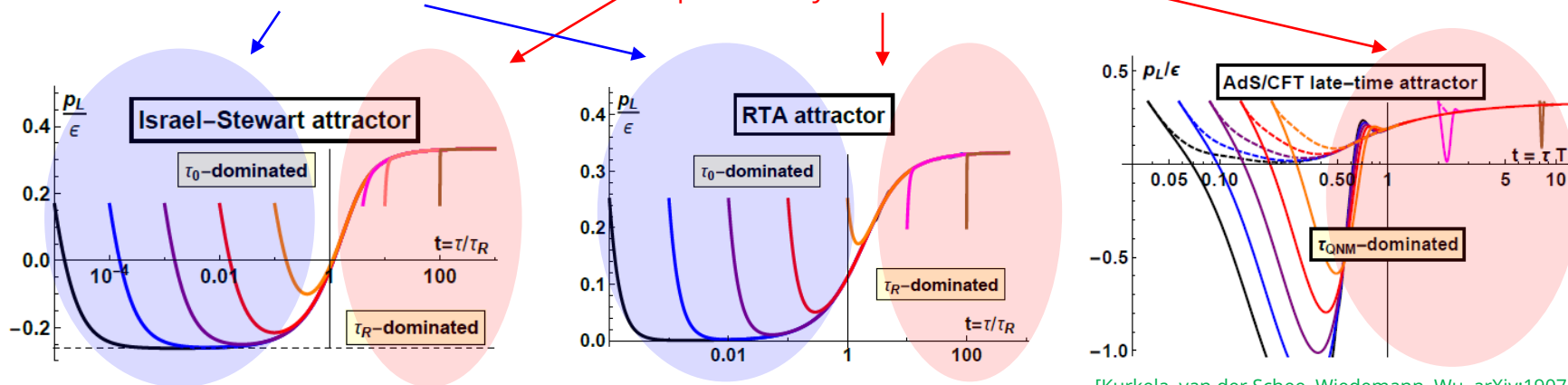
- Hydrodynamization seems universal, independent of coupling strength & micro. theories  $\Rightarrow$  imply the weak-coupling scenario is still valid at intermediate couplings
- Rescaling of time may enable us to study hydro attractor in a universal manner
- ( • At least when the symmetry is high enough. May be OK in HIC ...)

# 2. Two origins of attractor: early/late time attractors

**Idea:** Carefully check the “speed” of attraction to discriminate the origin of attractors

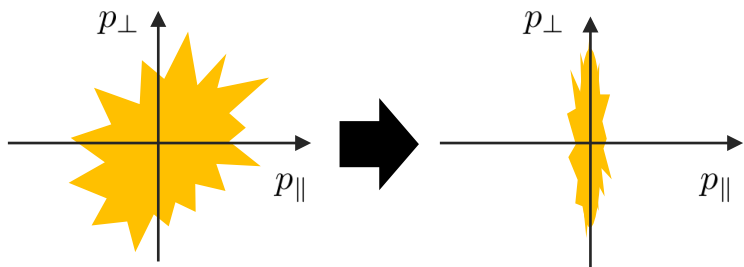
Power-law “slow” early-time attractor

Exponentially-fast late-time attractor



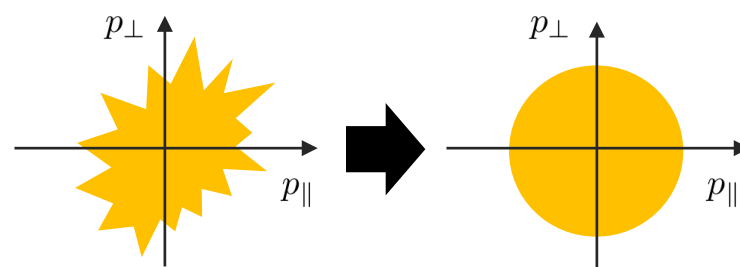
[Kurkela, van der Schee, Wiedemann, Wu, arXiv:1907.08101]

- Origin of attractor = How to forget initial information
- Two ways to forget:  
Expansion (effective at early-times)



vs

Collisions (effective at late-times)



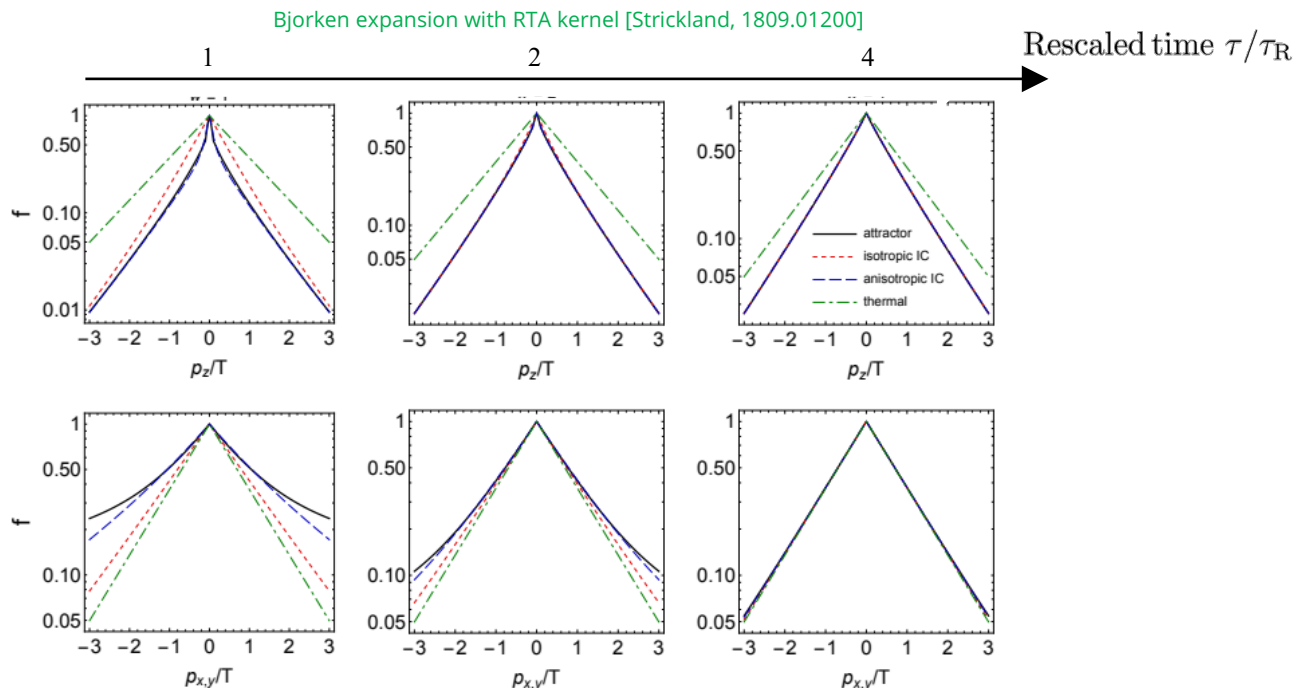
- Which is dominant depends on microscopic theory. cf. Collision effect win in the strong-coupling limit

**Lesson:** There are in general two origins of the hydrodynamic attractor in HIC

[Blaizot, Yan, 1712.03856] [Brewer, Ke, Yan, Yin, 2212.00820]  
[Heller, Jefferson, Spalinski, Svensson, 2003.07368]

# 3. Attractors beyond $T_{\mu\nu}$

**Idea:** Check attractor behaviors of distribution function and higher moments with kinetics



- Attractor for distribution function exists  $\Rightarrow$  Faster convergence for lower momentum ( $pT$ )  
 $\Rightarrow$  Momentum dependence of hydro behavior
- Similar attractor behaviors for higher moments  
cf. recent study with AMY kinetics [Almaalol, Kurkela, Strickland, 2004.05195]

Talk by Monnai, Mon. 13:50~

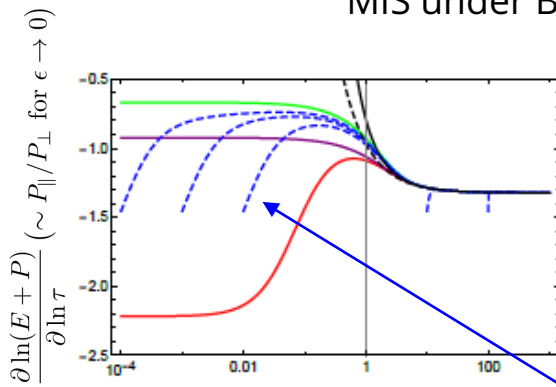
**Lesson:** Attractor does exist beyond  $T_{\mu\nu}$ ,  
though the time-scale depends on momentum scale or the order of moments

# 4. Lower symmetries (non-conformal case)

**Idea:** Break the conformality by, e.g., finite mass, non-conformal EoS

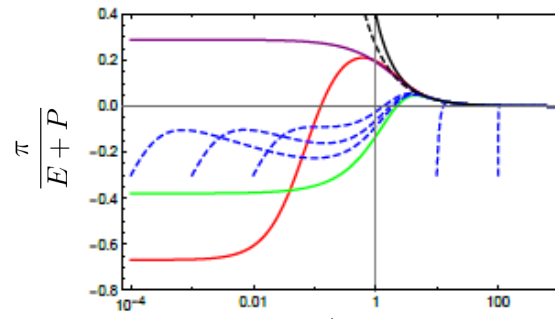
MIS under Bjorken flow with non-conformal EoS:  $\epsilon = 1/3 - c_s^2 = 0.01$

[Chen, Yan, 2109.06658]

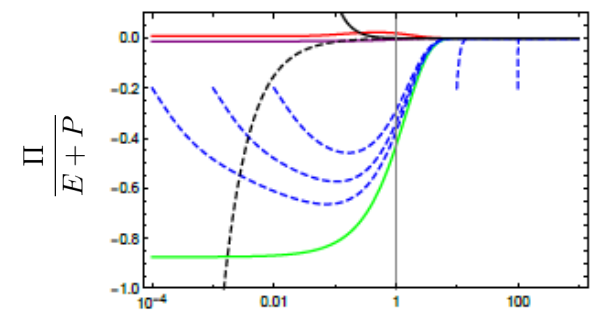


Rescaled time  $\tau/\tau_R$

Non-equilibrium evolution  
w/ various initial cond.



$\tau/\tau_R$



$\tau/\tau_R$

- Attractor does exist for non-conformal case as well
- Details of attraction can change, e.g., disappearance of early-time attractor in shear  $\pi$

[Chattopadhyay, Jaiswal, Du, Heinz, Pal, 2107.05500 & 2107.10248]  
[Alalawi, Strickland, Pal, 2210.00658]

early attractor  $\approx$  free streaming of total  $P_{\parallel} = P + \Pi - \pi$

$\Rightarrow P_{\parallel}$  has an early-time attractor, but  $\pi$  and  $\Pi$  do not necessarily to have separately

(in the conformal limit,  $\pi$  has an early-attractor, as  $\Pi = 0$  and thus behaves essentially the same as  $P_{\parallel}$ )

**Lessons:** • Hydrodynamization does occur with lower symmetries,

cf. for other symmetry consideration, see, e.g., with transverse profile: [1907.08101, 2020.135901, 2102.11785]

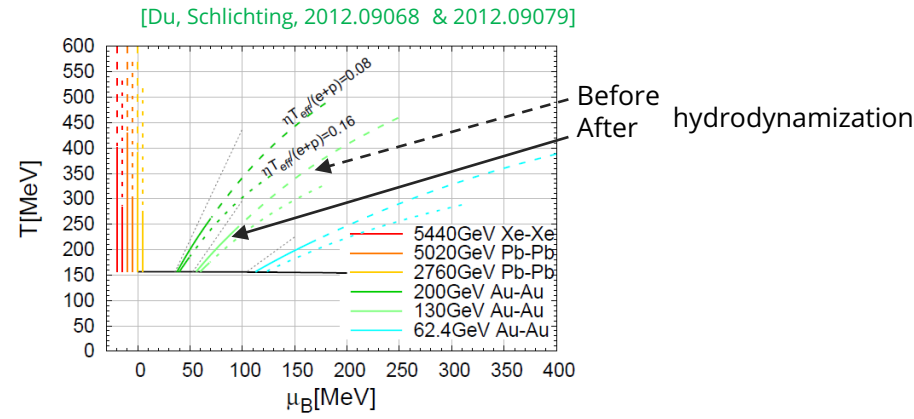
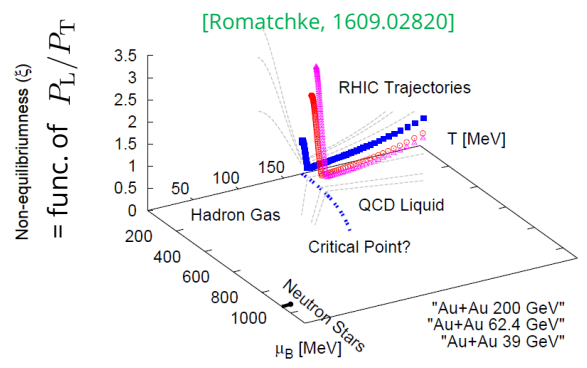
other flows: Gubser [1711.01745, 1804.04771, 1911.06406, 2020.135481]  
Hubble [2104.12534]

- The degree of symmetry can affect hydrodynamization  $\Rightarrow$  Needs further study

# 5. Implications to QCD-phase-diagram study in HIC

**Idea:** That hydrodynamics is applicable does NOT mean equilibrium

- Only out-of-equilibrium QCD phase diagram can be explored in HIC



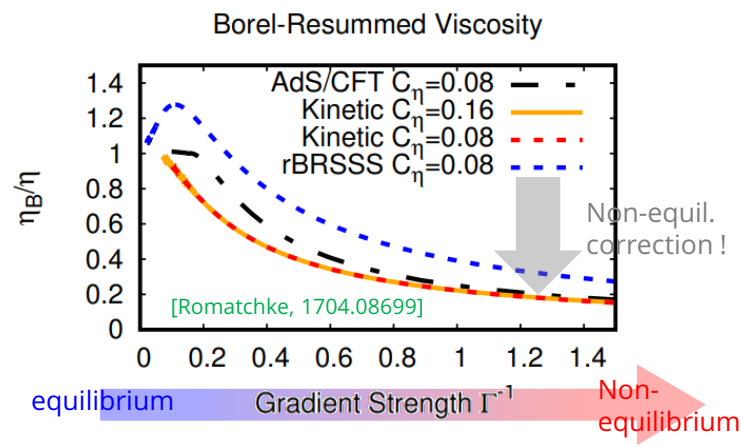
⇒ Isotropization (and thus thermalization) never achieved in HIC

⇒ low energy collisions may not even hydrodynamize

- Non-equilibrium effects can modify matter properties

example) "Non-equilibrium" correction to viscosity

⇐ determined by matching hydro attractor to viscous hydro



**Lessons:**

- Open question: need to think what we are really measuring in HIC
- In any case, interesting opportunity to study non-equilibrium QCD

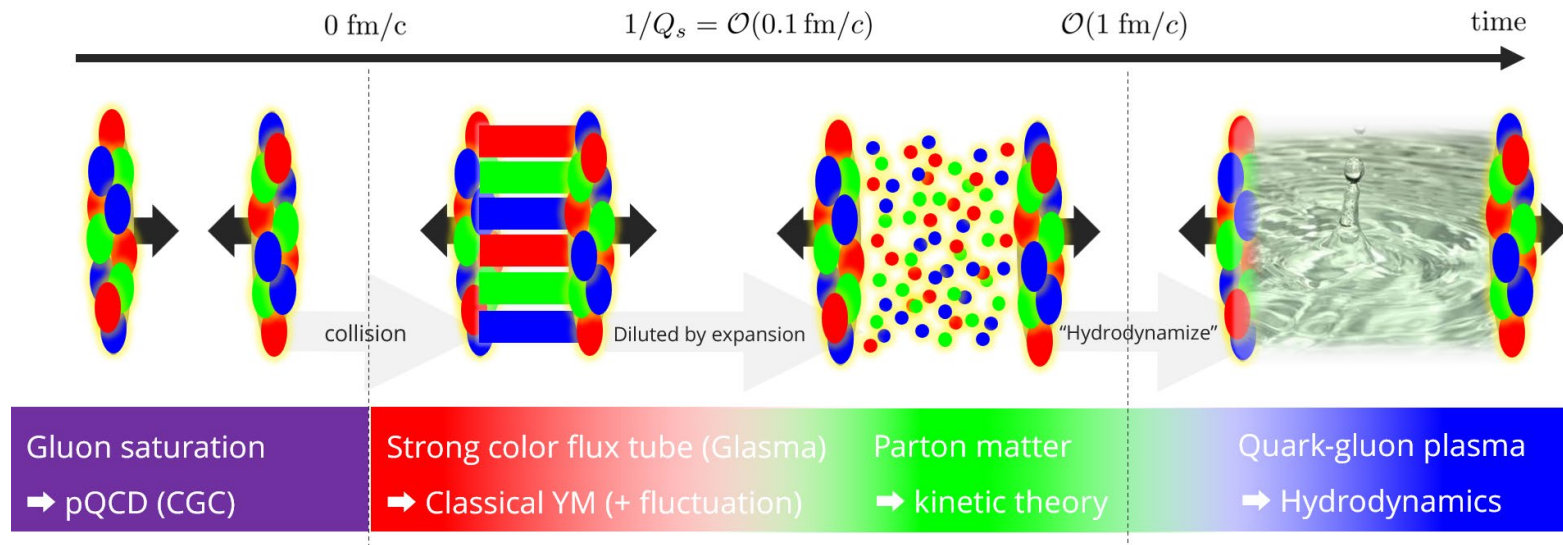


I. Progress ~2015

II. Progress after 2015

**III. Summary**

# Summary: Our understanding as of 2023



## Significant progress over the past 10 years ⇒ No longer a black box

- Establishment of the weak coupling picture & proposal of hydrodynamization ~2015
- Many progresses after 2015
  - Classical YM: 3D glasma simulation, quark production, ...
  - Kinetic theory: inclusion of quarks,  $K\phi MP\phi ST$ , ...
  - Hydrodynamization: weak vs strong coup., early/late-time attractors, attractors beyond  $T_{\mu\nu}$ , non-conformal case, implication to QCD-phase-diagram study in HIC, ...
- Have rich connection to others: In HIC ⇒ corr./fluc. observables, intense fields, small system, ...

Plenary by Sakai, Tue. 10:00~

Plenary by Yang, Tue. 11:40~

Plenary by Kanakubo, Wed. 10:00~

Other area ⇒ cold atom, early Universe, intense lasers, ...

See reviews: [Berges, Heller, Mazeliauskas, Venugopalan, 2005.12299]  
[Fedotov, Ilderton, Karbstein, King, Seipt, HI, Torgimsson, 2203.00019]

- But still lots of open questions ⇒ Needs your insights !