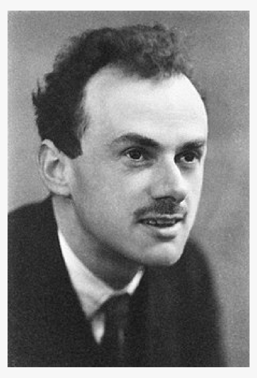
Analytical & numerical study of strong-field QED and its application to high-energy physics

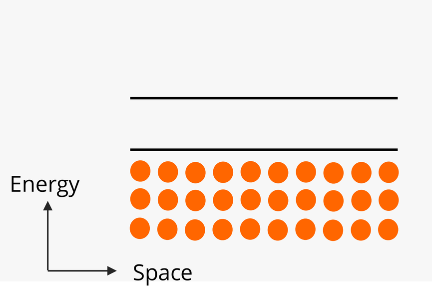
Hidetoshi TAYA
(Theoretical physics; High-energy particle and nuclear physics)

Introduction to strong-field QED

1. Non-trivial response of the vacuum by strong field



The vacuum is NOT an empty pace but has a structure similar to semi-conductor



✔ **The vacuum should exhibit some responses**
when shined by strong light (= electromagnetic field), similarly to semi-conductor

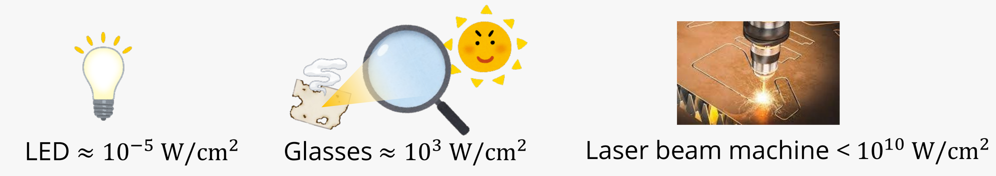
✔ **Why interesting ?** Review: [Fedotov, Ilderton, Karbstein, King, Seipt, HT, Torgrimsson (2023)]

(1) Non-trivial: Our vacuum has rich physics !
(2) Fundamental: The response can be used to diagnose the vacuum & any physical processes occur on top of the vacuum
(3) Timeliness: within the experimental reach in the future
(4) Interdiciplinarity: connect various disciplines of physics

2. Availability of strong EM field

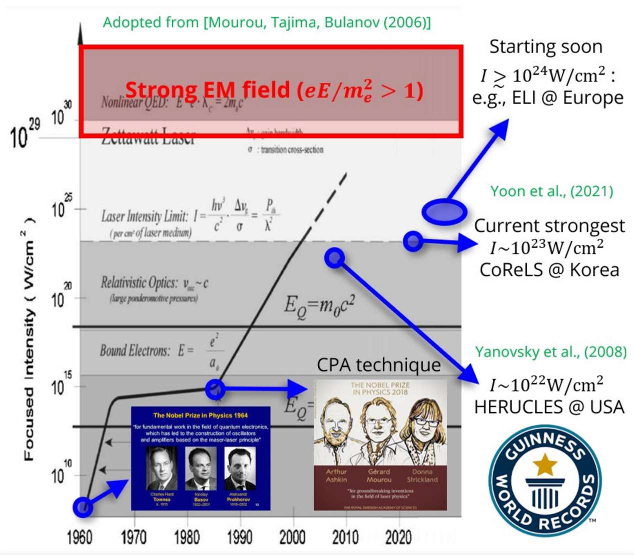
✔ **How strong is needed ?**
⇒ Field strength comp. to the gap size = extremely strong !

$$eE > m_e^2 = (0.511\ \mathrm{MeV})^2 \approx O(10^{28}\ \mathrm{W/cm^2})$$



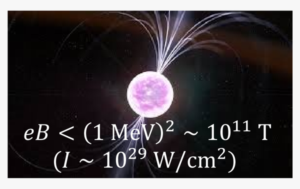
✔ **Realization in nature**

High-power laser experiments



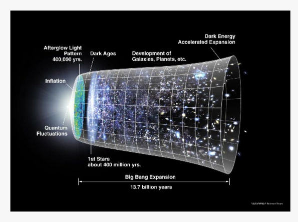
Also in other extreme systems

Magnetar



Suzaku (2005~) IXPE, XL-Calibur (2022~)

Early Universe



3. About this project and presentation

✔ **Purpose of this project**
(1) Develop a better formulation to discuss the vacuum physics by strong EM field
(2) Use (1) to predict something new
(3) Propose possible experimental setups and/or signals of (2)

✔ **About this presentation**
Explain what I have accomplished regarding (1) - (3) during SPDR (2021 - 2023)

References:
[1] Taya, Ironside, Phys. Rev. D 108, 096005 (2023) [arXiv:2308.11248]
[2] Fukushima, Shimazaki, Hidaka, Taya, Ann. Phys. 458, 169494 (2023) [arXiv:2305.11432]
[3] Taya, Yamada, JHEP 02, 048 (2023) [arXiv:2207.03831]
[4] Cao, Hattori, Hongo, Huang, Taya, PTEP 7, 071D01 (2022) [arXiv:2205.08051]
[5] Fedotov, Ilderton, Karbstein, King, Seipt, Taya, Phys. Rep. 1010, 1 (2023) [arXiv:2203.00019]
[6] Taya, Hongo, Ikeda, Phys. Rev. B 104, L140305 (2021) [arXiv:2105.12446]
[7] Du, Huang, Taya Phys. Rev. D 104, 056022 (2021) [arXiv:2104.12534]
[8] Taya, Fujimori, Misumi, Nitta, Sakai, JHEP 03, 082 (2021) [arXiv:2010.16080]
[9] Hattori, Taya, Yoshida, JHEP 01, 093 (2021) [arXiv:2010.13492]

Achievement on (1): Theoretical formulation of strong-field QED

1. Background

✔ **Strong field supplies large energy to the Dirac sea**

- Non-perturbative treatment is indispensable

⇒ inapplicability of standard calculation-method = perturbative (free-field wavefunction)

- Need to use dressed wavefunction

✔ **Strategy: Use a math. Tech. of the exact WKB method**

2. Typical calc. of strong-field QED

✔ **Observable = Integral of wavefunctions**

Typically, of the form of

$$O = \langle \psi^\dagger \hat{O} \psi \rangle = \sum_{i\ \in \text{quantum number}} \psi_i^\dagger O_i \psi_i$$

where ψ_i is the so-called wavefunction which is determined by solving the so-called Schrodinger equation = ODE

$$i\hbar\partial_t \psi_i(t) = H(t)\psi_i(t)$$

✔ **The essential problem is to solve ODE**

3. Exact WKB method

[HT, Fujimori, Misumi, Nitta, Sakai (2021)] cf. [Voros (1983)] [Pham, Dillinger, Delabaere, Aoki, Koike, Takei, ...]

✔ **Exact WKB = non-pert. method to solve ODE = standard WKB + Borel resummation**

- Standard WKB = expansion in $\hbar$

$$\psi_{i,\pm}(t;\hbar) := \exp\left[\mp\frac{i}{\hbar}\int_{t_0}^{t} dt' E_i(t')\right] \times \sum_{n=0}^{\infty} \psi_{i,\pm}^{(n)}(t)\hbar^n$$

where E_i is the (instantaneous) eigenvalue of H

- However, $\psi_{i,\pm}^{(n)}$ is in general divergent

$$\psi_{i,\pm}^{(n)} \propto n!$$

⇒ zero radius of convergence

✔ **Need to make the series well-defined**

- Borel resum. = make a div. series well-defined

(i) Construct "Borel transformation":

$$B[\psi_{i,\pm}](t;\eta) := \sum_{n}^{\infty} \frac{\psi_{i,\pm}^{(n)}(t)}{n!}\eta^n$$

(ii) Laplace transformation gives "Borel sum":

$$\Psi_{i,\pm}(t;\hbar) := \int_0^\infty \frac{d\eta}{\hbar} e^{-\eta/\hbar} B[\psi_{i,\pm}](t;\eta)$$

⇒ $\Psi_{i,\pm}$ is well-def. & is a natural analytic cont. of $\psi_{i,\pm}$
⇐ Indeed, reduces to the original result if $\sum\int = \int\sum$

$$\Psi_{i,\pm} = \int_0^\infty \frac{d\eta}{\hbar} e^{-\eta/\hbar} \sum_n^\infty \frac{\psi_{\pm n}(t)}{n!}\eta^n \sim \sum_n^\infty \frac{\psi_{i,\pm}^{(n)}(t)}{n!} \int_0^\infty \frac{d\eta}{\hbar} e^{-\eta/\hbar}\eta^n = \sum_{n=0}^\infty \psi_{i,\pm}^{(n)}(t)\hbar^n$$

✔ **$\Psi_{i,\pm}$ gives a well-defined solution of ODE !**

Achievement on (2): Predictions of novel vacuum processes

1. Background

✔ **Analogies among various physical systems**

If smth happens in an analog system, it should happen in QED (or the vacuum) and vice versa

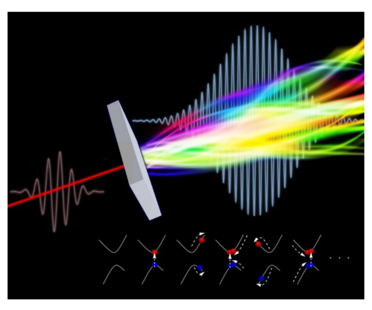
ex 1) QED vacuum ↔ Semi-conductor (materials w/ band)

ex 2) Strong EM field ↔ Other types of strong fields/forces

2. Predictions

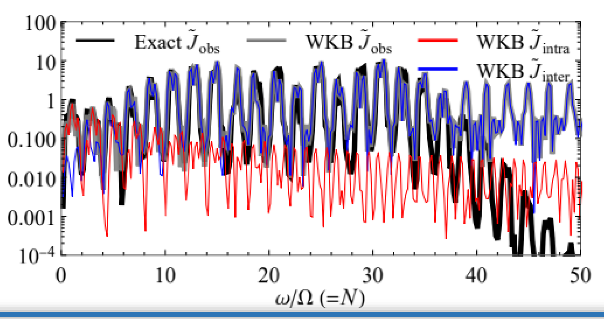
✔ **Ex 1: Vacuum high-harmonic generation**

- $\Omega \to$ smth $\to \omega = n \times \Omega$ (Input light freq. / Output)



- Of increasing interest in sold-state physics and material science
e.g., atto-second light (Nobel prize 2023)

- Predicted HHG spectrum for QED vacuum

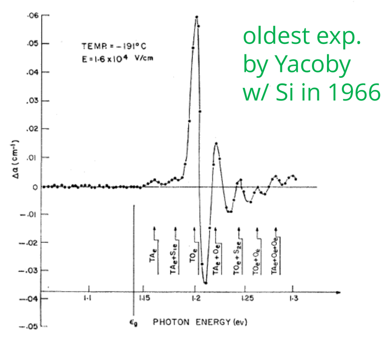


- plateau strc.
- cutoff law $N_{\rm cut} \sim \frac{2eE}{\hbar\Omega}$

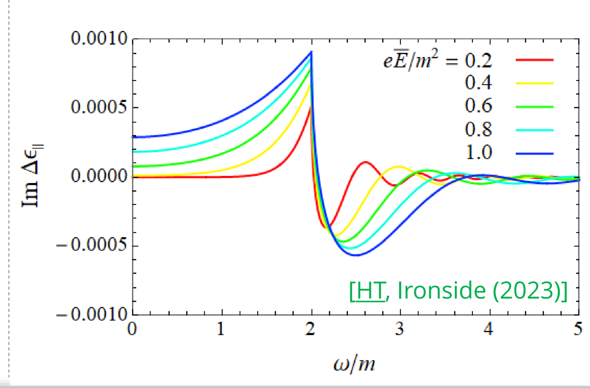
[HT, Hongo, Ikeda (2021)]

✔ **Ex 2: Electric permittivity of the vacuum**

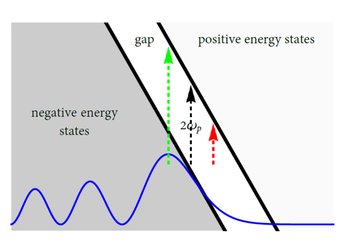
- Electroreflectance in materials
$\epsilon = \epsilon(E) \neq \epsilon(0)$



- Vacuum permittivity is not a const., either, under strong E field

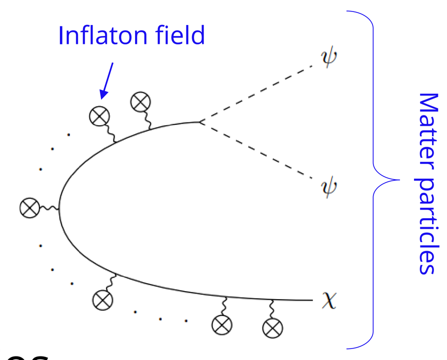


⇐ reflecting a tilted vac. strc. by E

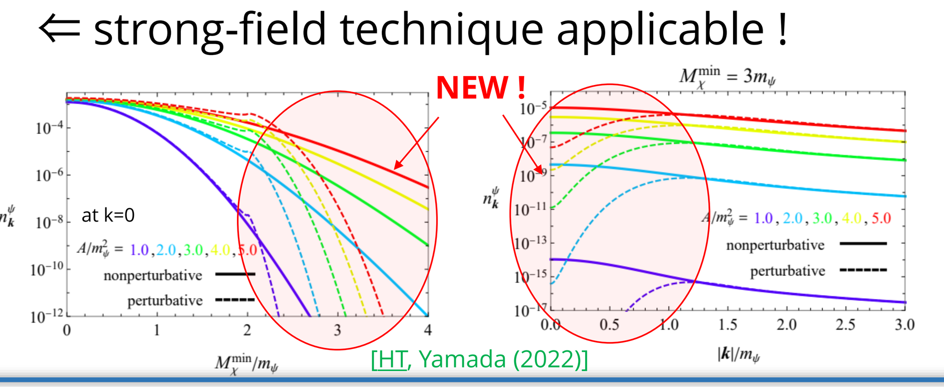


✔ **Ex 3: New decay channel in preheating**

- Inflation is driven by strong inflaton field
- Decay of inflaton = origin of matter in our Universe
- (instant) preheating: one of possible scenarios



⇐ strong-field technique applicable !



Achievement on (3): Proposal of a new laboratory system to test strong-field QED

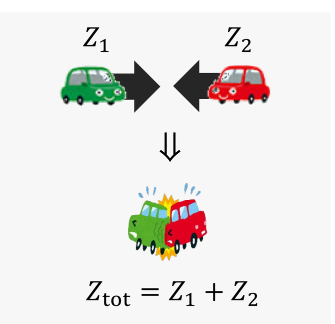
1. Background

✔ **No good system to test strong-field QED at the present**

- Laser: still too weak ...
- Extreme systems: not under control ...

✔ **Idea: Use heavy-ion collisions !**

Baryon stopping at middle-energies (planned at FAIR, NICA, HIAF, J-Parc-HI, ...)
⇒ Formation of a highly-charged matter for a long time



Rough order estimate:

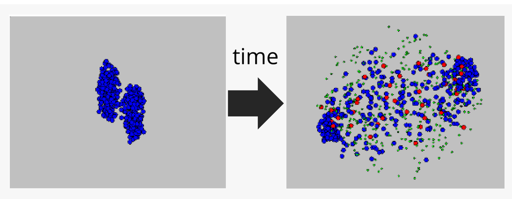
$$eE \sim \frac{(Z_1+Z_2)\alpha}{r^2} \sim \Lambda_{\rm QCD}^2 \sim (100\ \mathrm{MeV})^2$$

⇒ Strongest E field on Earth !
- supercritical to QED
- may also affect QCD/hadron

2. Numerical confirmation based on JAM (Jet AA Microscopic transport model)

✔ **Hadron transport model: JAM** [Nara, Otsuka, Ohnishi, Nitta, Chiba (2000)]

- An open source code to simulate heavy-ion collisions
- Based on the hadron cascade model
= superposition of elastic & inelastic collisions of hadrons, which move classically in space



From webpage of Hokkaido U., nuclear theory group

- Output: Phase-space distributions of hadrons (x, p)
⇒ EM field is obtained by calc. the retarded pot.

$$A^\mu(x^0, \boldsymbol{x}) = \frac{1}{4\pi}\int_{-\infty}^{\infty} d^3x' \frac{J^\mu(x^0 - |\boldsymbol{x}-\boldsymbol{x}'|, \boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

(obtained by JAM)

✔ **Results** [HT, Ohnishi, Nishimura (in preparation)] cf. [HT, Fujii, Itakura (2014)] [HT, Fujimori, Misumi, Nitta, Sakai (2021)]

"phase diagram" of strong-field QED & sensitivity region of HIC