Electric permittivity of the vacuum in a strong constant electric field

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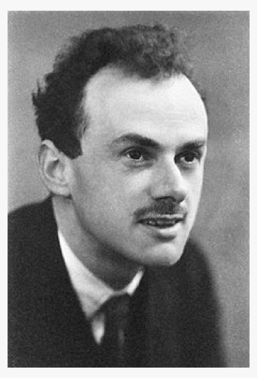
Based on a work (in preparation for submission) with Charlie Ironside (Curtin U.)

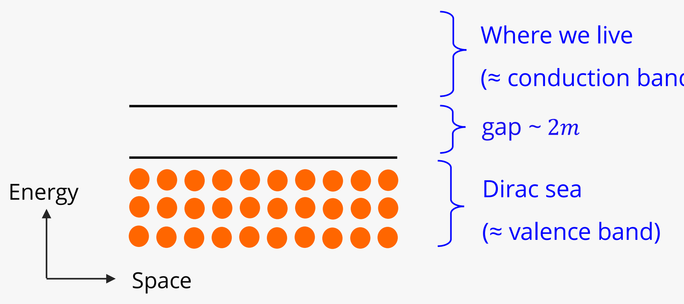
1. INTRODUCTION

1-1. The vacuum ≠ not an empty space



Dirac (1930)

The vacuum has a structure, similar to semi-conductor



✔ The vacuum should exhibit some responses when shined by strong light (= EM field), similarly to semi-conductor

✔ Why interesting ?

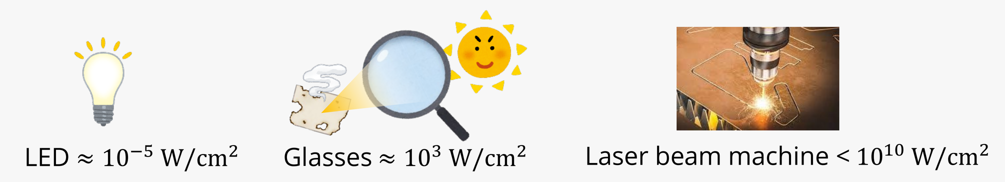
(1) Non-trivial: vacuum has rich physics !
(2) Fundamental: the response can be used to diagnose the vacuum
(3) Timeliness: within experimental reach (see 1-2)
(4) Interdiciplinarity: connect high-energy phys. to others

1-2. Recent availability of strong EM field

✔ How strong is needed ?

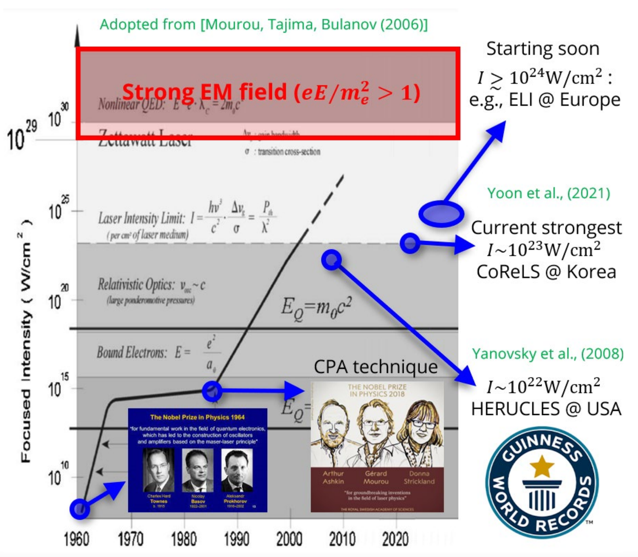
⇒ Field strength comp. to the gap size = extremely strong !

$$eE \;> m_e^2 = (0.511\ \mathrm{MeV})^2 \;\approx\; O(10^{28}\ \mathrm{W/cm^2})$$



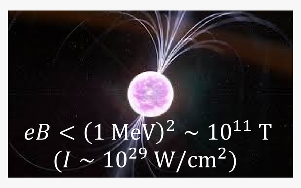
✔ Experimental prog. to create (observe) strong EM field

High-power laser



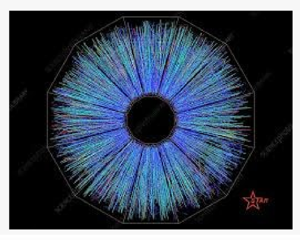
Also in other extreme systems

Magnetar (in astro phys)



Suzaku (2005~)
IXPE, XL-Calibur (2022~)

Heavy-ion coll. (particle & nuclear phys)

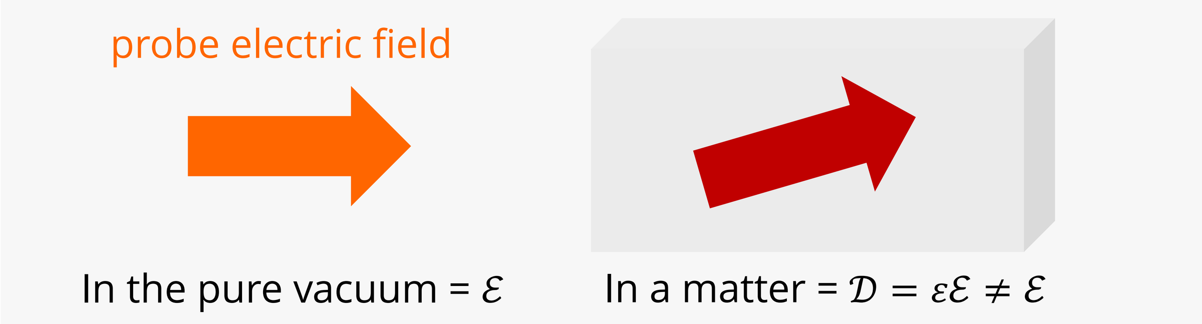


RHIC (2000)
LHC (2012)

1-3. This work: electric permittivity

✔ Electric permittivity ε

- One of the fundamental quantities to characterize the optical property of a matter



Microscopically, $\varepsilon \neq 1$ comes from polarizability of the matter

$$\mathcal{D} = \mathcal{E} + \int \mathrm{d}t\, J_{\rm pol}$$

- The electric permittivity of the vacuum ε has been studied well for equilibrium situations (e.g., B field, null field), Toll (1960), Baier-Breitenlohner (1967), Hattori-Itakura (2013), ...
  but NOT well for E field, which is genuinely non-equilibrium and so the calculation is more difficult
- **So, the goal is to calculate the electric permittivity of the vacuum in the presence of a strong electric field**

2. THEORY

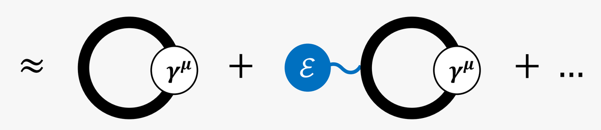
2-1. What I computed

Want to calculate the electric permittivity ε

✔ QED in the presence of a constant strong field $\bar{E}$
plus a weak spatially homogeneous probe field $\mathcal{E}(t)$

✔ Calculate the polarization current $J_{\rm pol}$ (in-in expect. value) under the non-perturbative influence by the strong $\bar{E}$

$$J_{\rm pol}(\bar{E},\mathcal{E}) = \langle 0;\mathrm{in}|\bar{\psi}(\bar{E},\mathcal{E})\gamma^\mu\psi(\bar{E},\mathcal{E})|0;\mathrm{in}\rangle$$
$$= \langle 0;\mathrm{in}|\bar{\psi}_0(\bar{E})\gamma^\mu\psi_0(\bar{E})|0;\mathrm{in}\rangle$$
$$+ \langle 0;\mathrm{in}|\bar{\psi}_1(\bar{E})\gamma^\mu\psi_0(\bar{E}) + \bar{\psi}_0(\bar{E})\gamma^\mu\psi_1(\bar{E})|0;\mathrm{in}\rangle \times \mathcal{E} + O(\mathcal{E}^2)$$



2-2. Some details

✔ Kramers-Kronig relation

Causality: the response $\mathcal{D}$ must be followed by $\mathcal{E}$
⇒ Re ε and Im ε are not independent with each other
⇐ can be confirmed with analyzing $J_{\rm pol}$

Frequency of $\mathcal{E}$

$$\mathrm{Re}\,\varepsilon(\omega) = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{+\infty}\mathrm{d}\omega'\,\frac{1}{\omega'-\omega}\,\mathrm{Im}\,\varepsilon(\omega')$$

Calculation of Im ε is easier than Re ε
⇒ Calculate Im ε first, then get Re ε via KK relation

✔ Relation to the Schwinger effect

- The vacuum is unstable against the particle prod. in the presence of $\bar{E}$ and $\mathcal{E}$
  ⇒ (dynamically-assisted) Schwinger effect Dunne-Gies-Schutzhold (2009), HT (2017)
- The number of pairs $N = \langle 0;\mathrm{in}\,|\,\hat{N}\,|\,0;\mathrm{in}\rangle$ has the direct relevance to Im ε

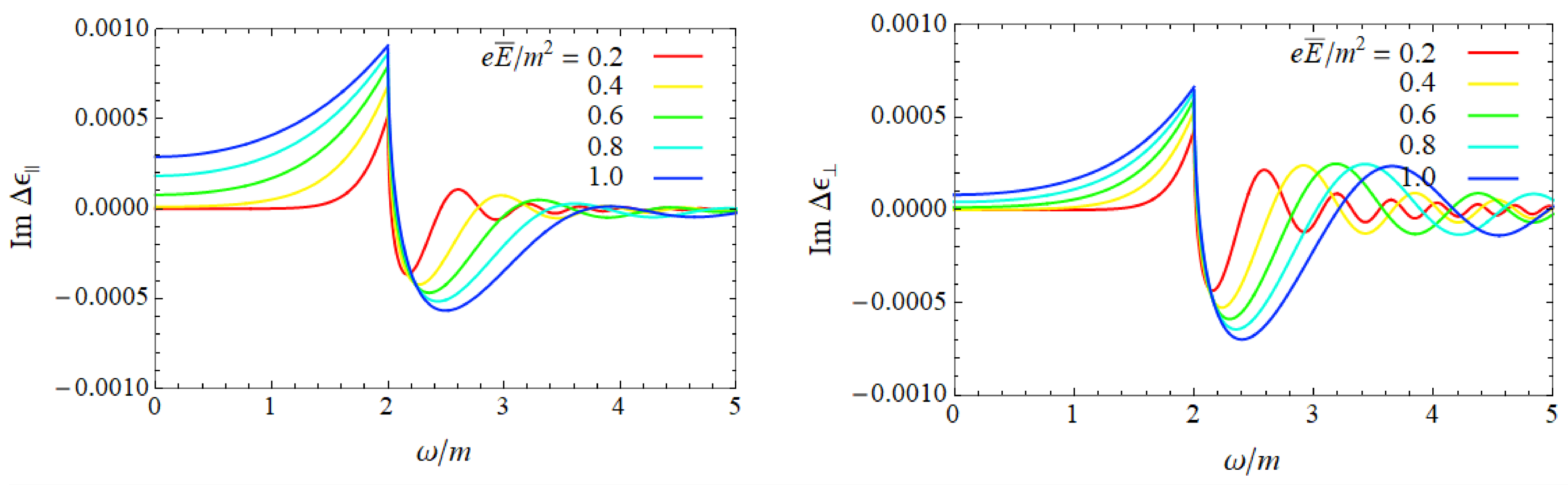
$$\text{Dielectric energy loss: } \frac{\mathrm{d}U_1}{\mathrm{d}t} = \mathcal{E}\frac{\mathrm{d}\mathcal{D}}{\mathrm{d}t} = \frac{1}{2}\omega\mathcal{E}^2\,\mathrm{Im}\,\varepsilon$$

$$\text{Energy used in part. pro. : } \frac{\mathrm{d}U_2}{\mathrm{d}t} = \omega\frac{N(\mathcal{E}\neq 0) - N(\mathcal{E}=0)}{VT}$$

⇒ can prove $U_1 = U_2$

3. RESULTS

3-1. Imaginary part Im $\Delta\varepsilon$ ($\Delta\varepsilon := \varepsilon(\bar{E}\neq 0) - \varepsilon(\bar{E}=0)$)



✔ Non-trivial oscillation in ω

- Exponential tail below the mass gap $\omega < 2m$
- Damped oscillation above the mass gap $\omega < 2m$

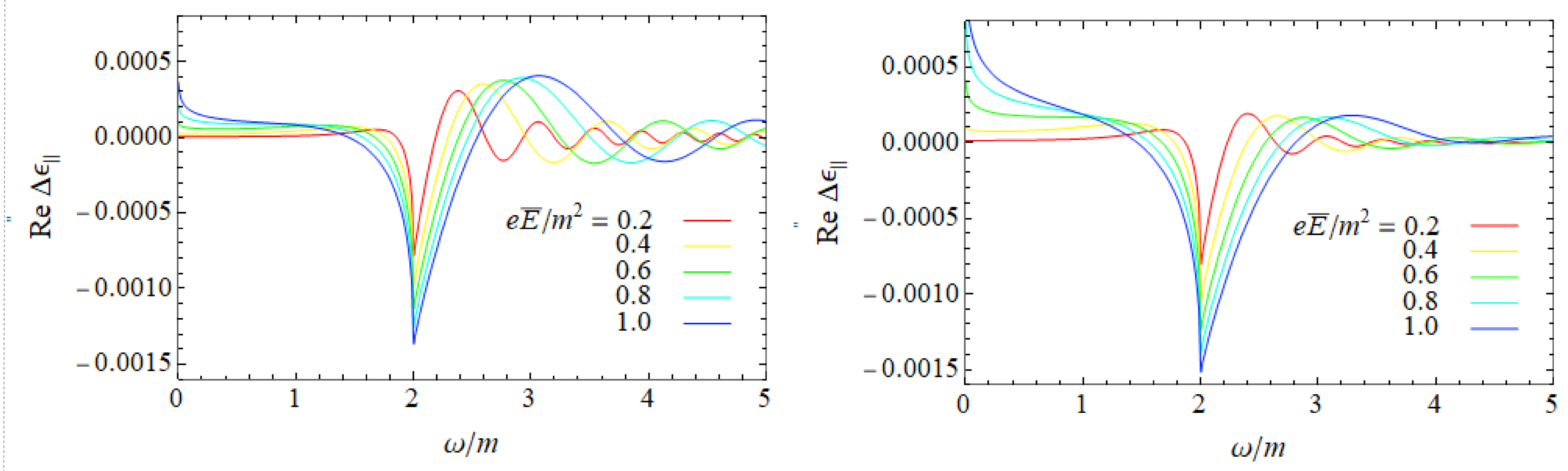
✔ Birefringence

Im $\Delta\varepsilon_\perp \neq$ Im $\Delta\varepsilon_\parallel$, though the basic features are the same

✔ Discontinuity at $\omega = 0$

due to the non-pert. effect Im $\Delta\varepsilon \propto \mathrm{e}^{-\#\,m^2/e\bar{E}}$, which was missing previously

3-2. Real part Re $\Delta\varepsilon$



✔ Similar features to Im $\Delta\varepsilon$

- Oscillation in ω
- Birefringence

✔ Log divergence at $\omega = 0$

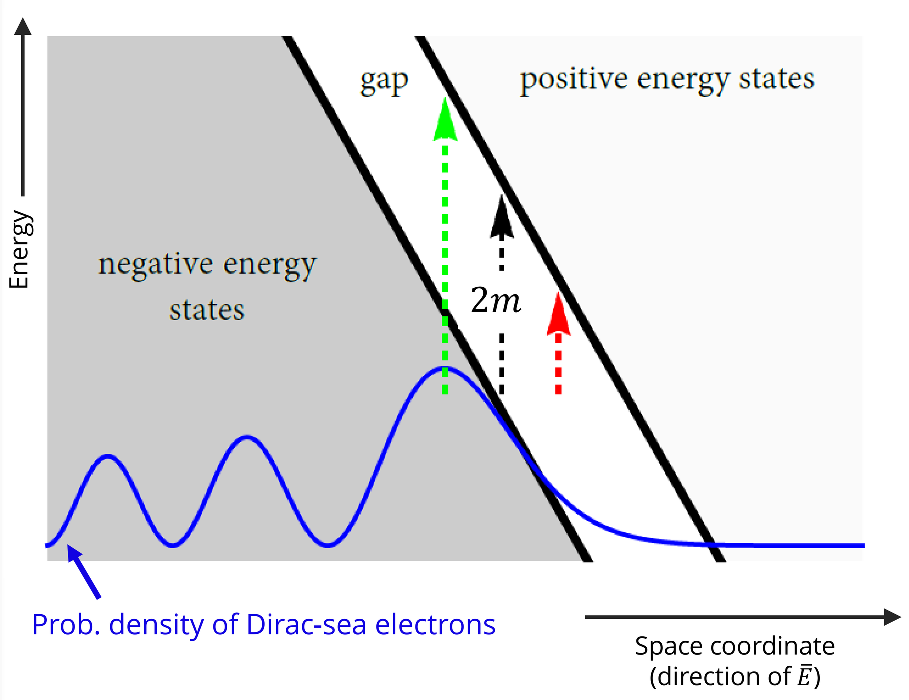
- due to the non-perturbative discontinuity of Im $\Delta\varepsilon$ at $\omega = 0$
- For $e\bar{E} \ll m^2$, it is negligible and the result agrees with the previously known Baier-Breitenlohner (1967)

$$\mathrm{Re}\,\Delta\varepsilon(\omega=0, e\bar{E}\ll m^2) = \frac{\alpha}{45\pi}\left(\frac{e\bar{E}}{m^2}\right)^2 \times \begin{cases} 6 & (\parallel) \\ 2 & (\perp) \end{cases}$$

3-3. Interpretation

✔ Reflecting the structure of the vacuum

Band structure of the vacuum



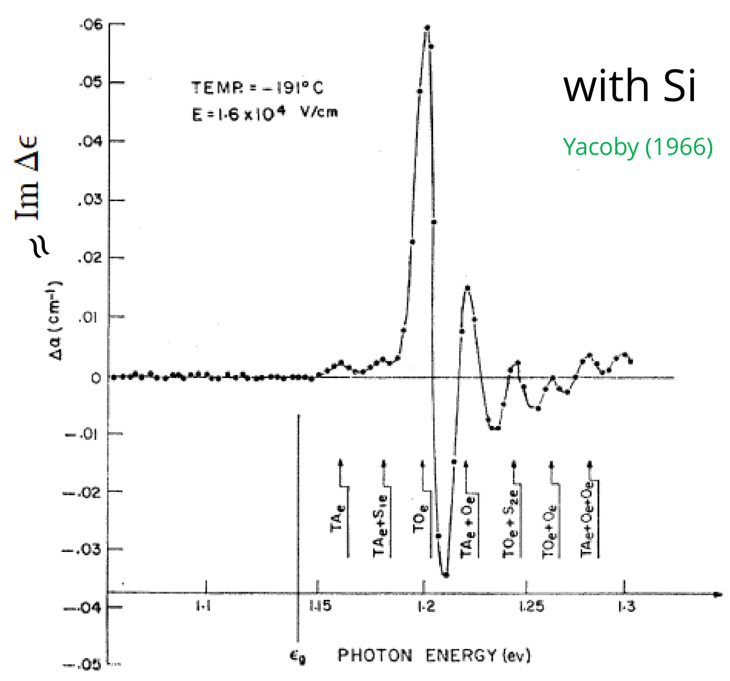
logic 1) The dist. of the Dirac-sea electrons is oscillating due to interference with the electrons reflected by the gap
logic 2) On the other hand, Im ε is related to the particle prod.
logic 3) Particle prod. occurs more at where the Dirac-sea electrons exist more
logic 4) The energy ω needed for the particle prod. is different depending on from where the particle prod. occurs

∴ Im ε exhibits an oscillating pattern in ω
⇒ A similar pattern for Re ε, as it is related to Im ε via the Kramers-Kronig relation

✔ A QED analog of the Franz-Keldysh effect in semi-conductor Franz (1958), Keldysh (1958)

Precisely the same pattern has already been observed in semi-cond. experiments

⇒ Dirac was correct that the vacuum is indeed like a semi-conductor !



4. SUMMARY

What I did

I calculated the electric permittivity of the vacuum in a constant strong electric field

What I wanted to tell you

✔ The vacuum is like a semi-conductor and can exhibit something interesting
✔ An intriguing oscillating pattern appears in the electric permittivity, which is reflecting the change of the vacuum structure in a strong electric field
✔ The slow-frequency limit is modified (e.g., log div.) due to non-perturbative effect by a strong electric field

What I may do next

✔ More general strong-field configurations
e.g.) spatial dependence, beyond static, incl. B field
✔ Magnetic permeability
✔ Import the wisdom of semi-cond. physics