Electric permittivity of the vacuum in a strong constant electric field

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RIKEN → Keio U. (from April)

HT, C. Ironside, PRD 108, 096005 (2023) [2308.11248]

This talk

Discuss the electric permittivity ϵ of the vacuum in a strong constant electric field

In the pure vacuum $\mathcal{D} = \mathcal{E}$

In an EM field $\mathcal{D} = \epsilon\mathcal{E} \neq \mathcal{E}$

- ϵ is no longer a const. $\epsilon = \epsilon(\bar{E})$ due to the vacuum polarization
- many studies since the early days, [Heisenberg-Euler (1936)] [Toll 1952] [Klein-Nigam (1964)] [Baier-Breitenlohner (1967)] … Review: [King-Heinzl 2015] [Ejlli et al. (PVLAS) 2020] …

  but is still worthwhile to be investigated ⇒ 3 motivations

Motivations (1/3)

(1) The current understanding is limited to weak/slow regime

The most famous formula = based on Euler-Heisenberg Lagrangian

$$\mathcal{D} = -\frac{\partial \mathcal{L}_{\text{Euler-Heisenberg}}}{\partial \mathcal{E}} \quad \Rightarrow \quad \epsilon = \frac{\alpha}{45\pi}\left(\frac{e\bar{E}}{m^2}\right)^2 \times \begin{cases} 6 & (\parallel) \\ 2 & (\perp) \end{cases}$$

[Baier-Breitenlohner (1967)]

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[Baier-Breitenlohner (1967)]

Problem 1: Valid only in the weak limit $e\bar{E} \ll m^2$

power corrections could be included $(e\bar{E})^n$, BUT

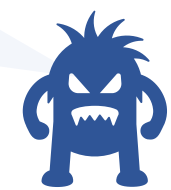
- is factorially divergent ⇒ does not necessarily improve the formula
- non-pert. factor like $e^{1/e\bar{E}}$ can never be included

cf. [Heinzl-Schroder (2006)]

Problem 2: Valid only in the slow limit $\omega \ll m$

- not possible to discuss ω dependence
- the physics must be different above the pair-production threshold $\omega > 2m$

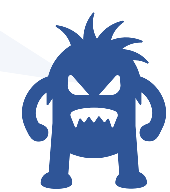
cf. [King-Heinzl-Blackburn (2023)]



Problem 3: Neglecting the imaginary part $\mathrm{Im}\,\mathcal{L}_{\text{Euler-Heisenberg}}$

ϵ (in the coordinate space) must be real, so one must set $\mathrm{Im}\,\mathcal{L}_{\text{Euler-Heisenberg}}$

⇒ pair production and "non-equilibrium-ness" of E field are completely dismissed



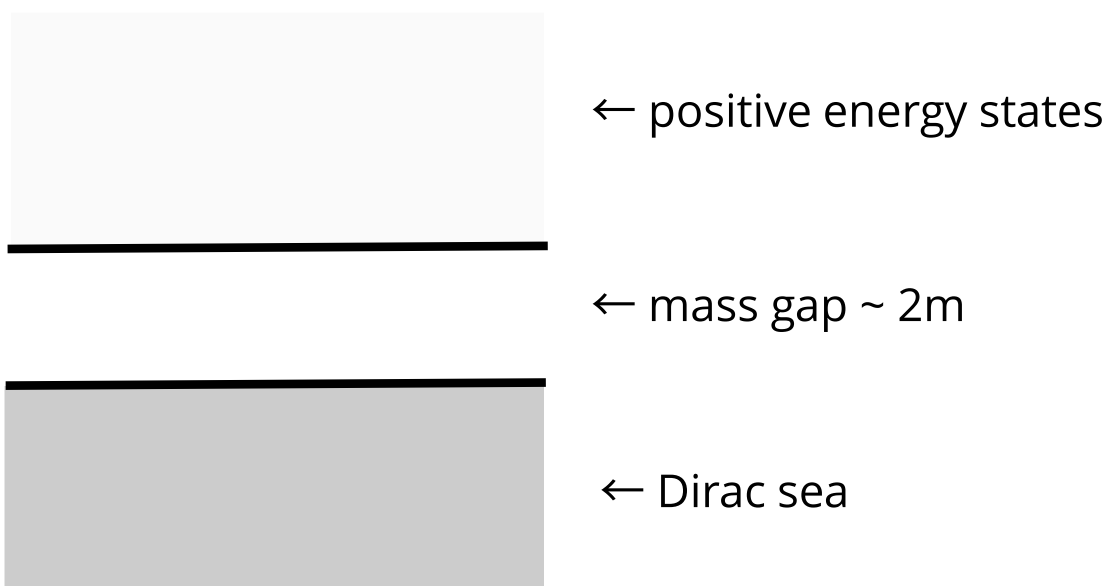
Q: What happens if I go beyond those limitations ?

Motivations (2/3)

(2) As a signature of non-trivial QED vacuum structure in E field

The QED vacuum (= the Dirac sea) has a non-trivial electron dist. in an E field, which can leave observable imprints

QED vacuum at $\overline{E} = 0$



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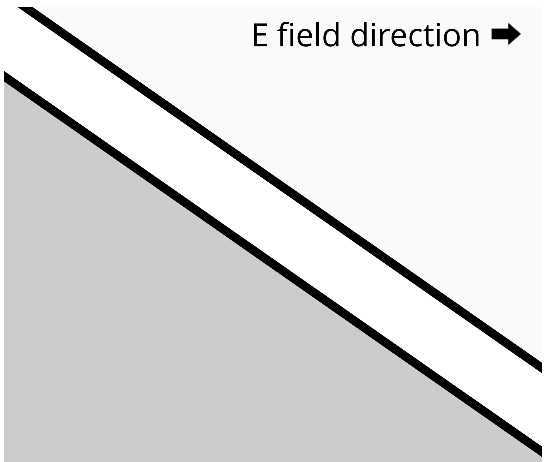

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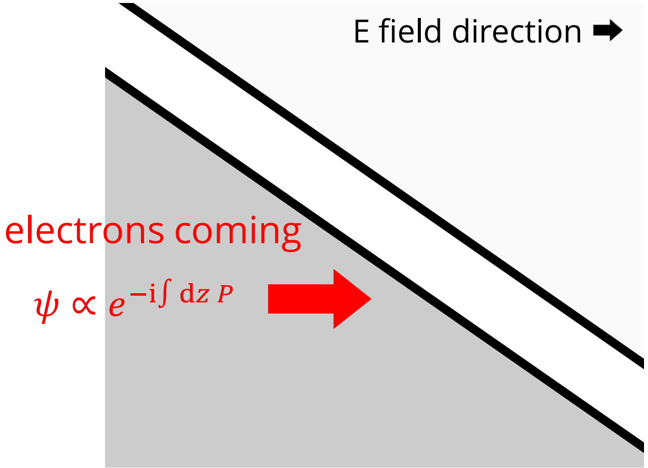

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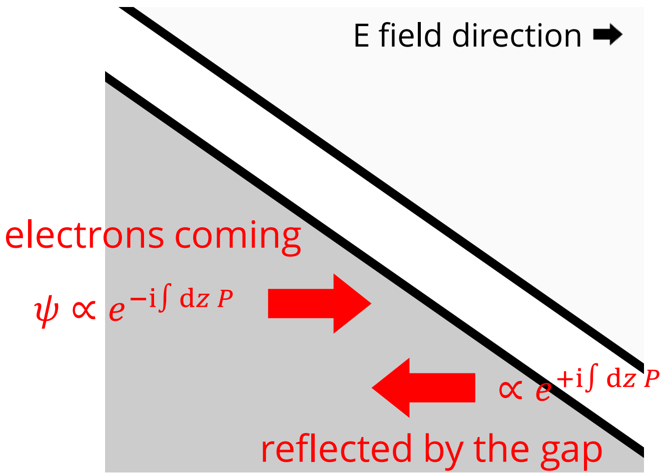

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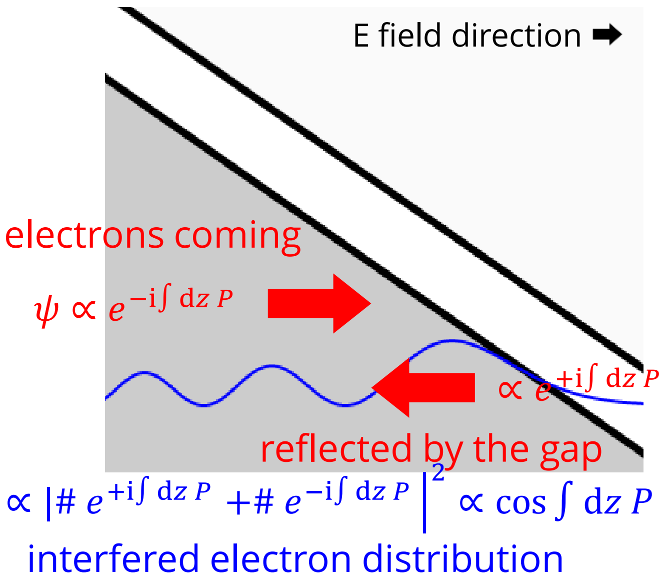

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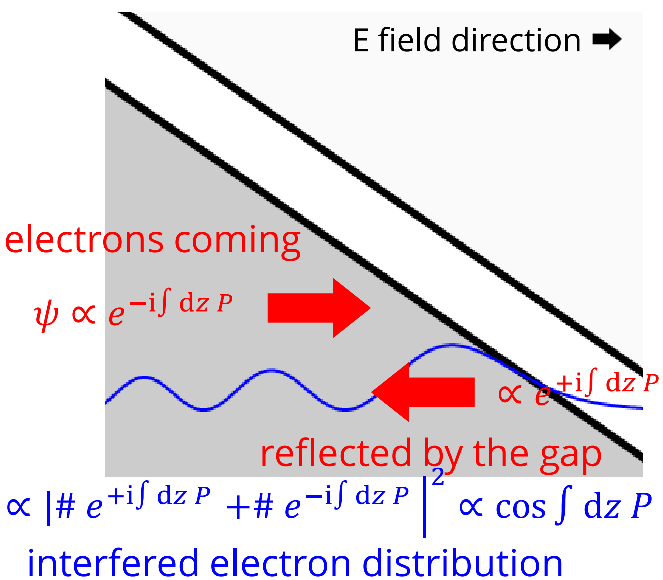
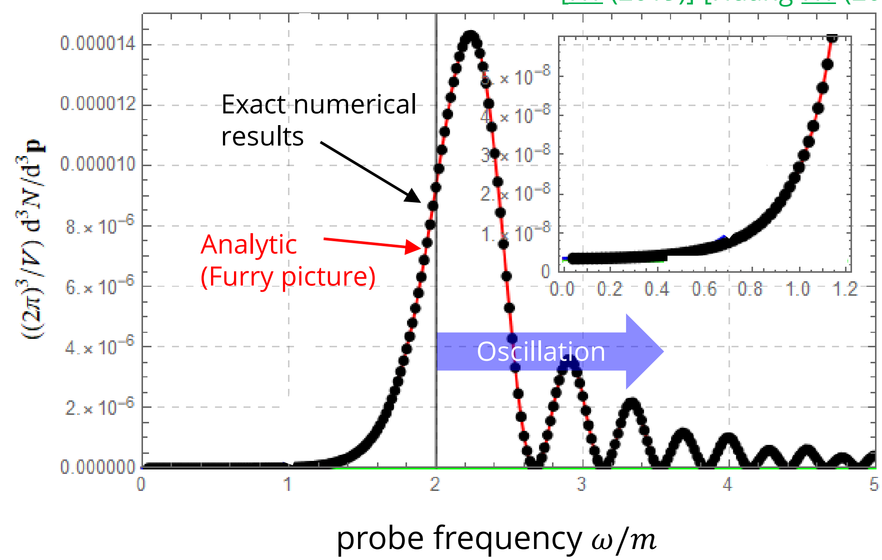


$\Rightarrow$

Observable imprint

Ex.) spectrum of the dynamically-assisted Schwinger (= pair prod. in const $\bar{E}$ + fast probe $\mathcal{E}$)
[HT (2019)] [Huang-HT (2020)]



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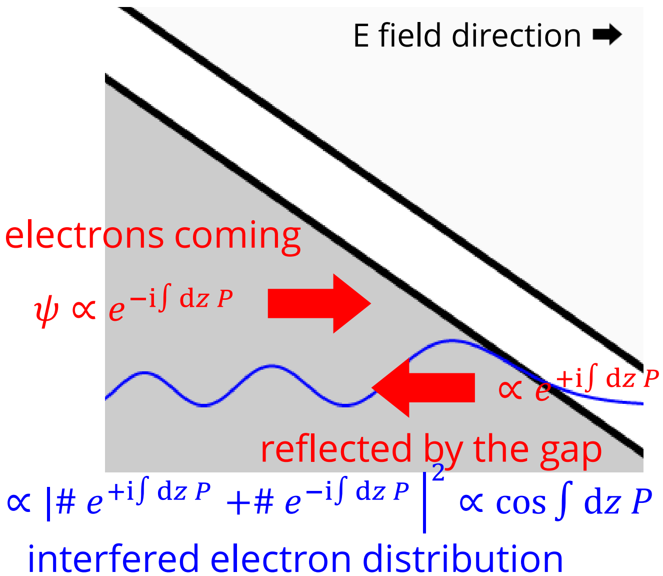

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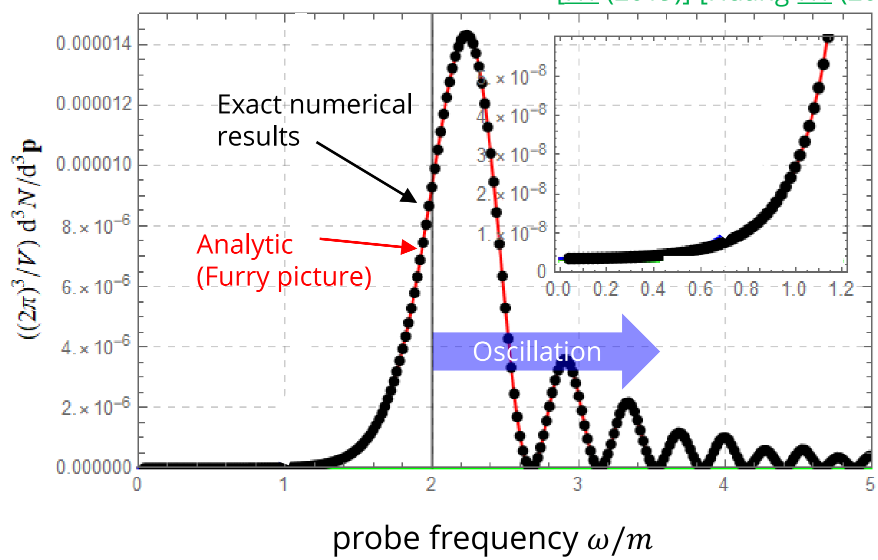

Q: What happens to the electric permittivity ?

Note: The motivation (1) (in particular, going beyond $\omega \ll m$) is important to achieve this

Motivations (3/3)

(3) Pursue analogy between strong-field QED and semicond. phys

- Ground-state structure of semicond. = The QED vacuum

  ⇒ the QED vacuum should response against external field in a similar way
  to a semiconductor and vice versa

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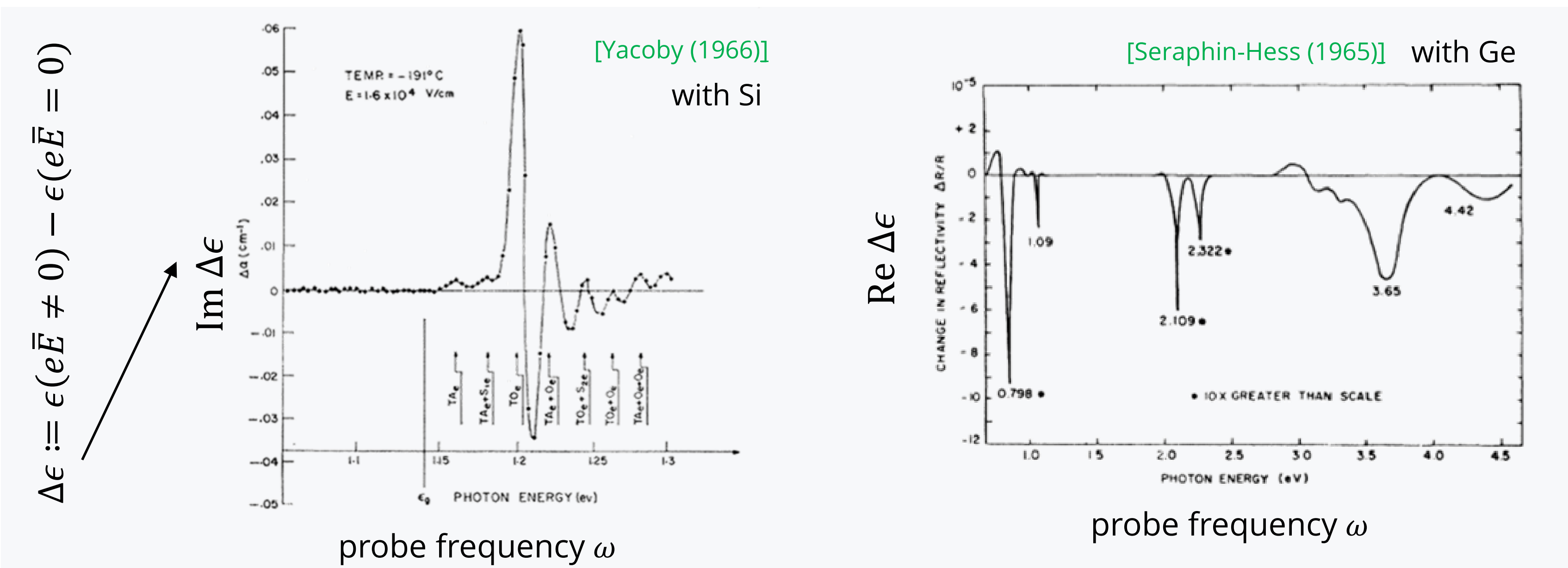
- Nontrivial oscillating change in ϵ (i.e., motivation (2)) has already been observed more than 50 yrs ago in semicond. physics !
  (⇒ the Franz-Keldysh effect and electroreflectance) [Franz (1958)] [Keldysh (1958)]



Q: Natural to expect this change in QED. To what extent is this true ?

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(2) As a signature of non-trivial QED vacuum structure in E field

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II. Theory

Linear response theory based on in-in formalism of QFT

+ Kramers-Kronig rel. + a "phenomenological" IR regularization

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Will give positive statements to the motivations (1) - (3)

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Theory (1/5)

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Step 1: Definitions of $\mathcal{D}$ and ϵ

- Total flux $D = E + P(\bar{E}, \mathcal{E}) = E + P_0(\bar{E}) + P_1(\bar{E})\mathcal{E} + \cdots$

  ($E = \bar{E} + \mathcal{E}$)

  $= \bar{E} + P_0(\bar{E}) + (1 + P_1(\bar{E}))\mathcal{E} + \cdots \Rightarrow \epsilon = 1 + P_1(\bar{E})$

  where $\mathcal{D} = (1 + P_1(\bar{E}))\mathcal{E}$

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  $(1 + P_1(\bar{E}))\mathcal{E} \to \mathcal{D}$

Step 2: Calculate P_1

- Ampere law: $-\dot{P} = J = \langle 0; \mathrm{in} | \bar{\psi}(\bar{E}, \mathcal{E}) \gamma^\mu \psi(\bar{E}, \mathcal{E}) | 0; \mathrm{in} \rangle$

$$= \langle 0; \mathrm{in} | \bar{\psi}_0(\bar{E}) \gamma^\mu \psi_0(\bar{E}) | 0; \mathrm{in} \rangle$$

$$+ \langle 0; \mathrm{in} | \bar{\psi}_1(\bar{E}) \gamma^\mu \psi_0(\bar{E}) + \bar{\psi}_0(\bar{E}) \gamma^\mu \psi_1(\bar{E}) | 0; \mathrm{in} \rangle \times \mathcal{E} + \mathcal{O}(\mathcal{E}^2)$$

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- Not in-out amplitude, but in-in ! cf. [Copinger-Fukushima (2018)]

  ⇒ crucial when pair creating (or in non-equil.): $|0; \mathrm{out}\rangle = |0; \mathrm{in}\rangle + (\text{pair states like } |e^+ e^-; \mathrm{in}\rangle)$

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  ($\langle 0;\mathrm{in}|\bar{\psi}_1\gamma^\mu\psi_0 + \bar{\psi}_0\gamma^\mu\psi_1|0;\mathrm{in}\rangle$ → P_1)

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Step 3: Calculate ψ_0 and ψ_1

- Solve Dirac eq. in E field:

  $[i\partial\!\!\!/ - e(\bar{A\!\!\!/} + \mathcal{A\!\!\!/}) - m]\psi = 0 \Rightarrow \psi = \psi_0(\bar{E}) + S_\mathrm{R}(\bar{E})e\mathcal{A\!\!\!/}\psi_0(\bar{E}) + \mathcal{O}(\mathcal{E}^2)$

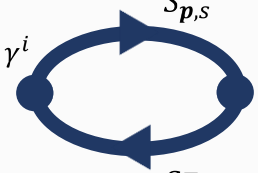
  ($S_\mathrm{R}(\bar{E})e\mathcal{A\!\!\!/}\psi_0(\bar{E})$ → ψ_1)

Theory (2/5)

Linear response theory based on in-in formalism of QFT

Step 4: Collect everything + massaging …

$$\epsilon_{ij}(\omega) = 1 + \frac{1}{\omega^2}\int_{-\infty}^{+\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau}\Theta(\tau)\Pi_{\mathrm{ij}}(\tau) \simeq 1 +$$



where $\quad \Pi_{\mathrm{ij}}(\tau) \coloneqq 2e^2 \mathrm{Im\,tr} \sum_{s,s'} \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \gamma^i S^-_{\boldsymbol{p},s}(+\frac{\tau}{2}, -\frac{\tau}{2}) \gamma^j S^+_{\boldsymbol{p},s}(-\frac{\tau}{2}, +\frac{\tau}{2})$

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Point 1: No Feynman propagator ($\because$ in-in calculation)

Point 2: Manifestly causal $\quad \epsilon(t) \propto S_{\mathrm{R}}(t) \propto \Theta(t)$

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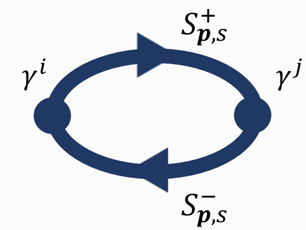

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Point 2: Manifestly causal $\epsilon(t) \propto S_{\mathrm{R}}(t) \propto \Theta(t)$

Step 5: Do the integrations ! ⇒ But it is not straightforward

Problem 1: UV divergence (∵ loop diagram)

Problem 2: IR divergence (∵ the LO treatment in $\mathcal{E}$ breaks down, since the non-linearity parameter $\xi \coloneqq e\mathcal{A}/m \simeq e\mathcal{E}/\omega \to \infty$)

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Idea: (1) Only the real part is UV divergent, and the imaginary part is finite and calculable

cf. Cutkosky rule

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(2) Due to causality, Re and Im are related with each other
(Kramers-Kronig relation) [Toll (1960)] See also [Heinzl, Schroeder (2006)] [Borysov et al. (2022)] for nBW [Aspnes (1967)] for semicond.

$$\epsilon_{ij}(\omega) = 1 + \frac{1}{\omega^2}\int_{-\infty}^{+\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau}\Theta(\tau)\Pi_{\mathrm{ij}}(\tau) \Rightarrow \ \mathrm{Re}\,\epsilon(\omega) = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{+\infty} \mathrm{d}\omega' \frac{1}{\omega' - \omega}\mathrm{Im}\,\epsilon(\omega')$$

Causality = the step function is the essence (for in-in response functions)
(not unitarity, unlike the optical theorem for in-out amplitudes)

∴ **It is sufficient to calculate the imaginary part, which does not suffer from UV div.**

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Step 7: Introduce a counter term and match it with Schwinger formula

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Idea: (1) In general, resummation gives a counter term $\Delta\epsilon$

$$\Rightarrow \ \epsilon \to \epsilon_{\rm reg} = \epsilon + \Delta\epsilon$$

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(3) Use relationship between $\mathrm{Im}\ \epsilon$ and the Schwinger effect (by $E = \bar{E} + \mathcal{E}$)

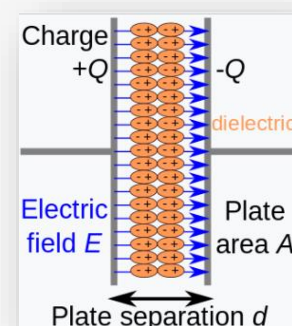
Intuitive explanation: analog of electromagnetism in matter

cf. see Landau-Lefshitz

- Dielectric energy loss (= decay of a probe) $\frac{\mathrm{d}U_{\mathrm{loss}}}{\mathrm{d}t} = \mathcal{E}\frac{\mathrm{d}\mathcal{D}}{\mathrm{d}t} = \frac{1}{2}\omega\mathcal{E}^2\ \mathrm{Im}\ \epsilon_{\mathrm{reg}}$
- Microscopically, the decay of a probe is caused by the pair production

⇒ Energy for pair prod. $\frac{\mathrm{d}U_{\mathrm{pp}}}{\mathrm{d}t} = \omega\frac{N(\mathcal{E}\neq 0) - N(\mathcal{E}=0)}{VT}$ must equal to U_{loss}

$$\Rightarrow \frac{N(\mathcal{E}\neq 0) - N(\mathcal{E}=0)}{VT} = \frac{1}{2}\mathcal{E}^2\ \mathrm{Im}\ \epsilon_{\mathrm{reg}}$$



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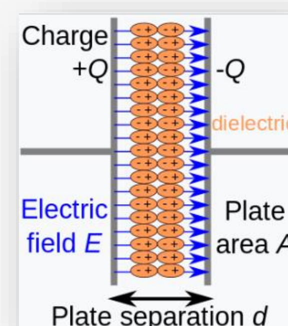


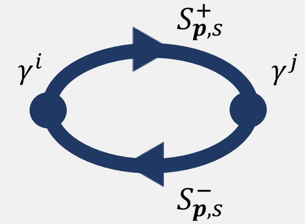
(4) LHS at $\omega \to 0$ can be calculated with the Schwinger formula, so $\Delta\epsilon$ can be fixed

$$N_{\rm Schwinger}(\bar{E}+\mathcal{E}) - N_{\rm Schwinger}(\bar{E}) \propto \left(\exp\left[-\pi\frac{m^2}{e(\bar{E}+\mathcal{E})}\right] - \exp\left[-\pi\frac{m^2}{e\bar{E}}\right]\right) = (\dots)\times\exp\left[-\pi\frac{m^2}{e\bar{E}}\right]\times\mathcal{E}^2$$

Theory (5/5)

Linear response theory based on in-in formalism of QFT

$$\epsilon_{ij}(\omega) = 1 + \frac{1}{\omega^2}\int_{-\infty}^{+\infty} \mathrm{d}\tau \, \mathrm{e}^{\mathrm{i}\omega\tau}\Theta(\tau)\Pi_{\mathrm{ij}}(\tau) \;\simeq\; 1 +$$



$$\text{Schwinger } N \;\leftrightarrow\; \mathrm{Im}\,\epsilon \;\overset{\text{KK}}{\leftrightarrow}\; \mathrm{Re}\,\epsilon$$

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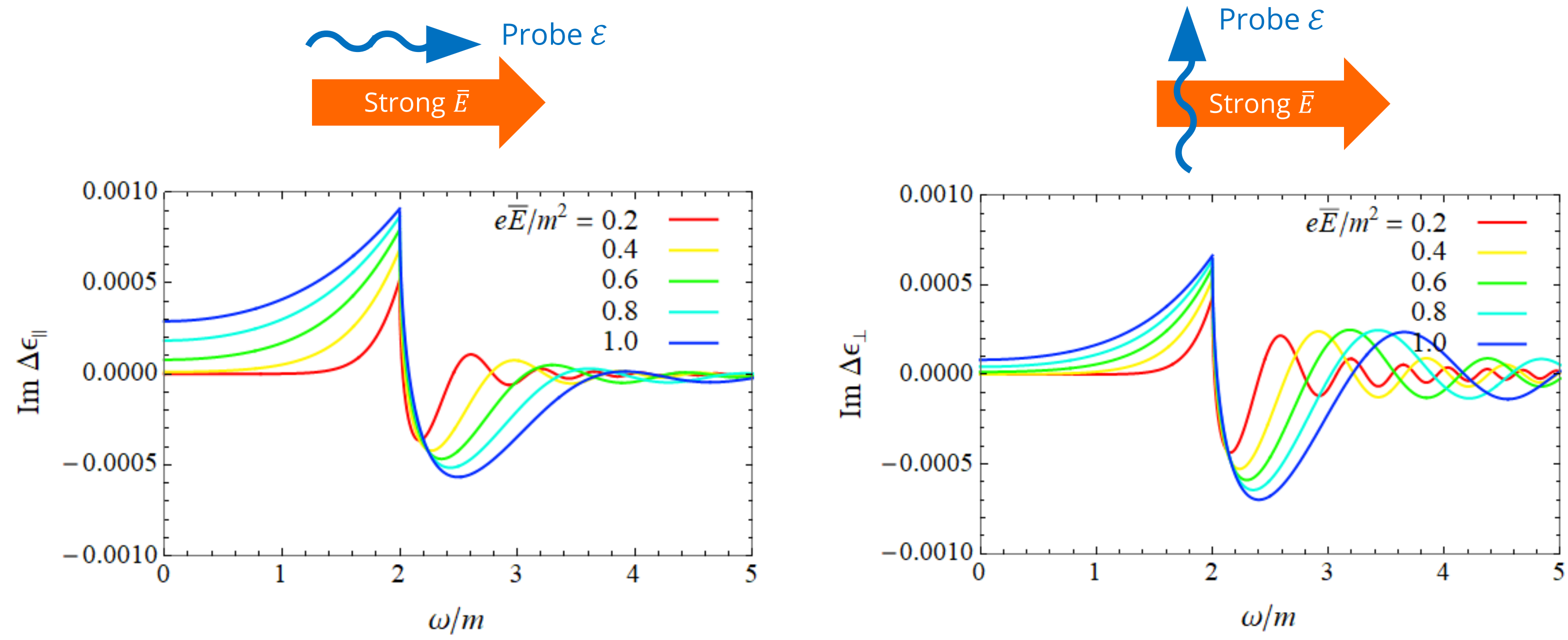
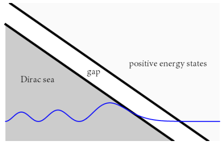
Results (1/4)

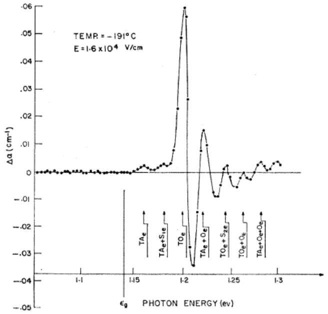
Imaginary part of the change $\Delta\epsilon = \epsilon(\bar{E} \neq 0) - \epsilon(\bar{E} = 0)$



- Oscillation, as expected from the tilted vacuum



- Essentially the same as what have observed in semiconductor physics



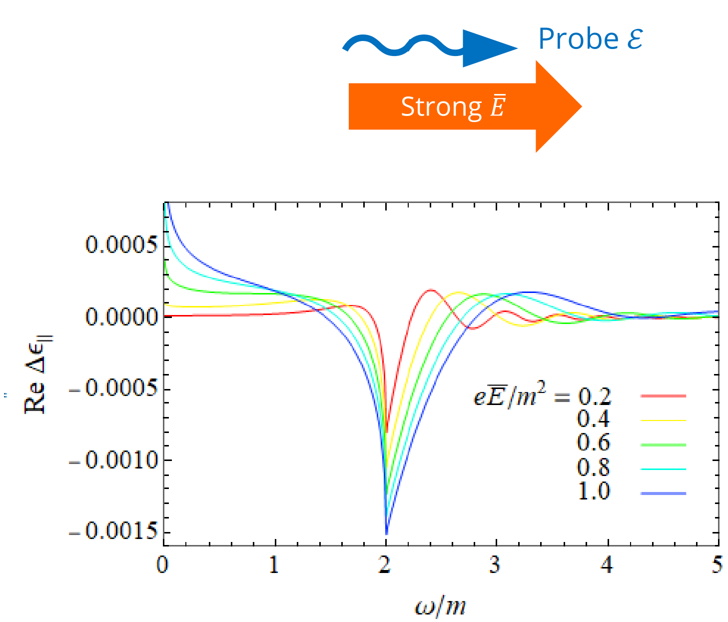
- Birefringent ($\mathrm{Im}\,\Delta\epsilon_{\parallel} \neq \mathrm{Im}\,\Delta\epsilon_{\perp}$) but basically the same

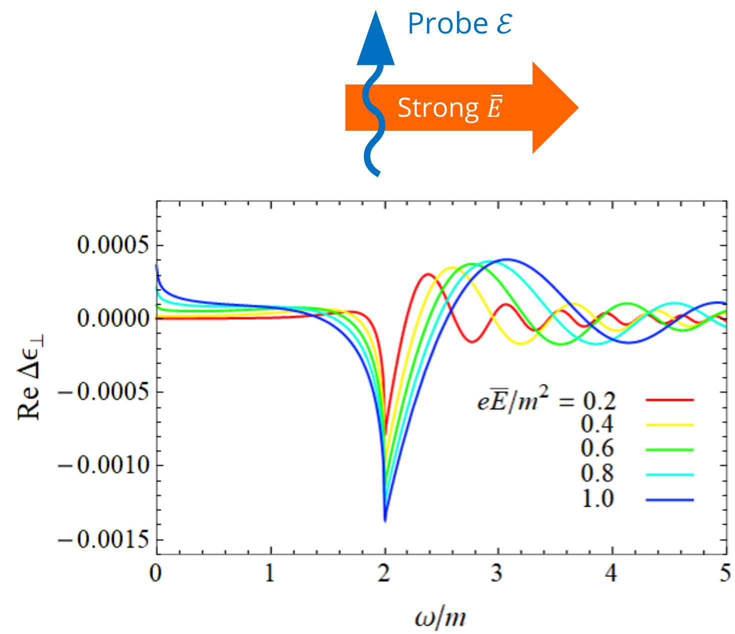
- Non-vanishing even at $\omega \to 0$ due to the non-perturbative pair production (Schwinger)

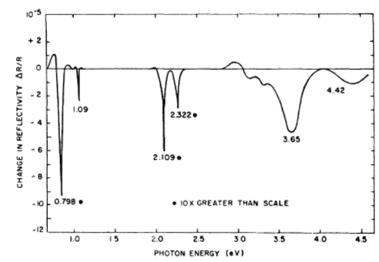
$$\because\ \mathrm{Im}\,\epsilon(\omega \to 0) \propto (N_{\mathrm{Schwinger}}(\bar{E} + \mathcal{E}) - N_{\mathrm{Schwinger}}(\bar{E})) \propto (\exp\left[-\pi \frac{m^2}{e(\bar{E}+\mathcal{E})}\right] - \exp[-\pi \frac{m^2}{e\bar{E}}]) = (\mathrm{finite}) \times \exp\left[-\pi \frac{m^2}{e\bar{E}}\right]$$

Results (2/4)

Real part of the change $\Delta\epsilon = \epsilon(\bar{E} \neq 0) - \epsilon(\bar{E} = 0)$







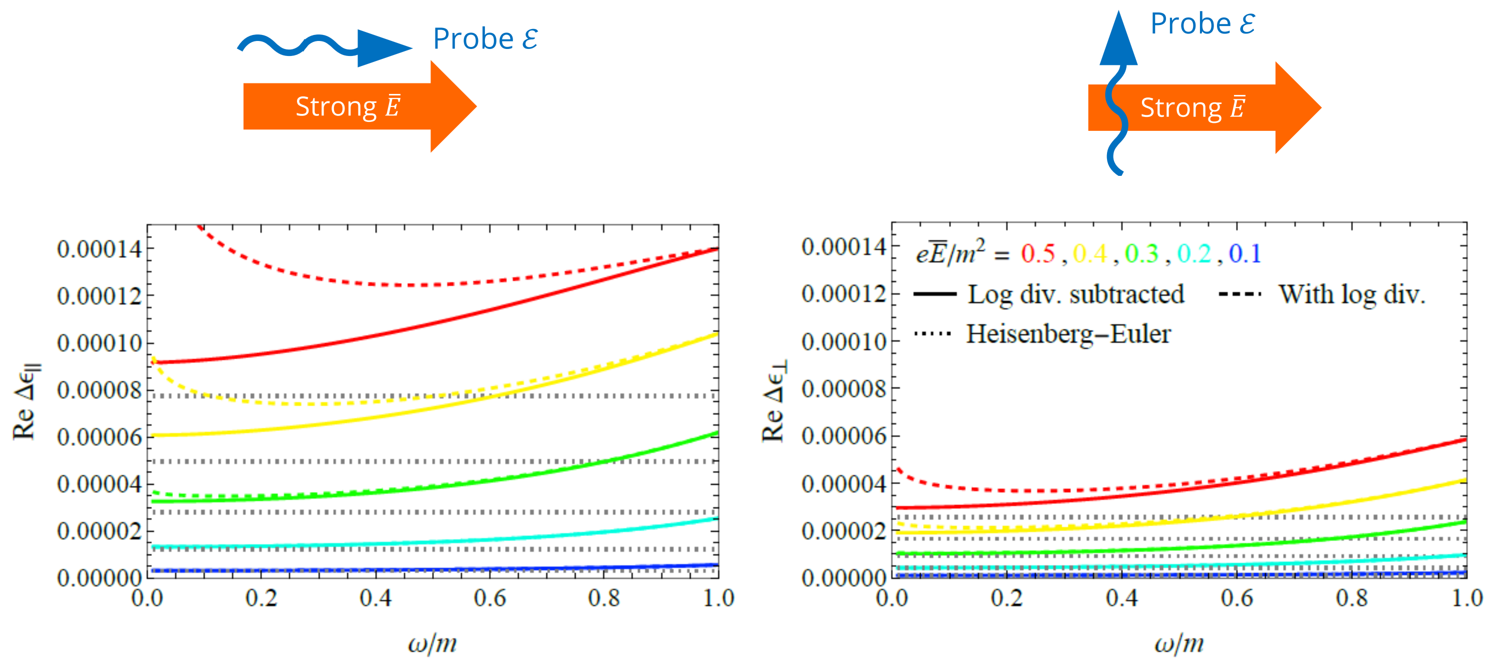
- Again oscillation, which is again consistent with semi-conductor
- Logarithmically divergent at $\omega \to 0$ due to the non-pert. pair prod.

$$\because \quad \mathrm{Re}\,\epsilon(0) = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{+\infty} d\omega' \frac{1}{\omega'}\mathrm{Im}\,\epsilon(\omega') \sim \frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega' \frac{1}{\omega'}\mathrm{Im}\,\epsilon(0) \sim (\text{log div.}) \times \exp\left[-\pi\frac{m^2}{e\bar{E}}\right]$$

$\Rightarrow$ a “matter” effect, which needs to be subtracted to compare with the Euler-Heisenberg result

Results (3/4)

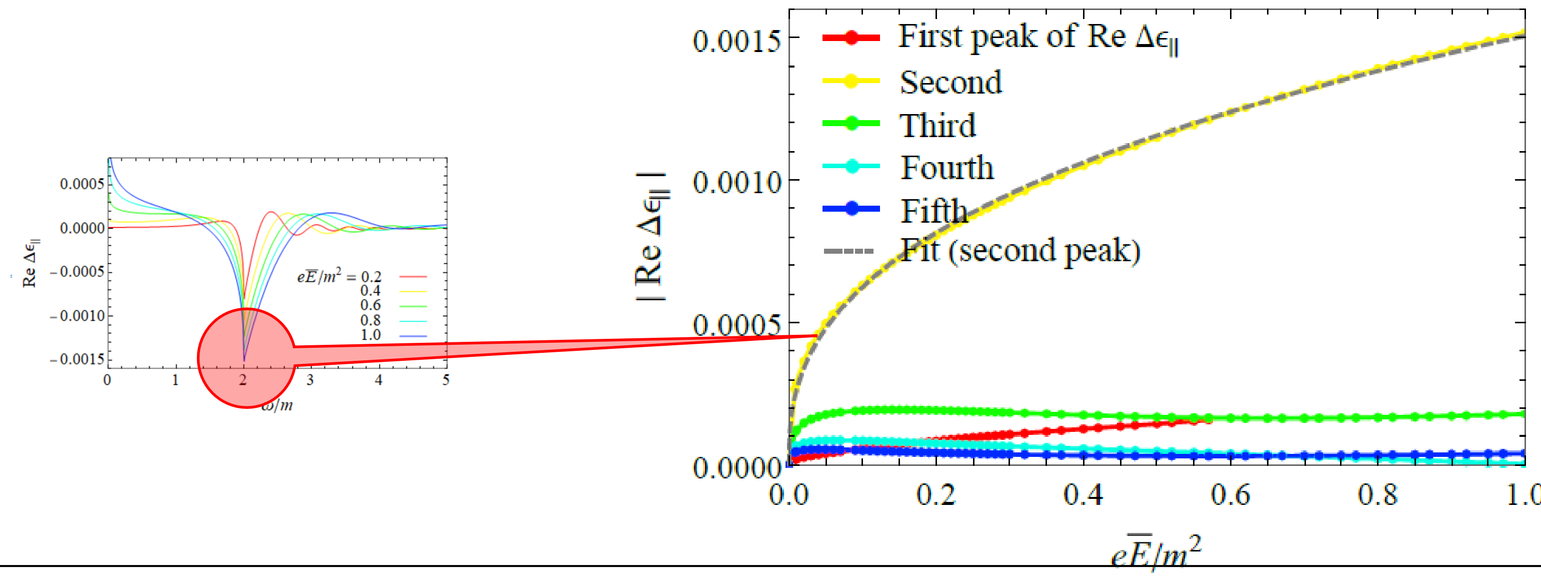
Comparison w/ EH result around $\omega \approx 0$



- After the log subtraction, the result is consistent with the EH result $\Delta\epsilon_{\rm EH} = \frac{\alpha}{45\pi}\left(\frac{e\bar{E}}{m^2}\right)^2 \times \begin{cases} 6 & (\parallel) \\ 2 & (\perp) \end{cases}$
- The agreement becomes very good for weak fields $e\bar{E}/m^2 \lesssim 0.2$
  but EH underestimates about O(>10%) for strong fields

Results (4/4)

Behaviors of the peaks



Numerical fit says (similar numbers for $\mathrm{Im}\,\epsilon_\perp$ and $\mathrm{Re}\,\epsilon_{\parallel,\perp}$)

$$\mathrm{Re}\,\Delta\epsilon_\parallel \approx (-1.4 \times 10^{-4}) \times \left(\frac{I}{1 \times 10^{23}\ \mathrm{W/cm^2}} \right)^{0.19}$$

where $I = \bar{E}^2/2$ is the focused intensity

- much larger than EH e.g., $\Delta\epsilon_{\mathrm{EH}} = O(10^{-7})$ for $I = O(1 \times 10^{23}\ \mathrm{W/cm^2})$ (PW laser)

  ⇒ High-frequency probe is useful to study ϵ (or vacuum birefringence, in general)

- weak E dependence $\Delta\epsilon \propto I^{\frac{1}{5}\sim\frac{1}{6}}$ ⇒ the peaks of $\Delta\epsilon$ can still be large for subcritical fields

  e.g., only one-order smaller $\Delta\epsilon = O(1 \times 10^{-5}) \gg \Delta\epsilon_{\mathrm{EH}}$ at GW scale

This talk

Discuss the electric permittivity ϵ of the vacuum in a strong constant electric field

I. Introduction

(1) The current understanding is limited to weak/slow regime

(2) As a signature of non-trivial QED vacuum structure in E field

(3) Pursue analogy between strong-field QED and semicond. phys

II. Theory

Linear response theory based on in-in formalism of QFT

+ Kramers-Kronig rel. + a “phenomenological” IR regularization

III. Results

Will give positive statements to the motivations (1) - (3)

IV. Summary

DONE

DONE

DONE

Summary

Discussed the electric permittivity ϵ of the vacuum in a strong constant electric field

3 motivations and answers to them:

(1) The current understanding is limited to weak/slow regime

⇒ Succeeded in developing such a theory:
Linear response theory based on in-in formalism of QFT
+ Kramers-Kronig rel. + a "phenomenological" IR regularization

(2) As a signature of non-trivial QED vacuum structure in E field

(3) Pursue analogy between strong-field QED and semicond. phys

⇒ Yes: a characteristic oscillating structure in ϵ and is analogous to semicond.