Electric permittivity of the vacuum in a strong constant electric field

Hidetoshi Taya

(RIKEN)

with Charlie Ironside (Curtin U.)

(paper in preparation)

This talk

Discuss the electric permittivity ϵ of the vacuum in a strong constant electric field

In the pure vacuum $\mathcal{D} = \mathcal{E}$

In an EM field $\mathcal{D} = \epsilon\mathcal{E} \neq \mathcal{E}$

Motivations

Why interesting ?

(1) Is a well studied topic, but incomplete yet

(2) Want to use as a tool to diagnose the structure of the QED vacuum

(3) Semi-conductor experiments of electroreflectance

Motivations (1/3)

Why interesting ?

(1) Is a well studied topic, but incomplete yet

(2) Want to use as a tool to diagnose the structure of the QED vacuum

(3) Semi-conductor experiments of electroreflectance

- Previous studies: typically for equilibrium situations (e.g., B field, null field)

[Toll (1960)] [Hattori-Itakura (2013)] ...

- E-field case exists, but not satisfactory enough

$$\mathcal{D} = -\frac{\partial \mathcal{L}_{\text{Euler–Heisenberg}}}{\partial \mathcal{E}}$$

$$\Rightarrow \ \mathrm{Re}\,\epsilon\,(\omega \ll m, e\bar{E} \ll m^2) = \frac{\alpha}{45\pi}\left(\frac{e\bar{E}}{m^2}\right)^2 \times \begin{cases} 6 & (\parallel) \\ 2 & (\perp) \end{cases}$$

[Baier-Breitenlohner (1967)]

☹ valid in the weak field limit

☹ valid for slow probes

☹ imaginary part ?

Q: What happens if I go beyond the weak & slow limit ?

Motivations (2/3)

Why interesting ?

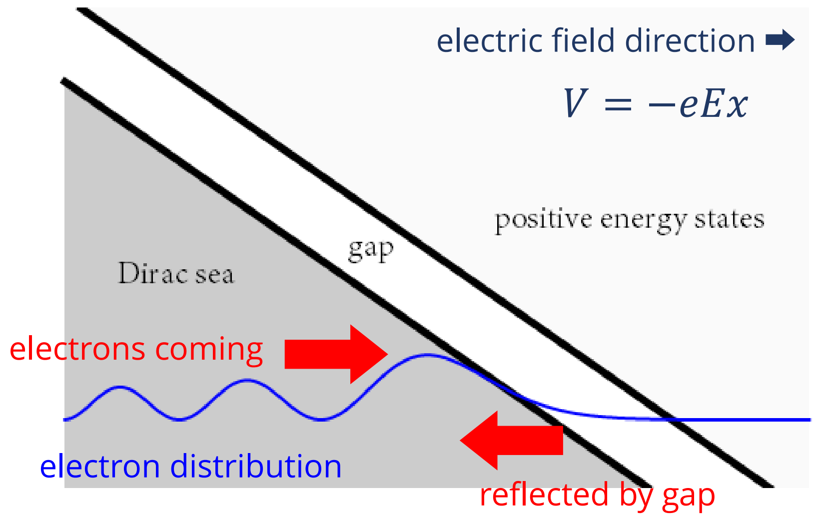
(1) Is a well studied topic, but incomplete yet

(2) Want to use as a tool to diagnose the structure of the QED vacuum

(3) Semi-conductor experiments of electroreflectance

In an E field, the QED vacuum is tilted

⇒ oscillating dist. due to interference



Q: How is the tilted QED-vacuum structure seen in the electric permittivity ?

Motivations (2/3)

Why interesting ?

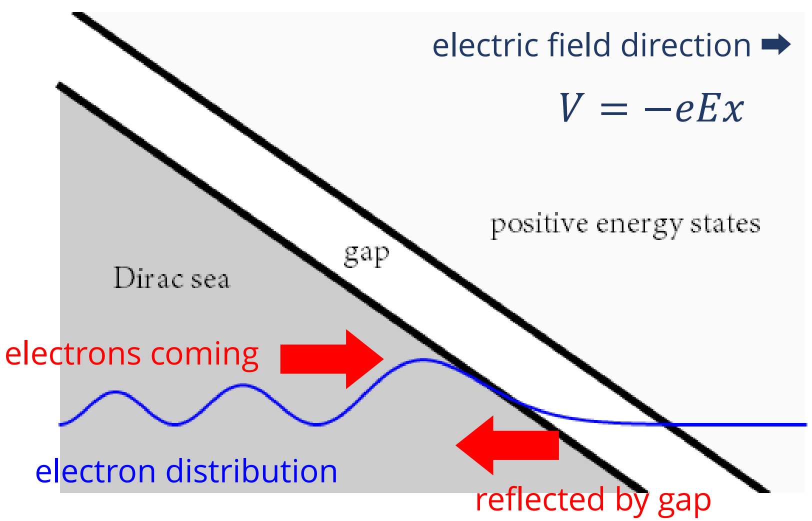
(1) Is a well studied topic, but incomplete yet

(2) Want to use as a tool to diagnose the structure of the QED vacuum

(3) Semi-conductor experiments of electroreflectance

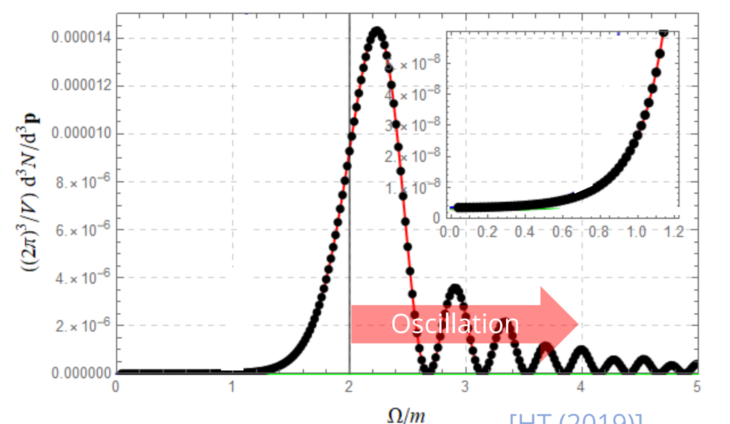
In an E field, the QED vacuum is tilted

$\Rightarrow$ oscillating dist. due to interference



Can leave observable imprints

- ex.) dynamically-assisted Schwinger (pair prod. in const $\bar{E}$ + fast $\mathcal{E}$)



[HT (2019)]
[Huang, HT (2019)]

Q: How is the tilted QED-vacuum structure seen in the electric permittivity ?

Motivations (3/3)

Why interesting ?

(1) Is a well studied topic, but incomplete yet

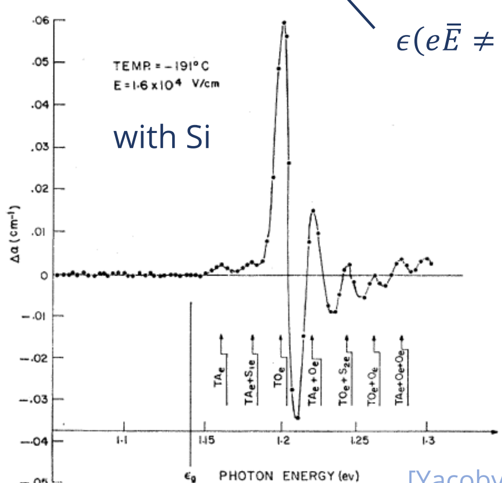
(2) Want to use as a tool to diagnose the structure of the QED vacuum

(3) Semi-conductor experiments of electroreflectance

Such an oscillating behavior in ϵ has been observed in semi-conductor materials

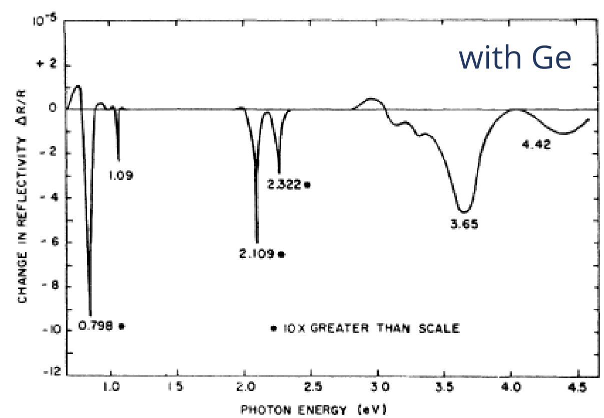
$\approx \mathrm{Im}\,\Delta\epsilon$

$\epsilon(e\bar{E} \neq 0) - \epsilon(e\bar{E} = 0)$



[Yacoby (1966)]

$\approx \mathrm{Re}\,\Delta\epsilon$



[Seraphin, Hess (1965)]

Q: Natural to expect semi-conductor-like behavior in QED. Is this true ?

Motivations

Why interesting ?

(1) Is a well studied topic, but incomplete yet

Q: What happens if I go beyond the weak & slow limit ?

(2) Want to use as a tool to diagnose the structure of the QED vacuum

Q: How is the tilted QED-vacuum structure seen in the electric permittivity ?

(3) Semi-conductor experiments of electroreflectance

Q: Natural to expect semi-conductor-like behavior in QED. Is this true ?

This talk: answer those questions

Theory (1/2)

Calculate the polarization via the linear response theory

(linear in the probe $\mathcal{E}$ but non-pert. in the strong field $\bar{E}$)

Theory (1/2)

Calculate the polarization via the linear response theory

(linear in the probe $\mathcal{E}$ but non-pert. in the strong field $\bar{E}$)

Setup: QED in the presence of a constant strong field $\bar{E}$ + a weak spatially homo. probe $\mathcal{E}(t)$

Theory (1/2)

Calculate the polarization via the linear response theory
(linear in the probe $\mathcal{E}$ but non-pert. in the strong field $\bar{E}$)

Setup: QED in the presence of a constant strong field $\bar{E}$ + a weak spatially homo. probe $\mathcal{E}(t)$

Step 1: Definition of $\mathcal{D}$ (or ϵ)

- Total flux $D = E + P(\bar{E}, \mathcal{E}) = E + P_0(\bar{E}) + P_1(\bar{E})\mathcal{E} + \cdots$

$\bar{E} + \mathcal{E}$

$$= \underbrace{\bar{E} + P_0(\bar{E})}_{\bar{D}} + \underbrace{(1 + P_1(\bar{E}))\,\mathcal{E}}_{\mathcal{D}} + \cdots$$

- So, identify $\epsilon = 1 + P_1(\bar{E})$

Theory (1/2)

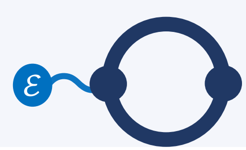
Calculate the polarization via the linear response theory
(linear in the probe $\mathcal{E}$ but non-pert. in the strong field $\bar{E}$)

Setup: QED in the presence of a constant strong field $\bar{E}$ + a weak spatially homo. probe $\mathcal{E}(t)$

Step 1: Definition of $\mathcal{D}$ (or ϵ)

- Total flux $D = E + P(\bar{E}, \mathcal{E}) = E + P_0(\bar{E}) + P_1(\bar{E})\mathcal{E} + \cdots$
  $$= \underbrace{\bar{E} + P_0(\bar{E})}_{\bar{D}} + \underbrace{(1 + P_1(\bar{E}))\,\mathcal{E}}_{\mathcal{D}} + \cdots$$
  $\bar{E} + \mathcal{E}$
- So, identify $\epsilon = 1 + P_1(\bar{E})$

Step 2: Calculate the polarization P_1

- In QFT, $\dot{P} = \langle 0; \mathrm{in} | \bar{\psi}(\bar{E}, \mathcal{E}) \gamma^\mu \psi(\bar{E}, \mathcal{E}) | 0; \mathrm{in} \rangle$
  $$= \langle 0; \mathrm{in} | \bar{\psi}_0(\bar{E}) \gamma^\mu \psi_0(\bar{E}) | 0; \mathrm{in} \rangle$$
  $$+ \boxed{\langle 0; \mathrm{in} | \bar{\psi}_1(\bar{E}) \gamma^\mu \psi_0(\bar{E}) + \bar{\psi}_0(\bar{E}) \gamma^\mu \psi_1(\bar{E}) | 0; \mathrm{in} \rangle} \times \mathcal{E} + \mathcal{O}(\mathcal{E}^2)$$
  gives P_1
- Diagrammatically, amounts to evaluate 

Theory (2/2)

- **Some details**

(1) Use of Kramers-Kronig relation

Causality $\Rightarrow$ $$\mathrm{Re}\,\epsilon(\omega) = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{+\infty} \mathrm{d}\omega' \frac{1}{\omega' - \omega}\mathrm{Im}\,\epsilon(\omega')$$

[Toll (1960)] [Heinzl, Schroeder (2006)] [Borysov et al. (2022)]

$\Rightarrow$ Sufficient to calculate the imaginary part
(Same approach has been adopted in semi-conductor) [Aspnes(1967)]

Theory (2/2)

- **Some details**

(1) Use of Kramers-Kronig relation

Causality $\Rightarrow$ $\mathrm{Re}\,\epsilon(\omega) = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{+\infty} \mathrm{d}\omega' \frac{1}{\omega' - \omega}\mathrm{Im}\,\epsilon(\omega')$

[Toll (1960)] [Heinzl, Schroeder (2006)] [Borysov et al. (2022)]

$\Rightarrow$ Sufficient to calculate the imaginary part
(Same approach has been adopted in semi-conductor) [Aspnes(1967)]

(2) $\mathrm{Im}\,\epsilon$ is directly related to the pair prod. via the dynamically-assisted Schwinger effect

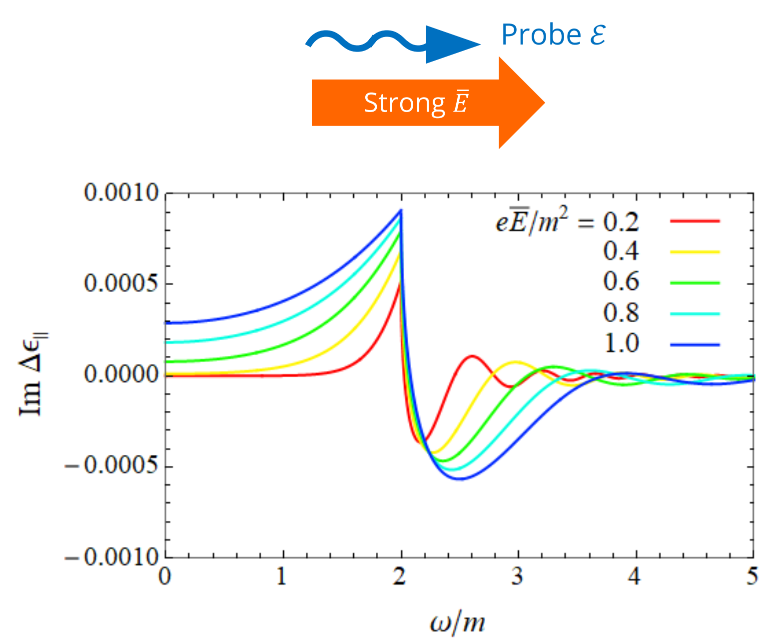
- $\mathrm{Im}\,\epsilon$ is related to the dielectric energy loss (= decay of probe) $\frac{\mathrm{d}U_1}{\mathrm{d}t} = \mathcal{E}\frac{\mathrm{d}\mathcal{D}}{\mathrm{d}t} = \frac{1}{2}\omega\mathcal{E}^2\,\mathrm{Im}\,\epsilon$
  See, e.g., textbook by Landau-Lefshitz
- Energy of probe used in the pair production $\frac{\mathrm{d}U_2}{\mathrm{d}t} = \omega\frac{N(\mathcal{E}\neq 0) - N(\mathcal{E}=0)}{VT}$
- Microscopically, the dielectric energy loss should be caused by the pair production

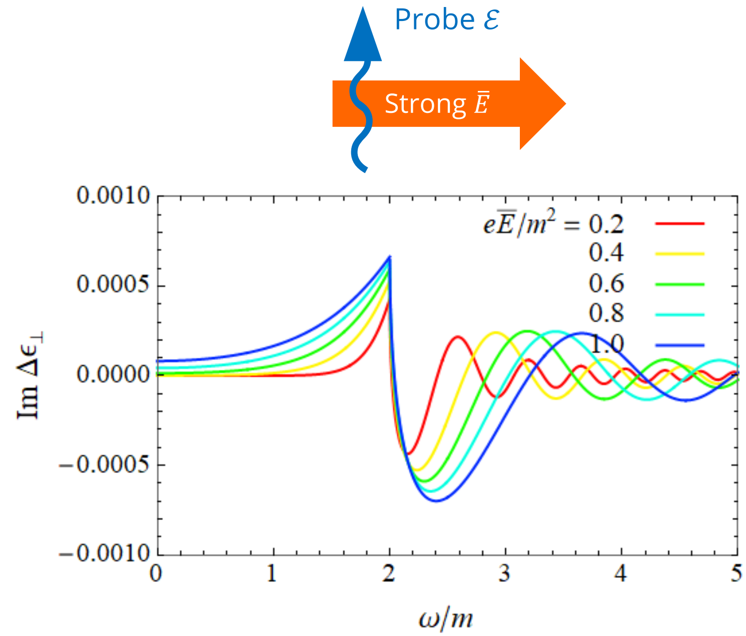
$\Rightarrow U_1 = U_2 \Rightarrow \frac{N(\mathcal{E}\neq 0) - N(\mathcal{E}=0)}{VT} = \frac{1}{2}\mathcal{E}^2\,\mathrm{Im}\,\epsilon$

$\therefore$ Dynamically assisted Schwinger $\leftrightarrow$ $\mathrm{Im}\,\epsilon \overset{\mathrm{KK}}{\leftrightarrow} \mathrm{Re}\,\epsilon$

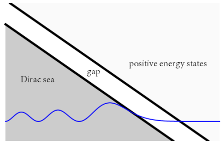
Results (1/2)

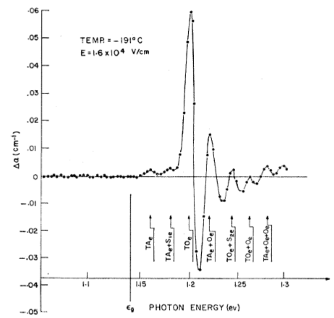
Imaginary part of the change $\Delta\epsilon = \epsilon(\bar{E} \neq 0) - \epsilon(\bar{E} = 0)$





- Oscillation, as expected from the tilted vacuum





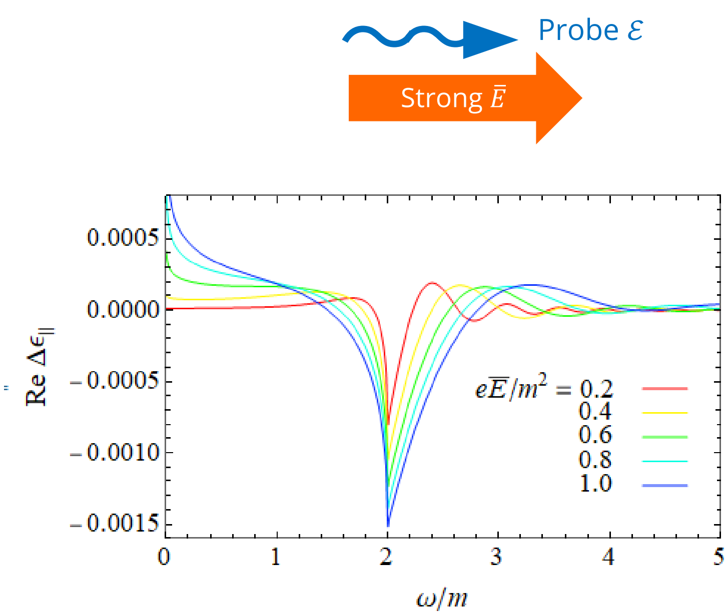
- Essentially the same pattern as semi-conductor observation
- Birefringent (Im $\Delta\epsilon_\parallel \neq$ Im $\Delta\epsilon_\perp$) but the basically the same
- Non-vanishing even at $\omega \to 0$ due to the strong-field non-perturbative effect

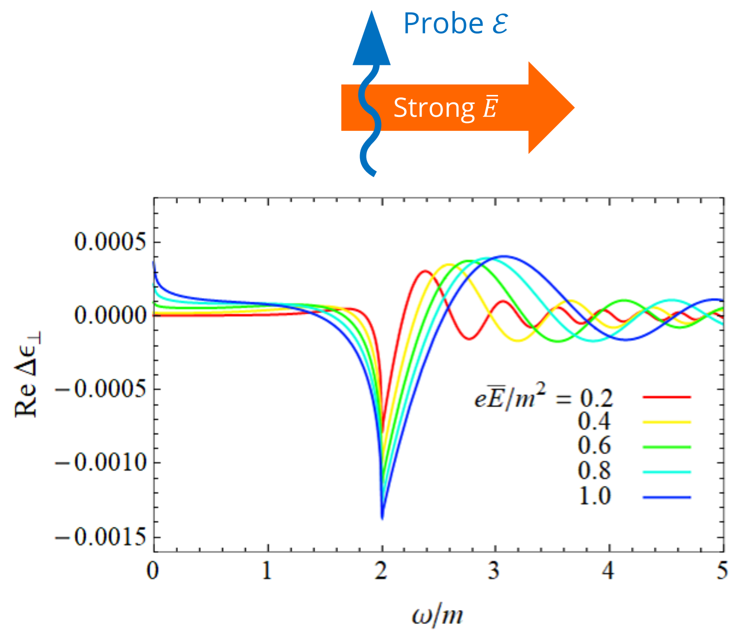
A simple explanation: In the slow limit, the Schwinger formula is valid

$$\Rightarrow \mathrm{Im}\,\epsilon \propto (N_{\mathrm{Schwinger}}(\bar{E} + \mathcal{E}) - N_{\mathrm{Schwinger}}(\bar{E})) \propto (\exp\left[-\pi \frac{m^2}{e(\bar{E}+\mathcal{E})}\right] - \exp[-\pi \frac{m^2}{e\bar{E}}]) = (\mathrm{finite}) \times \exp\left[-\pi \frac{m^2}{e\bar{E}}\right]$$

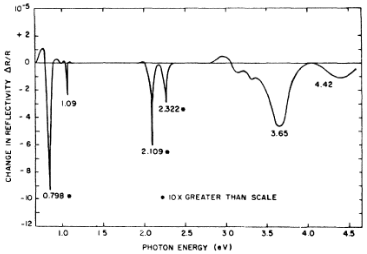
Results (2/2)

Real part of the change $\Delta\epsilon = \epsilon(\bar{E} \neq 0) - \epsilon(\bar{E} = 0)$





- Again oscillation, which is again consistent with semi-conductor



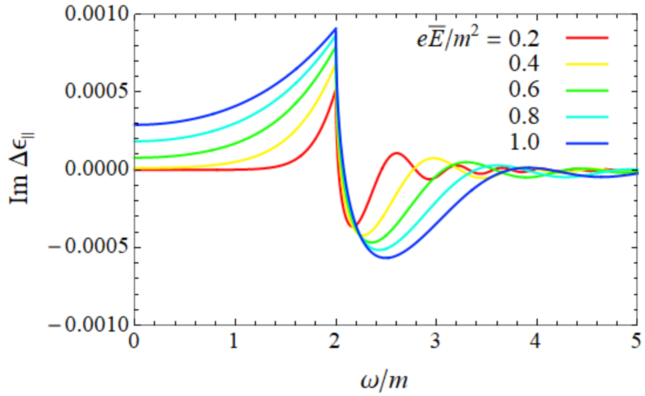
- Logarithmically divergent at $\omega \to 0$ due to the non-perturbative effect

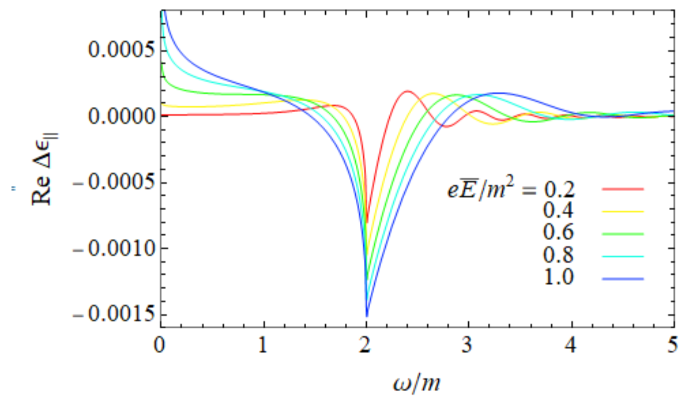
$$\because \quad \mathrm{Re}\,\epsilon(0) = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{+\infty} \mathrm{d}\omega' \frac{1}{\omega'} \mathrm{Im}\,\epsilon(\omega') \sim \frac{1}{\pi}\int_{-\infty}^{+\infty} \mathrm{d}\omega' \frac{1}{\omega'} \mathrm{Im}\,\epsilon(0) \sim (\mathrm{log\ div.}) \times \exp\left[-\pi \frac{m^2}{e\bar{E}}\right]$$

Note: agrees precisely with the known Euler-Heisenberg result at $\omega \to 0$ if the log div. was subtracted

This talk

- **I've calculated the electric permittivity of the vacuum in a constant strong E field**





(and similar plots for $\Delta\epsilon_{\perp}$)

- **My answer to the 3 questions:**

Q: What happens if I go beyond the weak & slow limit ?

A: Beyond slow $\Rightarrow$ Oscillation appears

Beyond weak $\Rightarrow$ Logarithmic divergence at $\omega \to 0$

Q: How is the tilted QED-vacuum structure seen in the electric permittivity ?

A: Oscillation in the high-frequency regime

Q: Natural to expect semi-conductor-like behavior in QED. Is this true ?

A: True.