Introduction to strong-field physics

Hidetoshi Taya

Research area: Theoretical physics (particle/nuclear theory)

Today's talk

Q: What happens if we make light (or "field" in general) stronger and stronger ?

LED

$I \sim 10^{-5}\ \mathrm{W/cm^2}$

or $eE \sim 10^{-1}\ \mathrm{eV/cm}$

$\sim (10^{-3}\mathrm{eV})^2$

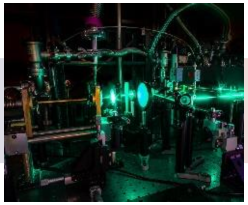
Laser welding

$10^{6}\ \mathrm{W/cm^2}$

$10^{4}\ \mathrm{eV/cm}$

$(10^{-1}\mathrm{eV})^2$

Cond.-mat (THz laser)



$10^{10}\ \mathrm{W/cm^2}$

$10^{6}\ \mathrm{eV/cm}$

$(1\ \mathrm{eV})^2$

Guiness record

GUINNESS WORLD RECORDS

CERTIFICATE

The highest intensity focused laser is the HERCULES Petawatt Laser at the University of Michigan, USA, which can produce a focused laser beam of 2×10^{22} W/cm² or 20 billion trillion watts per square centimetre

OFFICIALLY AMAZING

HERCULES @ USA (2008)

$10^{22}\ \mathrm{W/cm^2}$

$10^{12}\ \mathrm{eV/cm}$

$(1\ \mathrm{keV})^2$

Our limit at the present

???

?

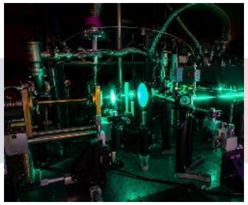
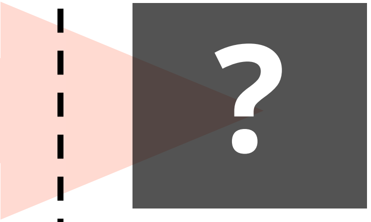
Daily life

Industry

Science

Today’s talk

Q: What happens if we make light (or “field” in general) stronger and stronger ?

LED	Laser welding	Cond.-mat (THz laser)	Guiness record	???
			 CERTIFICATE The highest intensity focused laser is the HERCULES Petawatt Laser at the University of Michigan, USA, which can produce a focused laser beam of 2x10²² W/cm² or 20 billion trillion watts per square centimetre OFFICIALLY AMAZING HERCULES @ USA (2008)	
$I \sim 10^{-5}\ \mathrm{W/cm^2}$	$10^6\ \mathrm{W/cm^2}$	$10^{10}\ \mathrm{W/cm^2}$	$10^{22}\ \mathrm{W/cm^2}$	**Our limit at the present**
or $eE \sim 10^{-1}\ \mathrm{eV/cm}$	$10^4\ \mathrm{eV/cm}$	$10^6\ \mathrm{eV/cm}$	$10^{12}\ \mathrm{eV/cm}$	
$\sim (10^{-3}\mathrm{eV})^2$	$(10^{-1}\mathrm{eV})^2$	$(1\ \mathrm{eV})^2$	$(1\ \mathrm{keV})^2$	

Daily life → Industry → Science →

A: Our vacuum shall be destroyed and gives us nontrivial responses

(≈ “something” even from “nothing”)

⇒ Purpose of this talk: Review such physics of strong light

Contents

1. Overview of strong-field physics

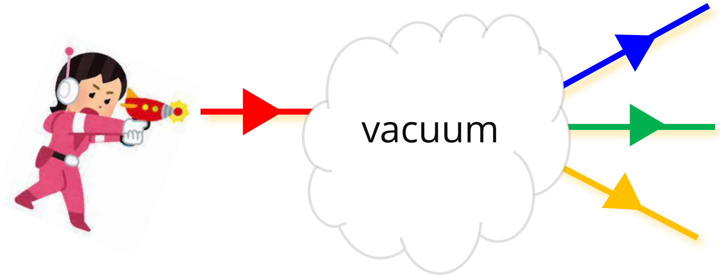
- becoming a hot topic in physics, thanks to the recent developments in exp.
- the Schwinger effect = a process to create “something” from “nothing”

2. Application of the Schwinger effect ⇒ High-harmonic generation from the vacuum

[**HT**, Hongo, Ikeda (2021)]

- first prediction of HHG from the vacuum
- a good example of “interdisciplinarity” of strong-field physics

  ⇒ your expertise should be useful in my area and vice versa !



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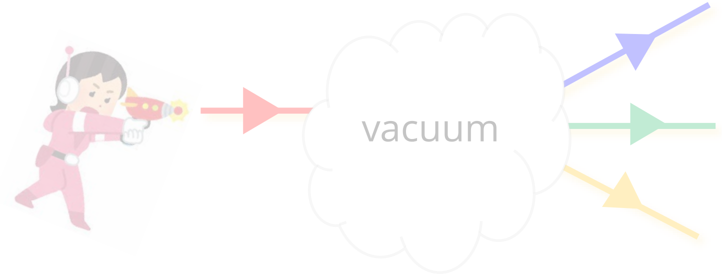
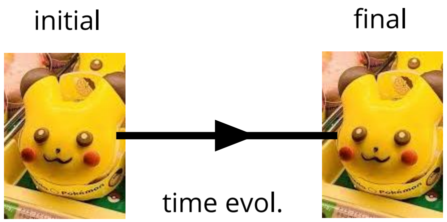


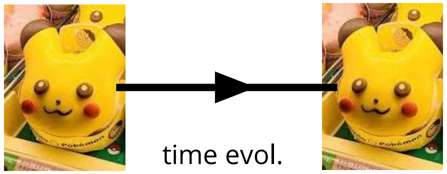
3. Summary

What if field becomes strong ?



No field

What if field becomes strong ?


initial

final

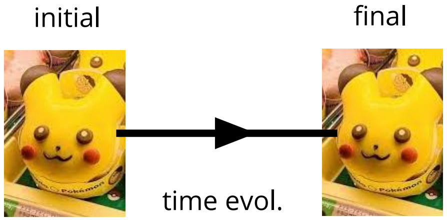
time evol.

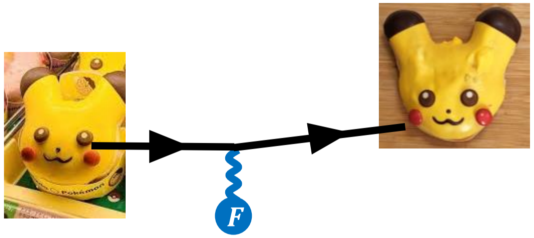

No field

Weak field

Strong field

What if field becomes strong ?





Only minor changes

⇒ Perturbative

⇒ Very well understood
in both exp.& theor.

ex.) Electron (anomalous) magnetic moment $a := \frac{g-2}{2}$
≈ Electron energy shift in a weak magnetic field

$a(\text{theor.}) = 1159652182.03 \ldots \times 10^{-12}$
$a(\text{exp.}) \quad = 1159652180.73 \ldots \times 10^{-12}$ [Aoyama, Kinoshita, Nio (2017)]

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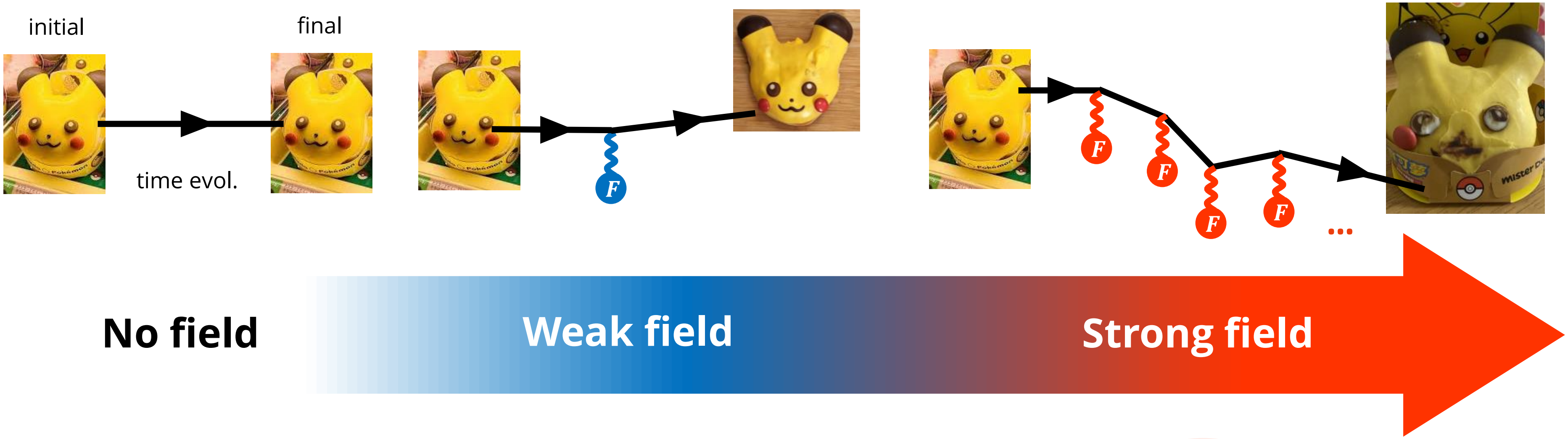

No field

Weak field

Strong field

Only minor changes

⇒ Perturbative

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Big change !

⇒ Non-Perturbative

⇒ Not understood well

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[Aoyama, Kinoshita, Nio (2017)]

If field becomes strong, physics becomes totally different & nontrivial

When is field “strong” ?

Strong-field condition:

To significantly modify the original system with typical energy Δ,
the field must be more energetic than Δ

⇒ Strong-field condition: $\boldsymbol{\Delta < (\text{energy scale of the field})}$

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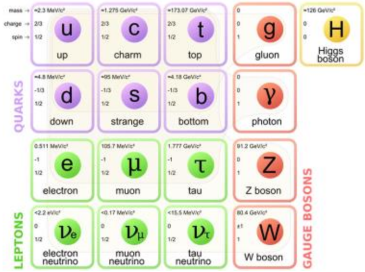
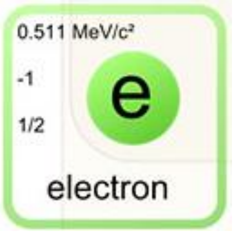
Estimate of the minimum field strength:

Standard Model



⇨

The (matter) particle having the minimum energy



- Charged ⇒ Couples to electromagnetic (EM) field
- $eE\ \mathrm{MeV}^2 > m_e^2 = (0.511\ \mathrm{MeV})^2 \approx O(10^{28}\ \mathrm{W/cm^2})$

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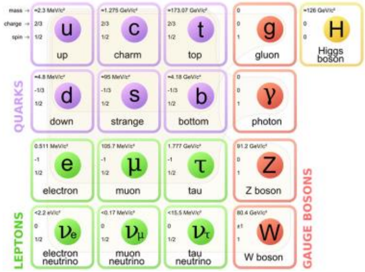

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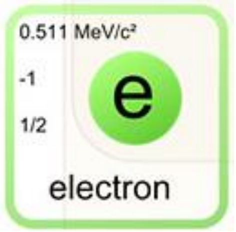

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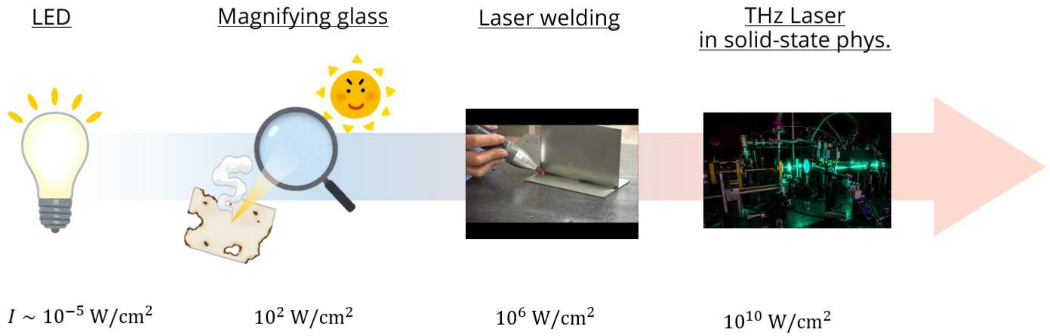
- $\boldsymbol{eE\ \mathrm{MeV}^2 > m_e^2 = (0.511\ \mathrm{MeV})^2 \approx O(10^{28}\ \mathrm{W/cm^2})}$

∴ **EXTREMELY strong**

⇒ Impossible to realize exp.
in 20th century



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Recent availability of strong EM fields

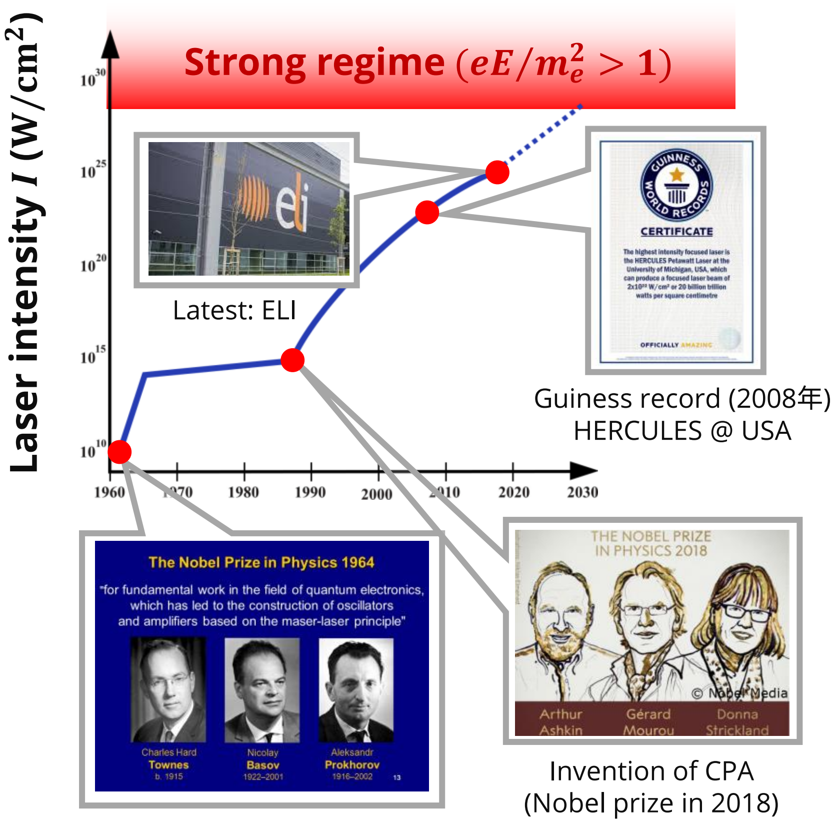
The situation changing: becoming able to create/observe strong light

∴ NOW is the BEST time to study strong-field physics

Recent availability of strong EM fields

The situation changing: becoming able to create/observe strong light

High-power laser



Latest: ELI

CERTIFICATE

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OFFICIALLY AMAZING

Guiness record (2008年)
HERCULES @ USA

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"for fundamental work in the field of quantum electronics, which has led to the construction of oscillators and amplifiers based on the maser-laser principle"

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Invention of CPA
(Nobel prize in 2018)

Invention of laser
(Nobel prize in 1964)

Extreme physical systems

- **Heavy-ion collisions**

  RIC (2000~), LHC (2012~),
  FAIR/NICA/HIAF/J-Parc-HI/… (20XX~)

  $I \sim 10^{35}\ \mathrm{W/cm^2}$
  $(eE, eB \sim m_\pi^2 \sim (140\ \mathrm{MeV})^2)$

- **Electron collider + Laser**

  Start soon: LUXE @ DESY, FACET-II @ SLAC

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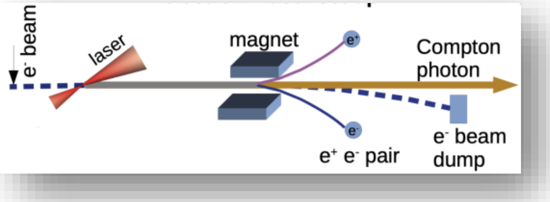

- **Magnetar**

  Suzaku (2005~2015), NICER (2017~)
  XL-Calibur (2018~), IXPE (2021~), …

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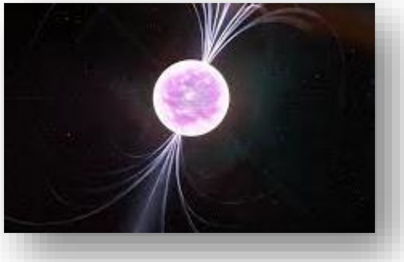

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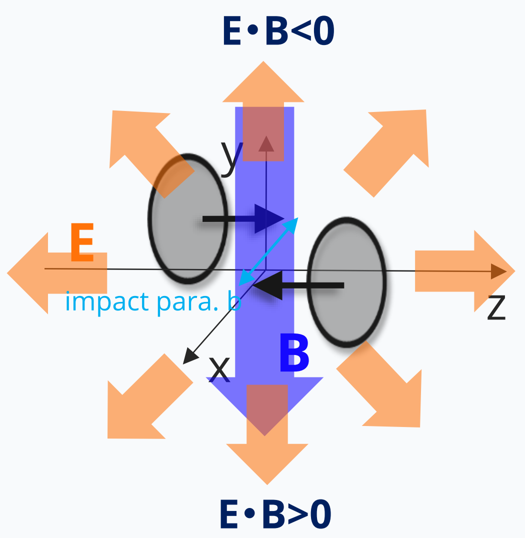
A bit more on heavy-ion collisions

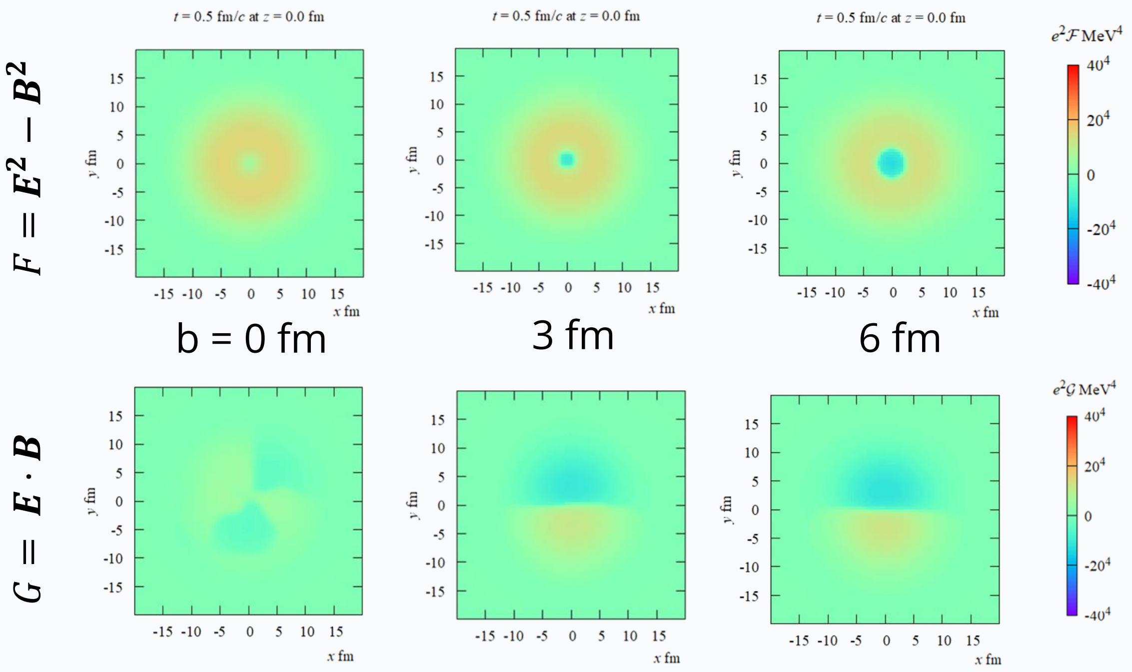
Low-energy ($\sqrt{s_{\rm NN}} = 2 - 10$) HIC is interesting among other strong-field systems

- Sales point: The only system that has supercritical $F := \boldsymbol{E}^2 - \boldsymbol{B}^2 > 0,\ G := \boldsymbol{E}\cdot\boldsymbol{B} \neq 0$

	high-power laser	**magnetar**	**High-energy HIC**	**Low-energy HIC**
Field profile	(usually) $F = G = 0$	$F < 0, G = 0$	$F < 0, G = 0$	$F > 0, G \neq 0$
strength	subcritical	supercritical	far-supercritical	supercritical
lifetime	super-long	super-long	super-short	long

- Numerical estimation of the EM profile in low-energy HIC @ $\sqrt{s_{\rm NN}}$ = 5.2 GeV





b = 0 fm   3 fm   6 fm

[HT, Nishimura, Ohnishi (2024)] [HT (in prep)]

- So, I'm interested in this and wanna study this further:
  ⇒ Chiral XXX ? Axion electrodynamics? Novel QCD phase? Let's discuss if interested ☺

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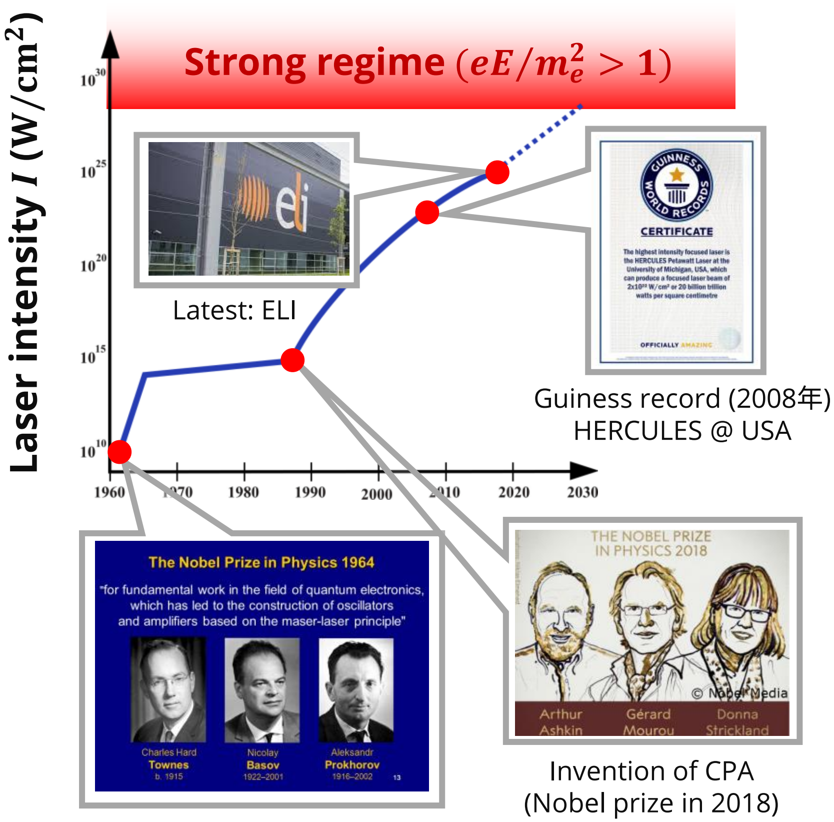

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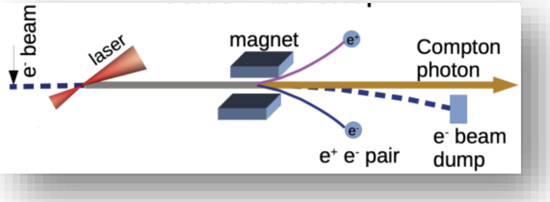

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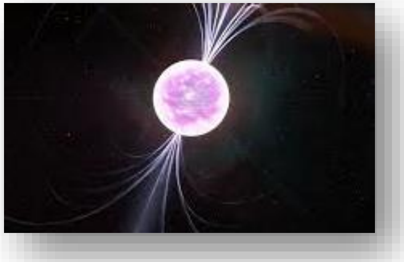


∴ NOW is the BEST time to study strong-field physics

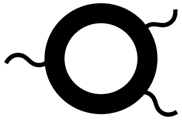
What can happen with strong EM field ?

When $eE > m_e^2$, many non-trivial phenomena have been predicted to occur

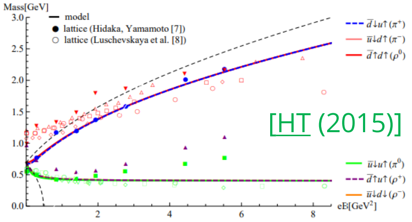
Review: [Fedotov, Ilderton, Karbstein, King, Seipt, HT, Torgrimsson, Phys. Rept. (2023)]

Birefringence of photon in vacuum

Photon splitting

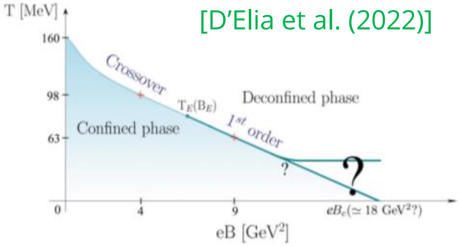


Hadron mass change (for $eB > \Lambda_{\mathrm{QCD}}^2$)



[HT (2015)]

QCD phase diagram (for $eB > \Lambda_{\mathrm{QCD}}^2$)

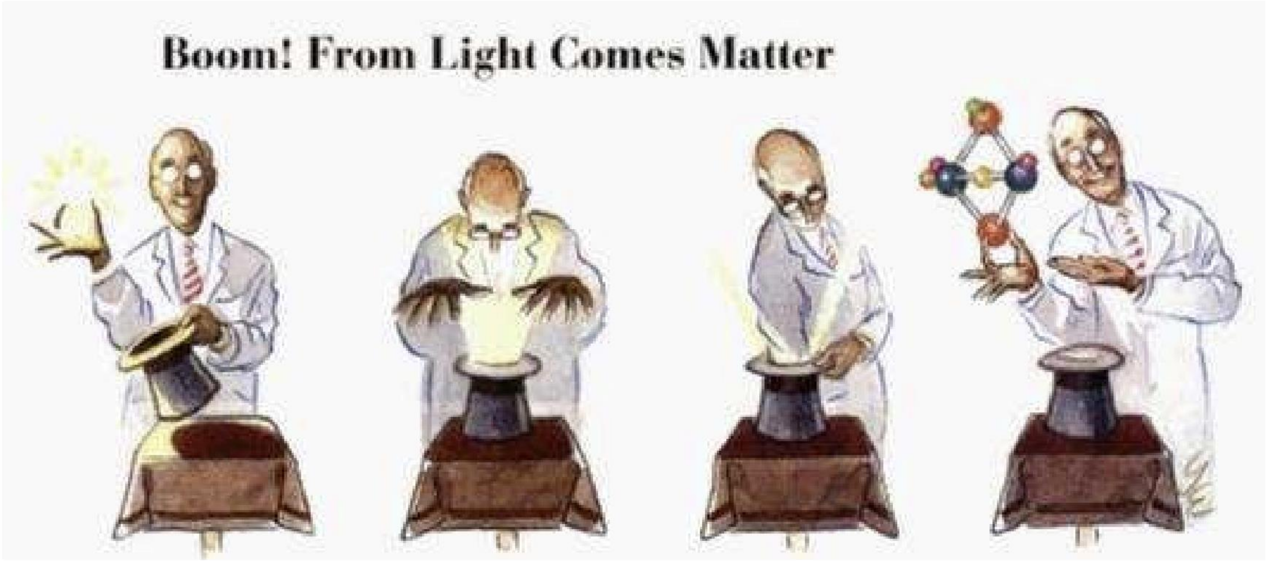


[D’Elia et al. (2022)]

Among others, the (Sauter-)Schwinger effect is the most intriguing

Sauter (1931), Schwinger (1951)

Particles are spontaneously produced from the vacuum (≈ “sthg” from “nothing”)



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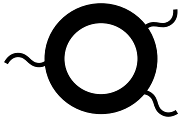

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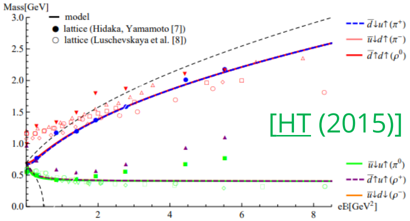

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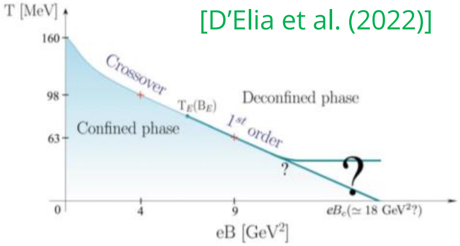

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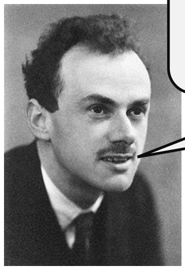
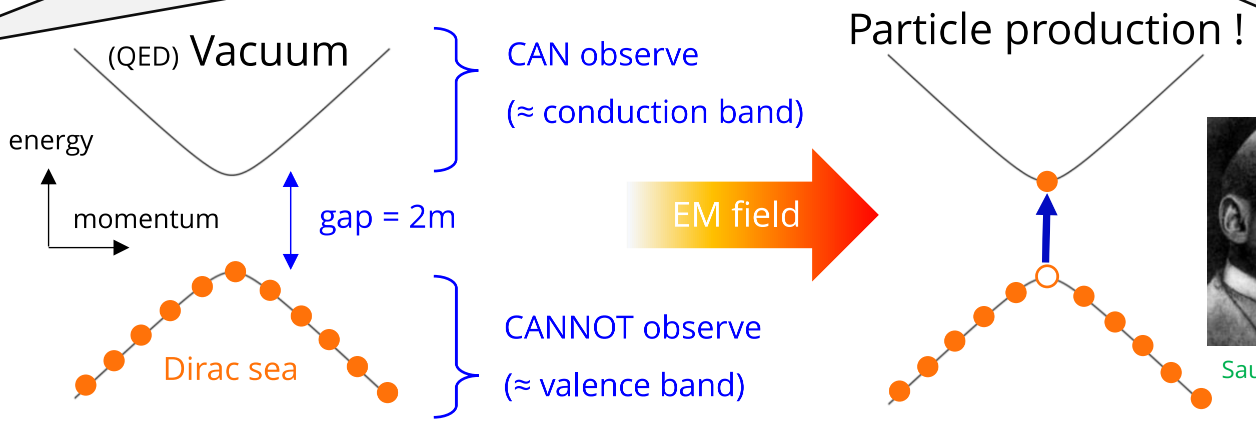
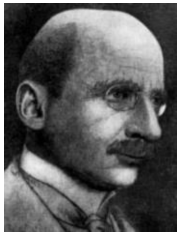


Dirac (1930)

Our vacuum is not an empty space,
but has a structure similar to semi-conductor

Then, real particles will be produced out of the vacuum by strong enough field





Sauter (1931)

Schwinger (1951)

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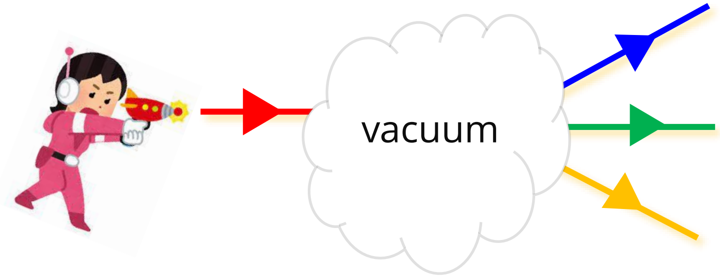

3. Summary

Analogue of the Schwinger effect

Analogous particle production occurs in various physical systems

- ex: Hawking radiation (strong gravity), dynamical Casimir effect (strong oscillation), (p)reheating in the early Universe (strong inflaton field), ...
- **Landau-Zener effect** in materials would be the most prominent Landau (1932), Zener (1932)

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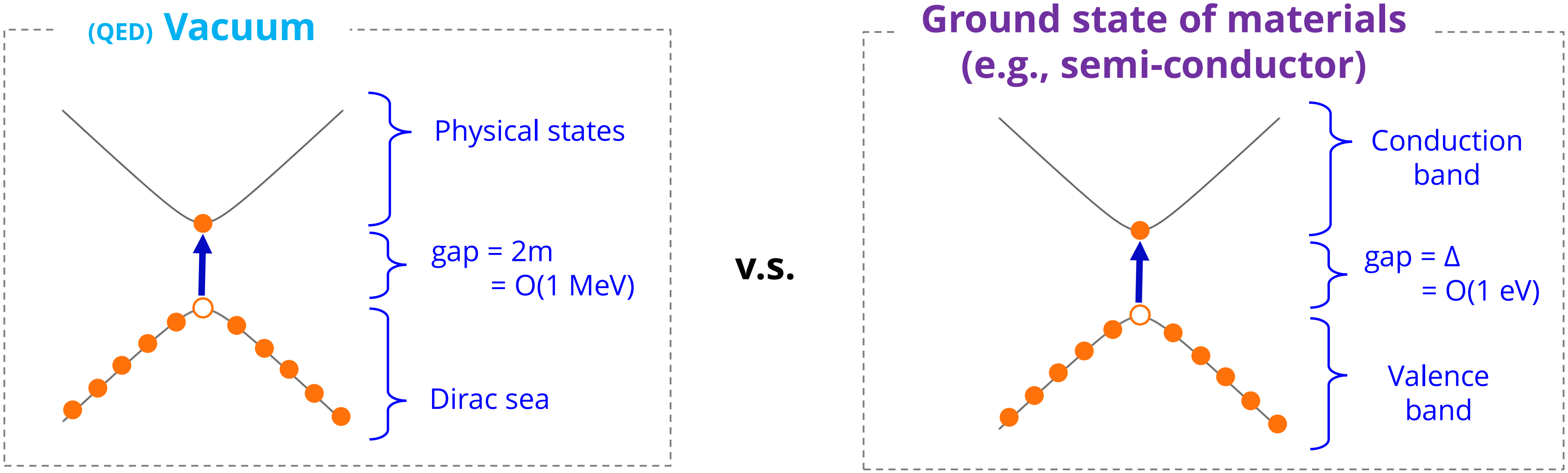

(QED) **Vacuum**

Physical states

gap = 2m
= O(1 MeV)

Dirac sea

v.s.

Ground state of materials
(e.g., semi-conductor)

Conduction band

gap = Δ
= O(1 eV)

Valence band

✔ The responses of the vacuum and ground state by an external EM field should be similar

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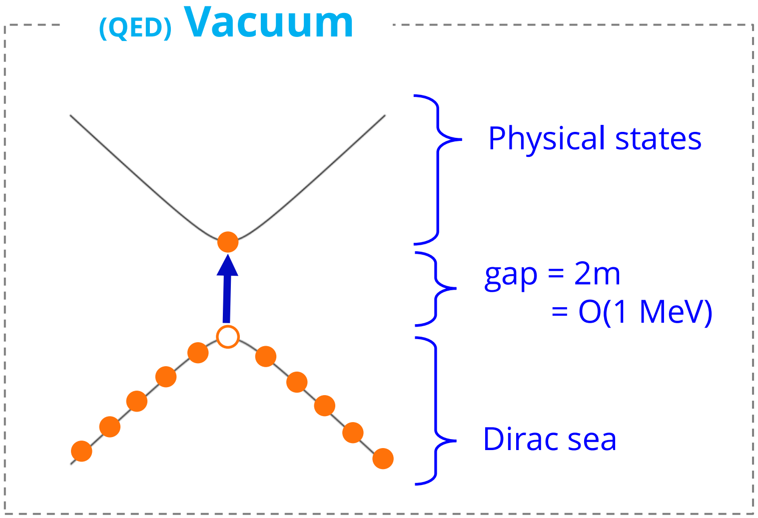

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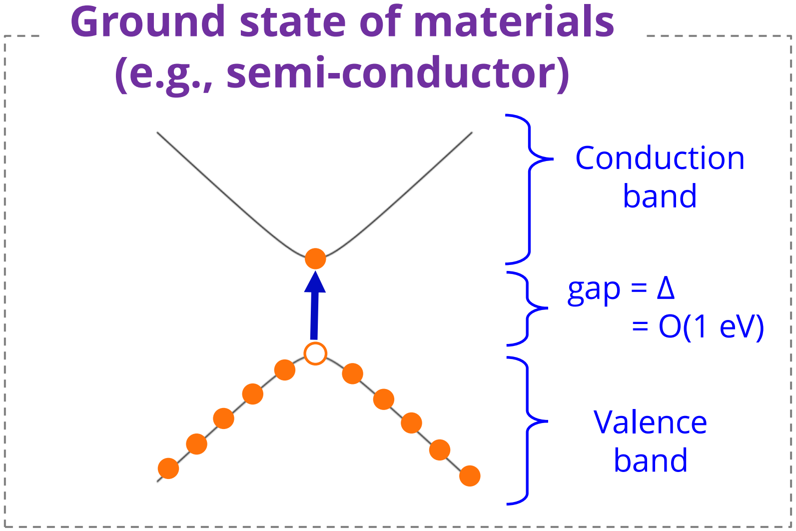


Ground state of materials (e.g., semi-conductor)

Conduction band

gap = Δ
= O(1 eV)

Valence band

✔ The responses of the vacuum and ground state by an external EM field should be similar

✔ Indeed, the particle prediction rate and the excitation rate obey the same exponential formula

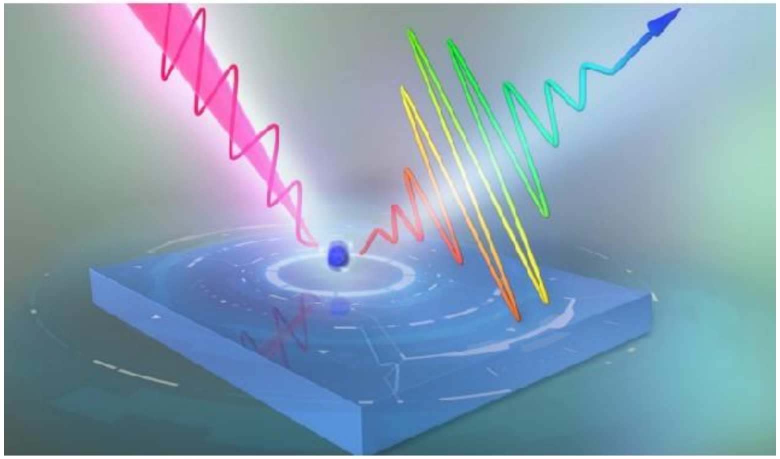
$$\Gamma_{\text{Schwinger}} = \# \exp[\# \, {}^{m^2}/_{eE}] \qquad \Gamma_{\text{Landau-Zener}} = \# \exp[\# \, {}^{\Delta^2}/_{eE}]$$

✔ Schwinger effect has not been verified yet

BUT, Landau-Zener transition has been observed and utilized, e.g., to design devices

⇒ Importing ideas from other areas of physics is quite to better understand the Schwinger effect (or strong-field physics in general)

As such idea: High-harmonic generation from the vacuum



- **HHG developed in optics & solid-state physics**
  - Observed with various materials: gas (1988), liquid (2014), solid (2011), ...
  - Many applications: laser pointer, high-speed switching, ...
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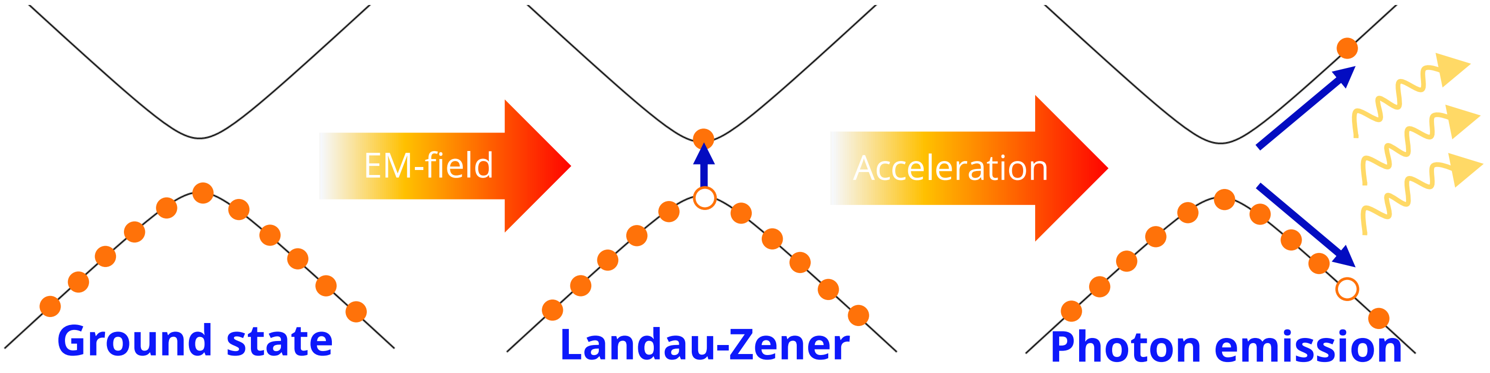


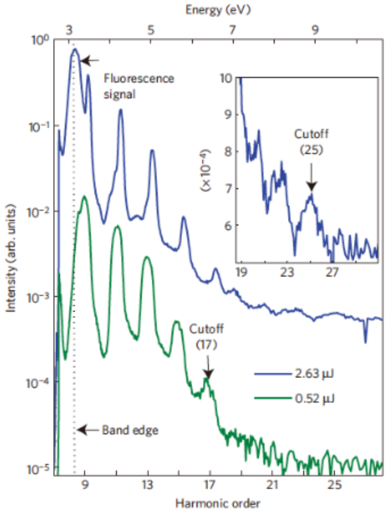
EM-field

Acceleration

Ground state

Landau-Zener

Photon emission



[Ghimire et al., (2011)]

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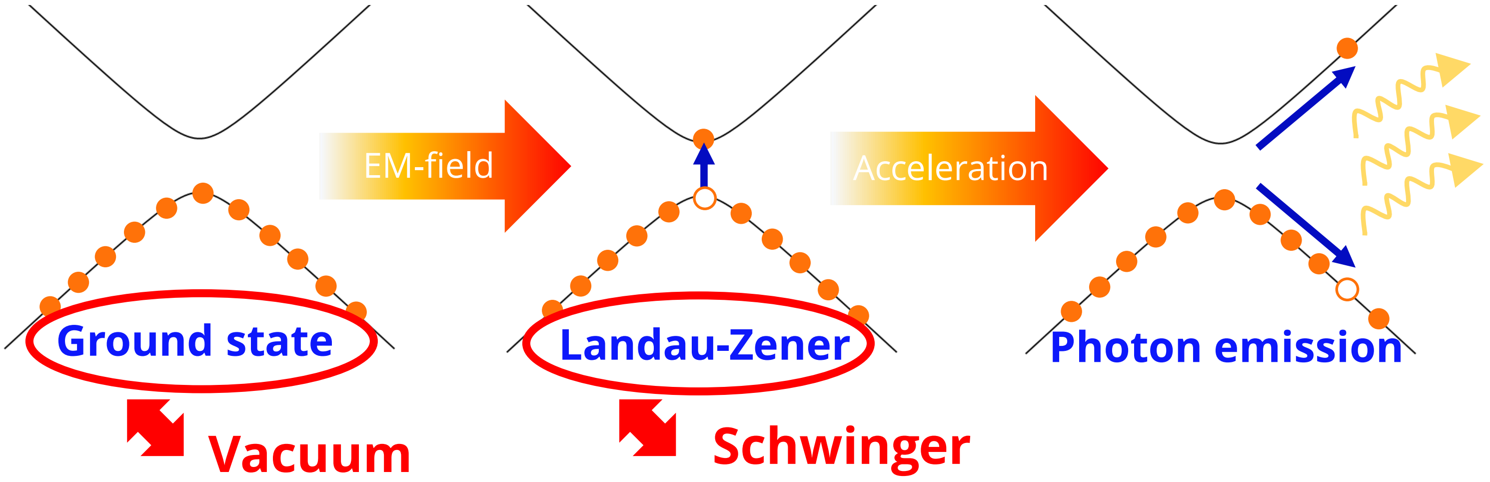

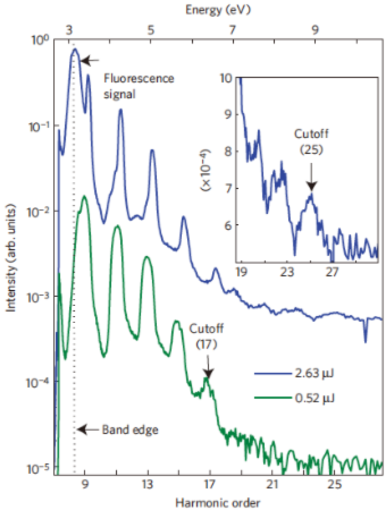

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Analogy b/w Schwinger & Landau-Zener ⇒ natural to expect HHG from the vacuum

How to formulate vacuum HHG ?

Observable: Photon spectrum ⇒ solving of a differential equation

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Maxwell eq. ⇒ Photon spectrum is determined by current

$$J^\nu = \partial_\mu F^{\mu\nu} \sim \partial^2 A^\nu \quad \Rightarrow \quad \omega \frac{\mathrm{d}N_{\mathrm{photon}}}{\mathrm{d}\omega} = |\omega \times \mathrm{F.T.}[J]|^2$$

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determined by the Schrodinger eq. (the Dirac eq.) <u>under the strong field</u>

$$i\hbar\partial_t \hat{\psi} = H\hat{\psi} = \left(H_{\mathrm{free}} + V_{\mathrm{strong\ field}}\right)\hat{\psi} \qquad (♠)$$

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∴ We can take two strategies to discuss vacuum HHG:

(1) Numerically solve Eq. (♠) **(sometimes called TDSE method)**

(2) Analytically solve Eq. (♠) by using some “wise” mathematical method

⇒ In this work: Exact WKB method

Exact WKB method

Exact WKB = **a wise method to solve ODE with a small parameter**

[Voros (1983)]
[Pham, Dillinger, Delabaere,
Aoki, Koike, Takei, ...]

= **"usual" WKB** + **Borel resum.**

[Jeffery (1924)] [Wentzel (1926)]
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[HT, Fujimori, Misumi, Nitta, Sakai (2021)]

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A perturbation theory w.r.t. $\hbar$ (or adiabatic approx.)

- Consider $0 = [\hbar^2\partial_t^2 + Q(t)]\phi(t) \overset{t\equiv\hbar\tau}{\Leftrightarrow} [\partial_\tau^2 + Q(\hbar\tau)]\phi(\tau)$

$$\Rightarrow \phi_\pm(t;\hbar) := \boxed{\exp\left[\mp\frac{\mathrm{i}}{\hbar}\int_{t_0}^{t} \mathrm{d}t'\sqrt{Q(t')}\right]} \times \boxed{\sum_{n=0}^{\infty}\psi_{\pm,n}(t)\hbar^n}$$

0th order = plane wave
$\sim \exp[\mp\frac{\mathrm{i}}{\hbar}\sqrt{Q}t]$

Perturbation w.r.t. $\hbar$

- WKB expansion makes sense if the perturbative part is convergent
- However, $\psi_{\pm,n} \sim n!$ in general (e.g., Airy function $Q(t) \propto t$)

⇒ WKB expansion has zero radius of convergence ⇒ ill-defined !!!

[HT, Fujimori, Misumi, Nitta, Sakai (2021)]

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A resummation scheme for factorially divergent ~n! series

- Consider the div. part of WKB expansion $\psi_\pm(t;\hbar) \coloneqq \sum_{n=0}^{\infty} \psi_{\pm,n}(t)\hbar^n$

  ① Construct "Borel transformation": $B[\psi_\pm](t;\eta) \coloneqq \sum_n^\infty \frac{\psi_{\pm,n}(t)}{n!}\eta^n$

  ② Laplace trans. gives "Borel sum": $\Psi_\pm(t;\hbar) \coloneqq \int_0^\infty \frac{\mathrm{d}\eta}{\hbar}\, \mathrm{e}^{-\eta/\hbar} B[\psi_\pm](t;\eta)$

- $\Psi_\pm$ is well-defined and is a natural analytic continuation of $\psi_\pm$

  ⇒ **$\Psi_\pm$ gives a well-defined version of the WKB solution !**

- Note 1: If $B[\psi_\pm]$ has singularities on the real axis, $\Psi_\pm$ is ill-defined
  ⇒ but, the "ill-defined-ness" can be used to quantify
  the Stokes phenomenon of the WKB solution

  Note 2: in practice, some approximations need to be used in Borel resum
  ⇒ in this work: only leading-order n! div. is considered (Airy approx.)

[HT, Fujimori, Misumi, Nitta, Sakai (2021)]

Result (1/2)

Setup: A monochromatic E-field $E(t) = E_0 \cos(\Omega t)$ applied onto QED vacuum

What I did: Compute harmonic spectrum **numerically** and **analytically with WKB**

Result (1/2): Numerical

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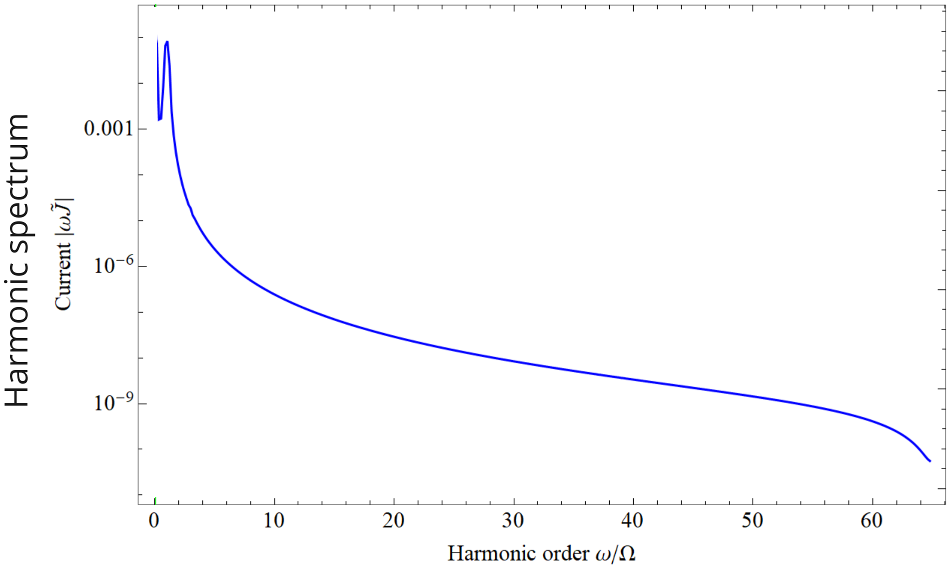
Weak field: $\frac{m\Omega}{eE_0} = 100$



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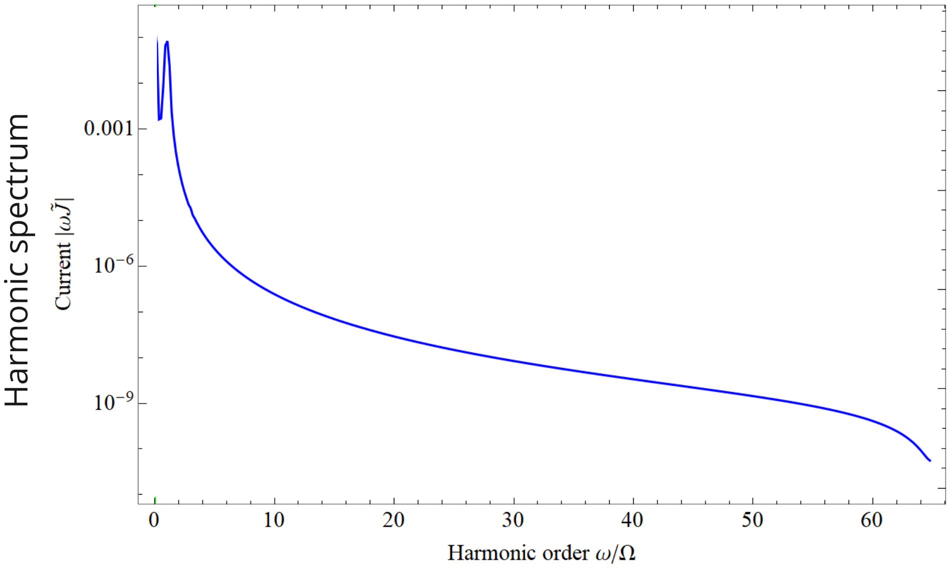
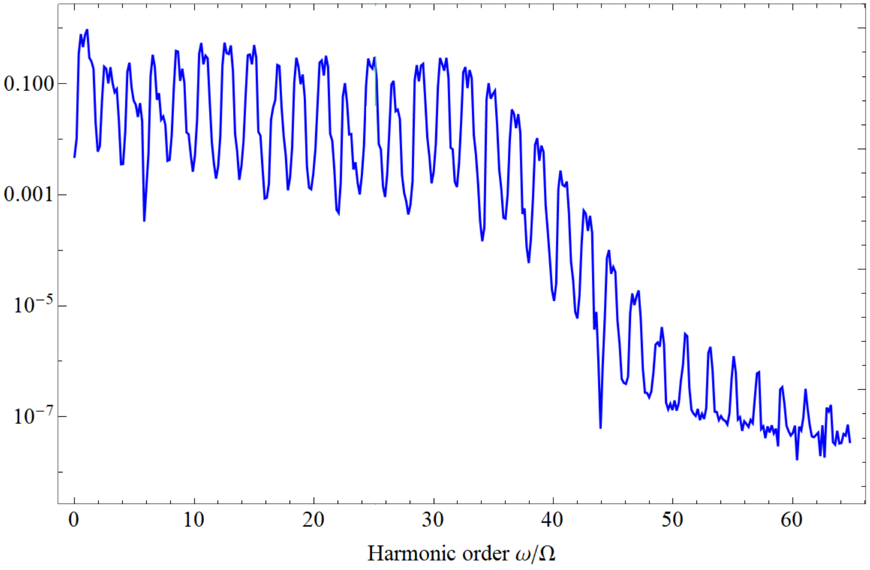


Strong field: $\frac{m\Omega}{eE_0} = 1/4$



- High harmonics appears from the vacuum when field becomes strong !
  - structure similar to solid HHG (e.g., plateau, cutoff)

Result (2/2): Analytical

Setup: A monochromatic E-field $E(t) = E_0 \cos(\Omega t)$ applied onto QED vacuum

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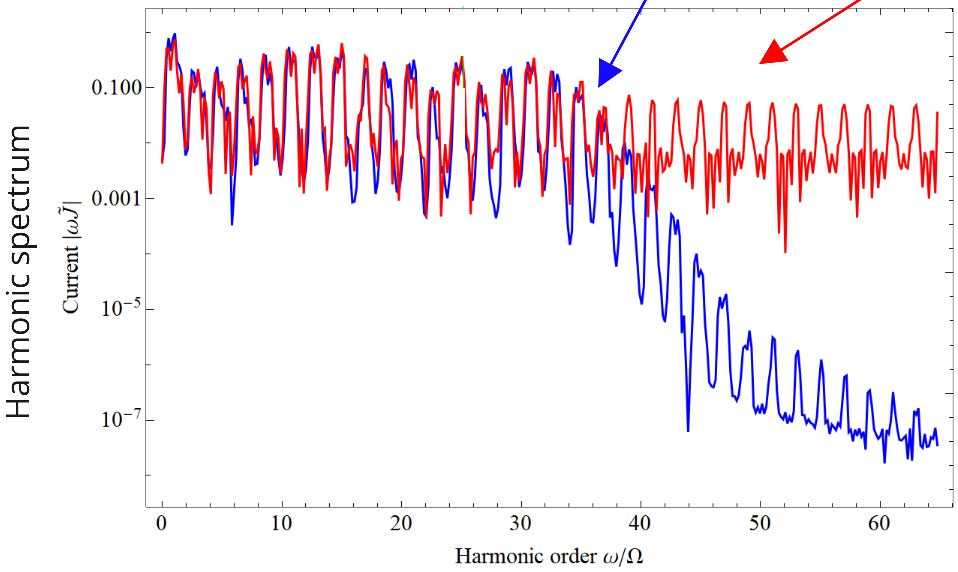
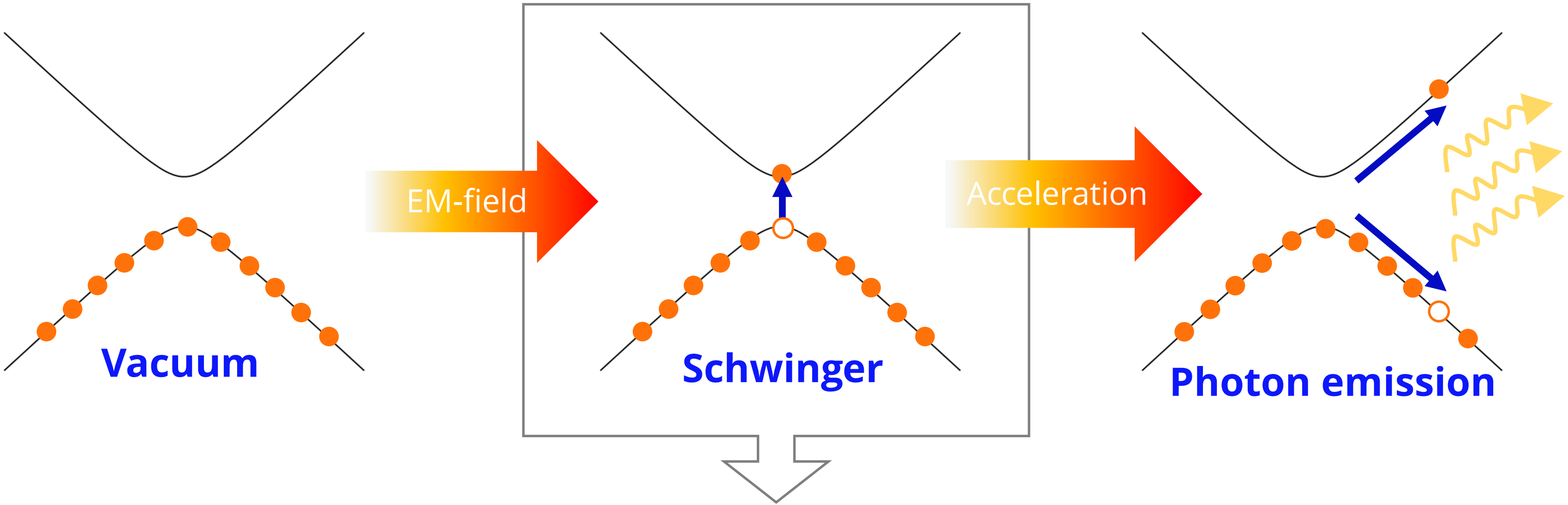


- High harmonics appears from the vacuum when field becomes strong !
  - structure similar to solid HHG (e.g., plateau, cutoff)

- Analytical WKB calculation is good at the plateau but bad after the cutoff

  ⇒ useful to get a deeper insight of HHG in the agreement region
  e.g., determination of the cutoff location (→ next slide)

Physics picture: Interband interference



- Total wavefunction = superposition of the two energy states:

$$\psi = \psi_+ + \psi_- = \#\,\mathrm{e}^{-\mathrm{i}\int \epsilon\,\mathrm{d}t} + \#\,\mathrm{e}^{+\mathrm{i}\int \epsilon\,\mathrm{d}t} \quad \text{w/} \quad \epsilon \sim \sqrt{m^2 + \boldsymbol{p}^2} \sim \sqrt{m^2 + (e\overset{\frac{eE_0}{\Omega}\cos(\Omega t)}{\overset{/\!/}{A}}(t))^2}$$

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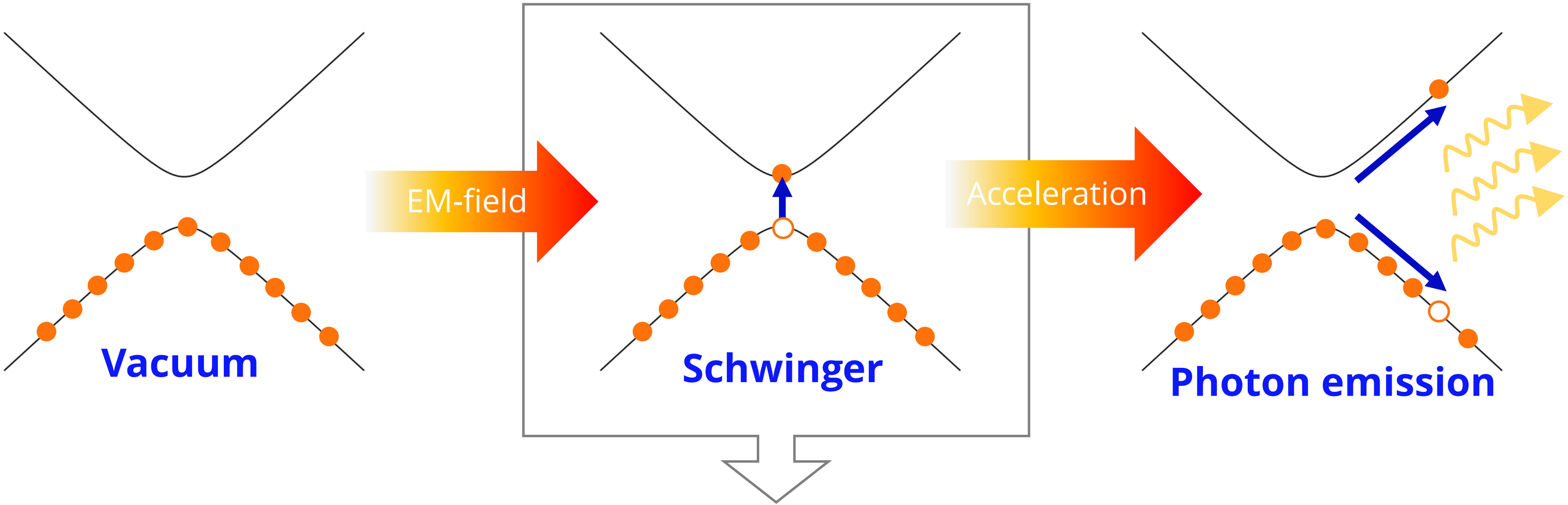

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⇒ current can be decomposed into intra- and inter-band contributions

$$J \sim \bar{\psi}\gamma\psi \sim \#\,\bar{\psi}_\pm\psi_\pm + \#\,\bar{\psi}_\pm\psi_\mp \sim \#\,\mathrm{e}^{i\int(\epsilon-\epsilon)\mathrm{d}t} + \#\,e^{i\int(\epsilon+\epsilon)\mathrm{d}t} \sim \# + \#\,\mathrm{e}^{2\mathrm{i}\int \epsilon\,\mathrm{d}t}$$

⇒ the intra-band interference has higher frequency ⇒ source of HHG

- is different from the naïve 3 step model in gas HHG, but is consistent w/ solid HHG

[Vampa et al. (2014)]

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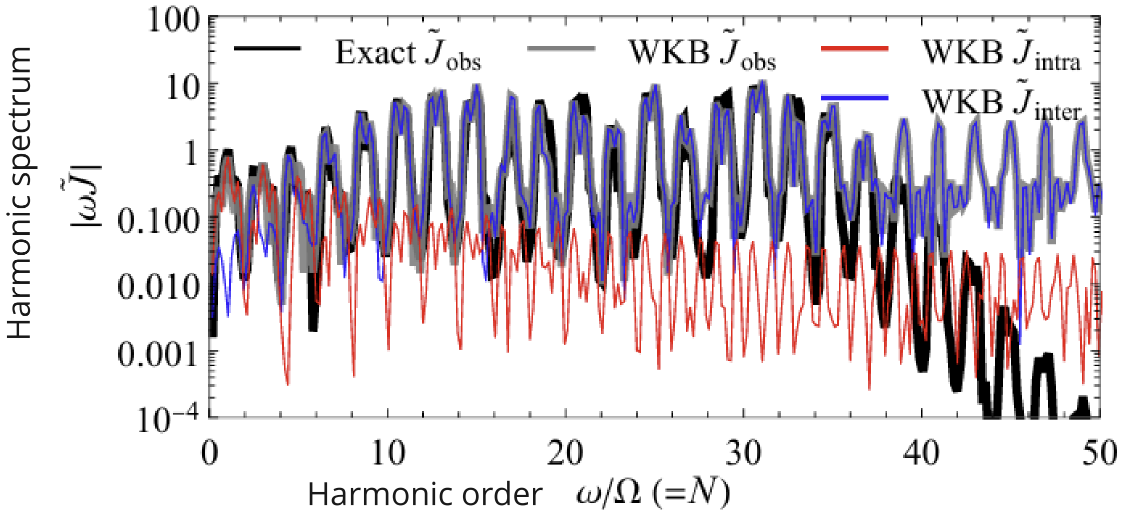

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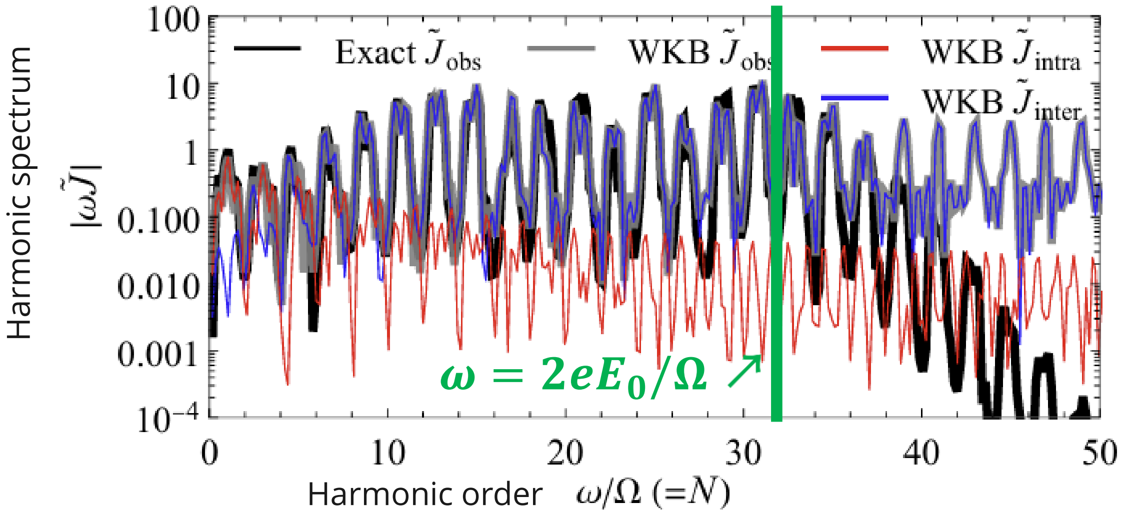

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[Vampa et al. (2014)]

- Cutoff law: $\mathrm{F.T.}[J_{\rm inter}] \sim \int \mathrm{d}t \exp\left[-\mathrm{i}\int \mathrm{d}t\,(\omega - 2\epsilon)\right] \Rightarrow \omega_{\rm cutoff} \sim \max_t 2\epsilon(t) \sim 2eE_0/\Omega$

- $\omega_{\rm cutoff} \propto E_0$ is consistent w/ solid HHG, while $\omega_{\rm cutoff} \propto \Omega^{-1}$ is our prediction

[Ghimire et al., (2011)]

Contents

1. Overview of strong-field physics

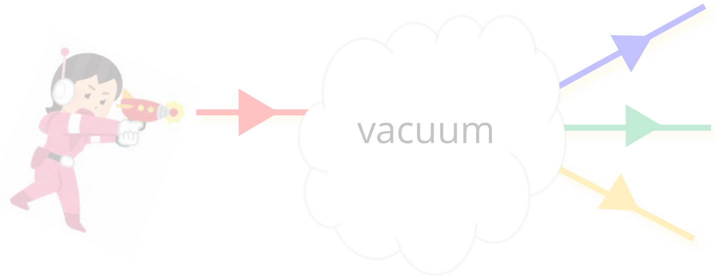
- becoming a hot topic in physics, thanks to the recent developments in exp.
- the Schwinger effect = a process to create "something" from "nothing"

2. Application of the Schwinger effect ⇒ High-harmonic generation from the vacuum

[HT, Hongo, Ikeda (2021)]

- first prediction of HHG from the vacuum
- a good example of "interdisciplinarity" of strong-field physics

  ⇒ your expertise should be useful in my area and vice versa !



3. Summary

Summary

Q: What happens if we make light (or "field" in general) stronger and stronger ?

LED

$I \sim 10^{-5}\ \mathrm{W/cm^2}$

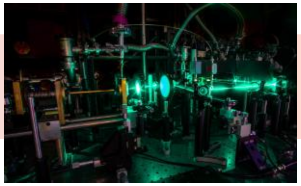
Magnifying glass

$10^{2}\ \mathrm{W/cm^2}$

Laser welding

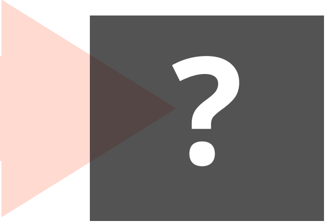
$10^{6}\ \mathrm{W/cm^2}$

THz Laser
in solid-state phys.



$10^{10}\ \mathrm{W/cm^2}$

???



$> 10^{28}\ \mathrm{W/cm^2}$

A: Our vacuum shall be destroyed and gives us nontrivial responses

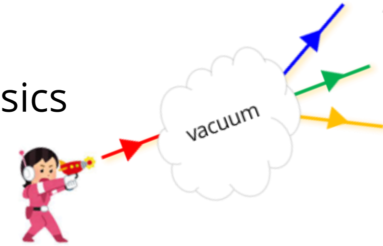
- Why interesting: Unexplored nonpert. regime ⇒ many nontrivial phenomena we've never seen
- Why timely: Recent availability of strong fields

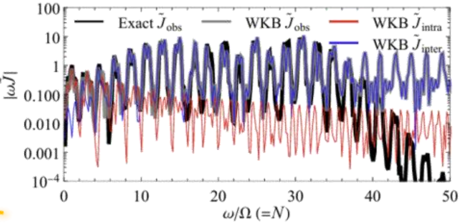
  e.g., high-power laser, magnetar, heavy-ion colls

Example phenomenon: HHG from the vacuum

- First prediction of HHG from the vauum
- An example of "interdisciplinarity" strong-field physics

  ⇐ borrowed ideas from
  optics, solid-state physics, & math





[**HT**, Hongo, Ikeda (2021)]

Exact-WKB recipe for Stokes phenomenon

Step 1: Draw a Stokes graph

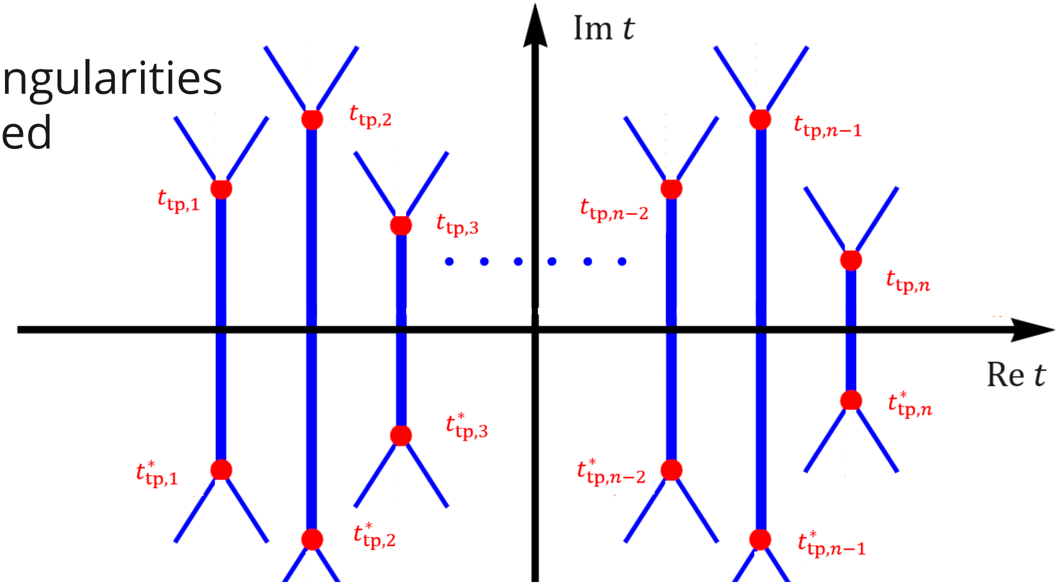
- Laplace trans. of $B[\psi_\pm](t;\eta)$ hits singularities (non-Borel summable) when located on Stokes lines in the t-plane

— : Stokes lines

$$\{t \in \mathbb{C} \mid 0 = \mathrm{Im}\, \mathrm{i} \int_{t_{\mathrm{tp}}}^{t} \mathrm{d}z\, \sqrt{Q(z)}\,\}$$

● : turning points

$$Q(t_{\mathrm{tp}}) = 0$$



Step 2: Compute Borel sum $\Psi_\pm$ at each Stokes region

- Borel sum is well-defined and computable in each region separated by Stokes lines

$$\Psi_\pm = \int_0^\infty \frac{\mathrm{d}\eta}{\hbar}\, \mathrm{e}^{-\eta/\hbar} B[\psi_\pm](t;\eta) \sim \exp\left[\mp \frac{\mathrm{i}}{\hbar} \int_{t_0}^{t} \mathrm{d}t' \sqrt{Q(t')}\right] \times (1 + O(\hbar))$$ at each Stokes region

Step 3: Compute the Stokes constants α and β

- Whenever crosses Stokes lines, $\Psi_\pm$ jumps discontinuously (**Stokes phenomenon**)

⇒ The discontinuity is given by the integral of singularities of $B[\psi_\pm]$

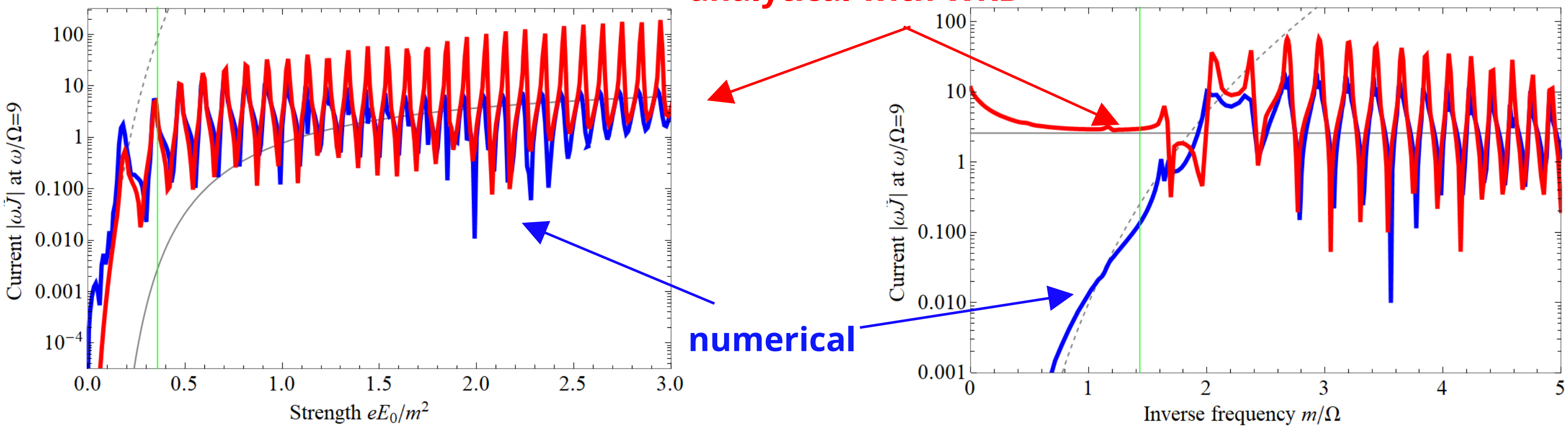
$$\Psi_+(\text{region } A) = \alpha\, \Psi_+(\text{region } B) + \beta\, \Psi_-(\text{region } B)$$

$$\sim \oint_{\substack{\text{sing.on} \\ \text{a Stokes line}}} \frac{\mathrm{d}\eta}{\hbar}\, \mathrm{e}^{-\eta/\hbar} B[\psi_\pm](t;\eta)$$

Application to high-harmonic generation (2/2)

- **WKB works more in the deep non-perturbative regime $E_0 \to \text{large}, \Omega \to \text{small}$**
- **Demonstration: magnitude of the harmonic peak at $\omega/\Omega = 9$**

Dependence on strength E_0 Dependence on frequency Ω



analytical with WKB

numerical

- **Lessons:**

(1) WKB makes it easier to analyze the non-perturbative regime

(2) Saturation & oscillation of the harmonic intensity

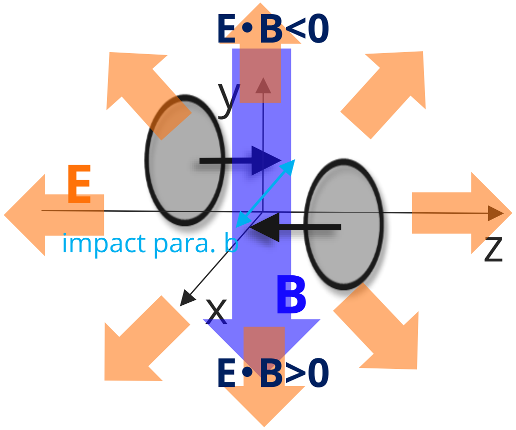
$\Rightarrow$ consistent with recent semi-conductor exp. [Xia et al., (2020)]

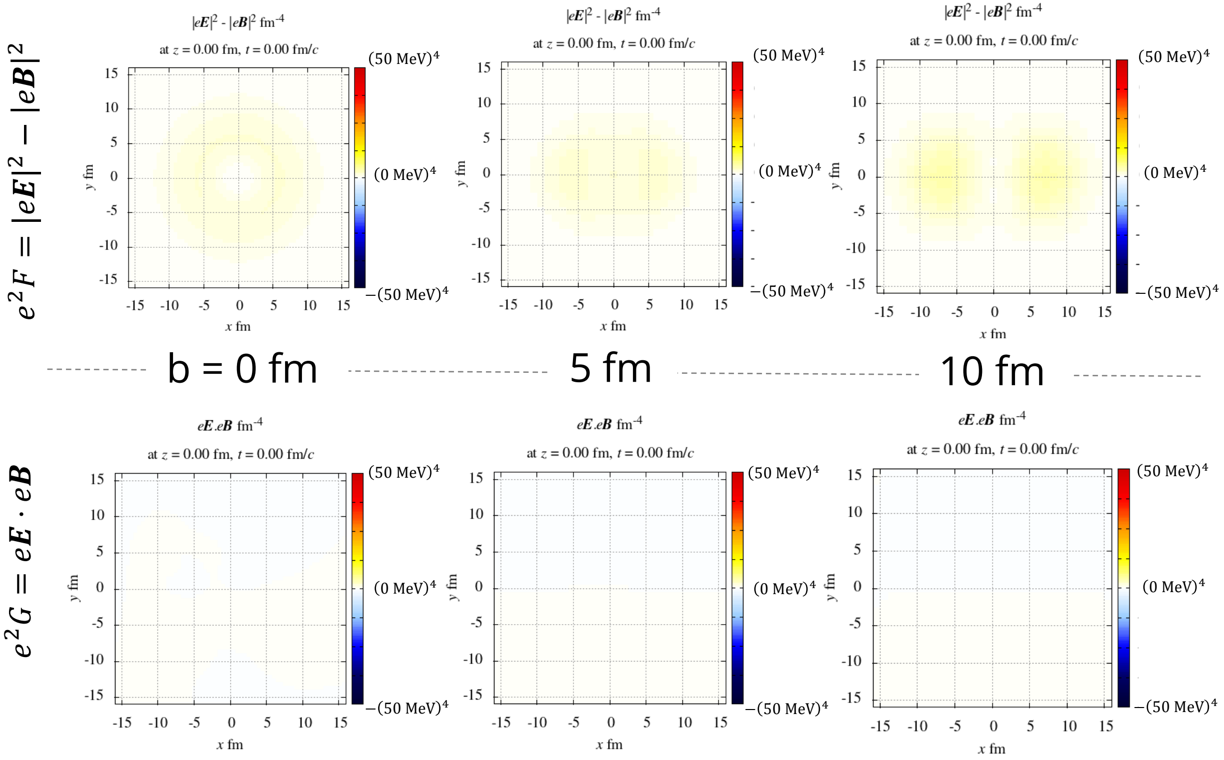
(but only E_0-dep. is measured and Ω-dep. is our prediction)

Peripheral case

[Nishimura (Osaka), Ohnishi (Kyoto), HT, in progress]

✔ Non-central collision ⇒ B & E•B are produced





Preliminary results

@ E_{CM} = 7.7 GeV

- $e\boldsymbol{E}\cdot e\boldsymbol{B} = O\left((50\ \mathrm{MeV})^4\right)$ ⇒ Non-negligible for QCD (Enough strong for electron and current quarks)
- Perhaps, a nice place to study chiral-anomaly-related stuffs

ex) chirality production via the Schwinger mechanism $N_5 = VT\dfrac{e\boldsymbol{E}\cdot e\boldsymbol{B}}{2\pi^2}\exp\left[-\pi\dfrac{m^2}{eE}\right]$