Strong-field physics in heavy-ion collisions

Hidetoshi Taya

(Keio University)

Today's talk

Q: What happens if we make light **(or "field" in general)** **stronger and stronger ?**

LED

$I \sim 10^{-5}\ \mathrm{W/cm^2}$

or $eE \sim 10^{-1}\ \mathrm{eV/cm}$

$\sim (10^{-3}\mathrm{eV})^2$

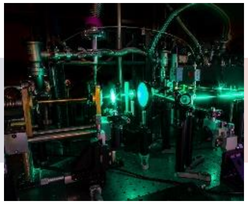
Laser welding

$10^{6}\ \mathrm{W/cm^2}$

$10^{4}\ \mathrm{eV/cm}$

$(10^{-1}\mathrm{eV})^2$

Cond.-mat (THz laser)



$10^{10}\ \mathrm{W/cm^2}$

$10^{6}\ \mathrm{eV/cm}$

$(1\ \mathrm{eV})^2$

Guiness record

HERCULES @ USA (2008)

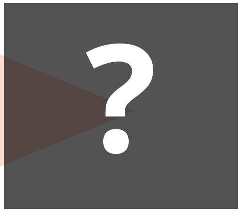
$10^{22}\ \mathrm{W/cm^2}$

$10^{12}\ \mathrm{eV/cm}$

$(1\ \mathrm{keV})^2$

Our limit at the present

???



Daily life

Industry

Science

Today's talk

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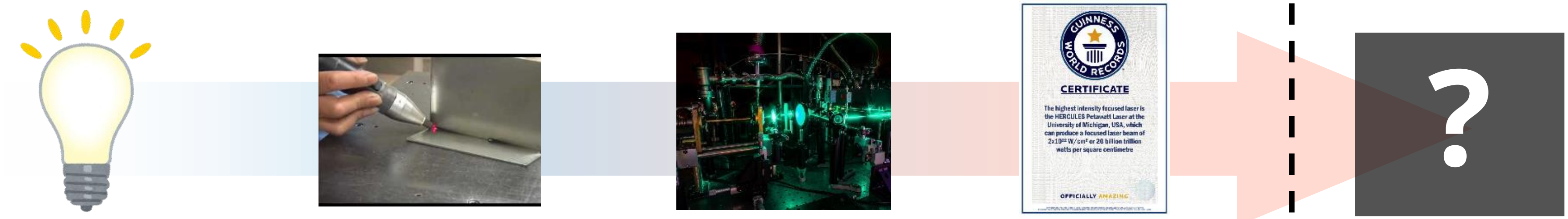
LED	Laser welding	Cond.-mat (THz laser)	Guiness record	???



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LED	Laser welding	Cond.-mat (THz laser)	Guiness record	
$I \sim 10^{-5}\ \mathrm{W/cm^2}$	$10^{6}\ \mathrm{W/cm^2}$	$10^{10}\ \mathrm{W/cm^2}$	$10^{22}\ \mathrm{W/cm^2}$	**Our limit at the present**
or $eE \sim 10^{-1}\ \mathrm{eV/cm}$	$10^{4}\ \mathrm{eV/cm}$	$10^{6}\ \mathrm{eV/cm}$	$10^{12}\ \mathrm{eV/cm}$	
$\sim (10^{-3}\mathrm{eV})^2$	$(10^{-1}\mathrm{eV})^2$	$(1\ \mathrm{eV})^2$	$(1\ \mathrm{keV})^2$	

Daily life → Industry → Science →

Purpose: Review physics of such strong field

Take-home messages:

(1) Once $eE >$ (typical energy scale), sthg extremely non-trivial occur (e.g., Schwinger effect ≈ "something" from "nothing")

(2) Such strong fields are now (or soon will be) within the exp. reach

(3) Of relevance to hadron/QCD physics, in particular, heavy-ion collisions

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- focus on the Schwinger effect with time-dependent E fields

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- Quark production is very fast ! [HT, 1609.06189] [HT, Ph. D thesis (2017)]

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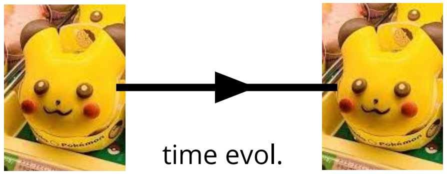
What if field becomes strong ?

initial final



time evol.

No field

What if field becomes strong ?

initial

time evol.

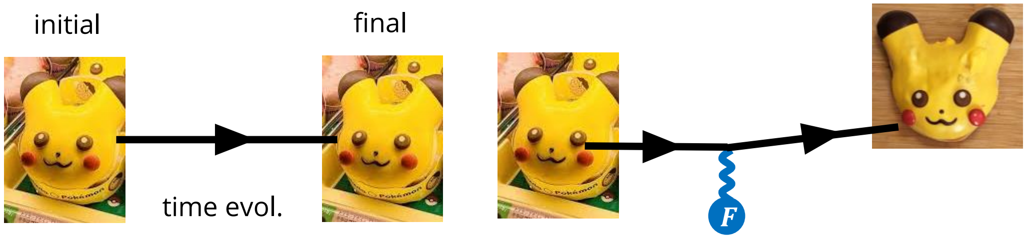
final

No field

Weak field ($eE, eB/m^2 \ll 1$)

Strong field ($eE, eB/m^2 \gg 1$)

What if field becomes strong ?



No field | **Weak field ($eE, eB/m^2 \ll 1$)** | **Strong field ($eE, eB/m^2 \gg 1$)**

Only minor changes

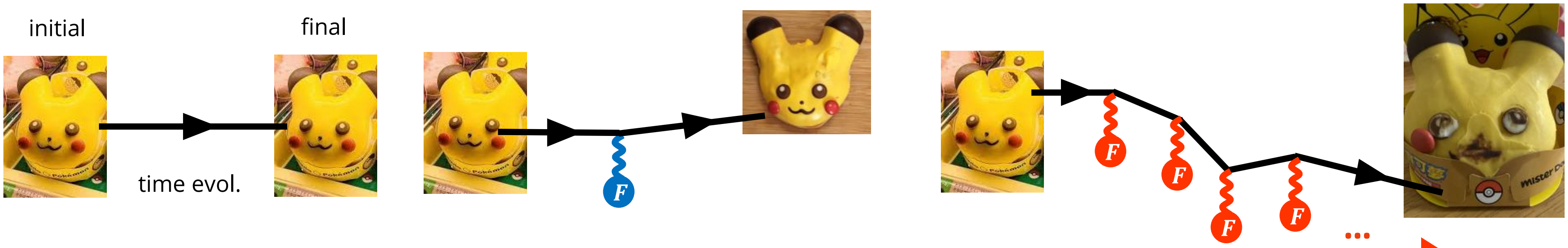
⇒ Perturbative

⇒ Very well understood
in both exp.& theor.

ex.) Electron (anomalous) magnetic moment $a := \frac{g-2}{2}$
≈ Electron energy shift in a weak magnetic field

$a(\text{theor.}) = 1159652182.03 \ldots \times 10^{-12}$
$a(\text{exp.}) = 1159652180.73 \ldots \times 10^{-12}$ [Aoyama, Kinoshita, Nio (2017)]

What if field becomes strong ?



No field → **Weak field ($eE, eB/m^2 \ll 1$)** → **Strong field ($eE, eB/m^2 \gg 1$)**

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Big change !

⇒ Non-Perturbative

⇒ Not understood well

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[Aoyama, Kinoshita, Nio (2017)]

If field becomes strong, physics becomes totally different & nontrivial

Novel processes with strong fields

✔ **In QED (for $eE, eB \,/\, m_e^2 \gg 1$)**

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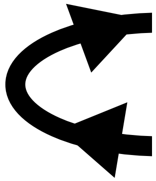

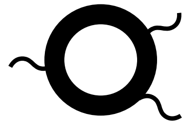

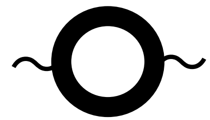

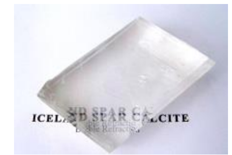

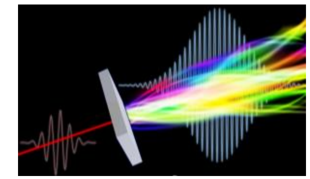

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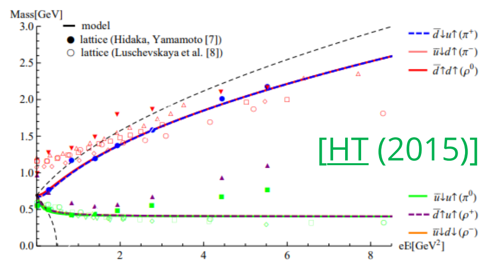

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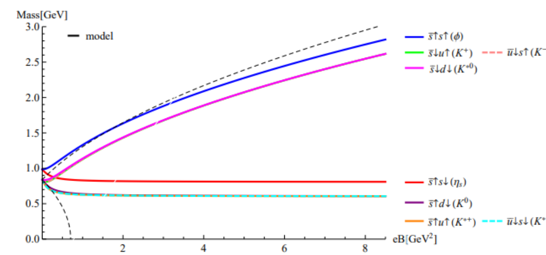

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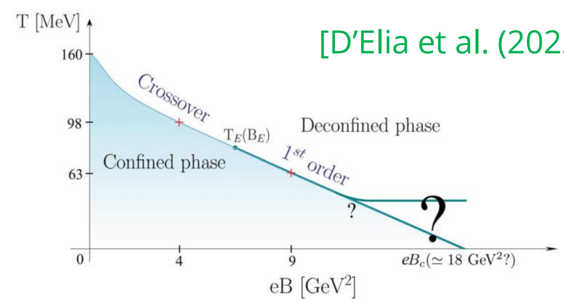
⇒ (inverse-)catalysis of chiral condensate, order of phase trans., novel phase, ...

[D'Elia et al. (2022)]



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⇒ Anomalous transports, spin polarization, early-stage dynamics of HIC (QGP formation), ...

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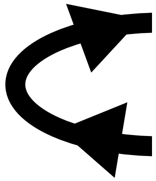

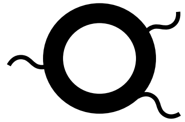

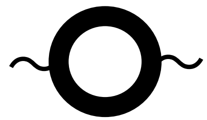

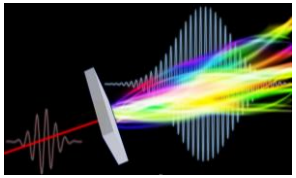

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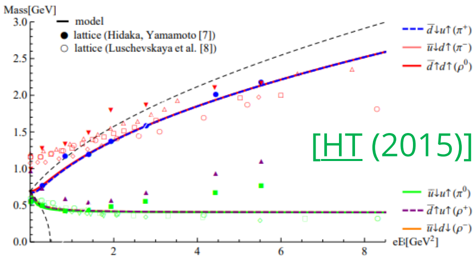

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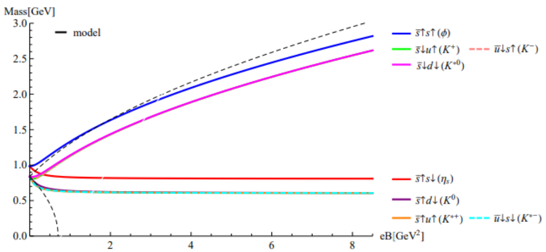

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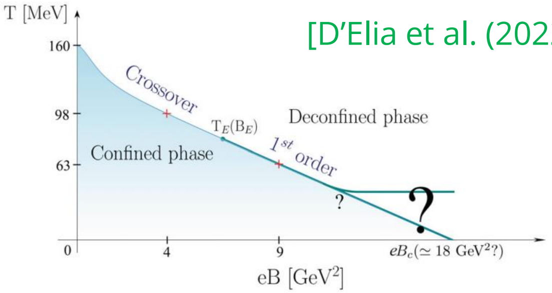

✔ **In other systems:** Gravity, Curved spacetime, Inflaton, Cond-mat analog, …

However, extremely strong fields needed

⇒ Experimentally impossible in the 20th century

$$eE_{\rm Guiness} \approx (1\ {\rm keV})^2 \ll m_e^2, \Lambda_{\rm QCD}^2$$

Recent availability of strong EM fields

The situation changing: becoming able to create/observe strong light

∴ NOW is the BEST time to study strong-field physics

Recent availability of strong EM fields

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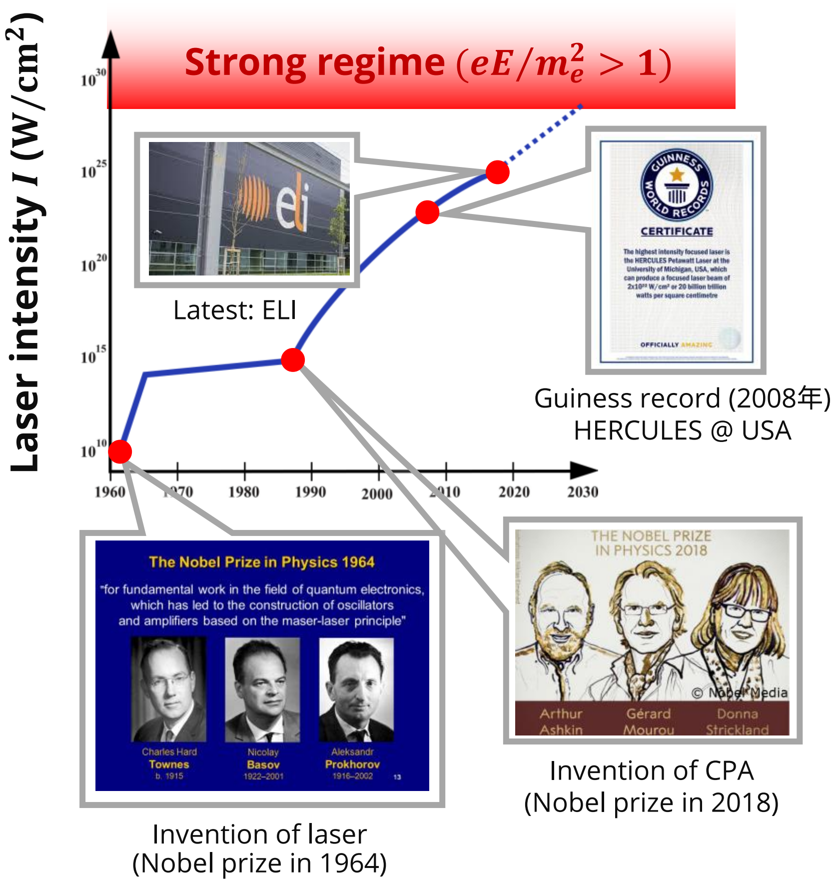
High-power laser



Extreme physical systems

- **Heavy-ion collisions**

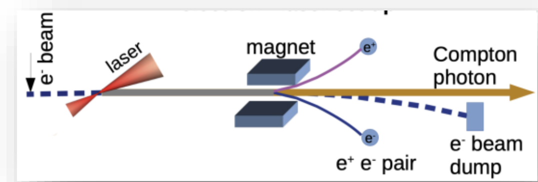
  RIC (2000~), LHC (2012~),
  FAIR/NICA/HIAF/J-Parc-HI/... (20XX~)

  $I \sim 10^{35}\ \mathrm{W/cm^2}$
  $(eE, eB \sim (100\ \mathrm{MeV} - 1\ \mathrm{GeV})^2)$

- **Electron collider + Laser**

  Start soon: LUXE @ DESY, FACET-II @ SLAC

  $I \sim 10^{29}\ \mathrm{W/cm^2}$
  $(eE, eB > m_e^2 \sim (1\ \mathrm{MeV})^2)$



- **Magnetar**

  Suzaku (2005~2015), NICER (2017~)
  XL-Calibur (2018~), IXPE (2021~), ...

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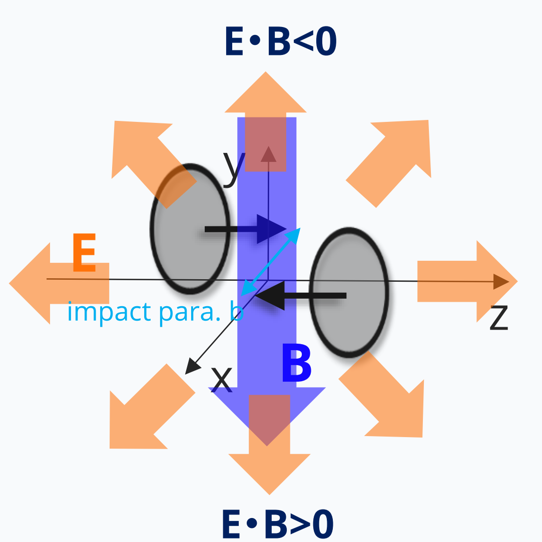
A bit more on heavy-ion collisions

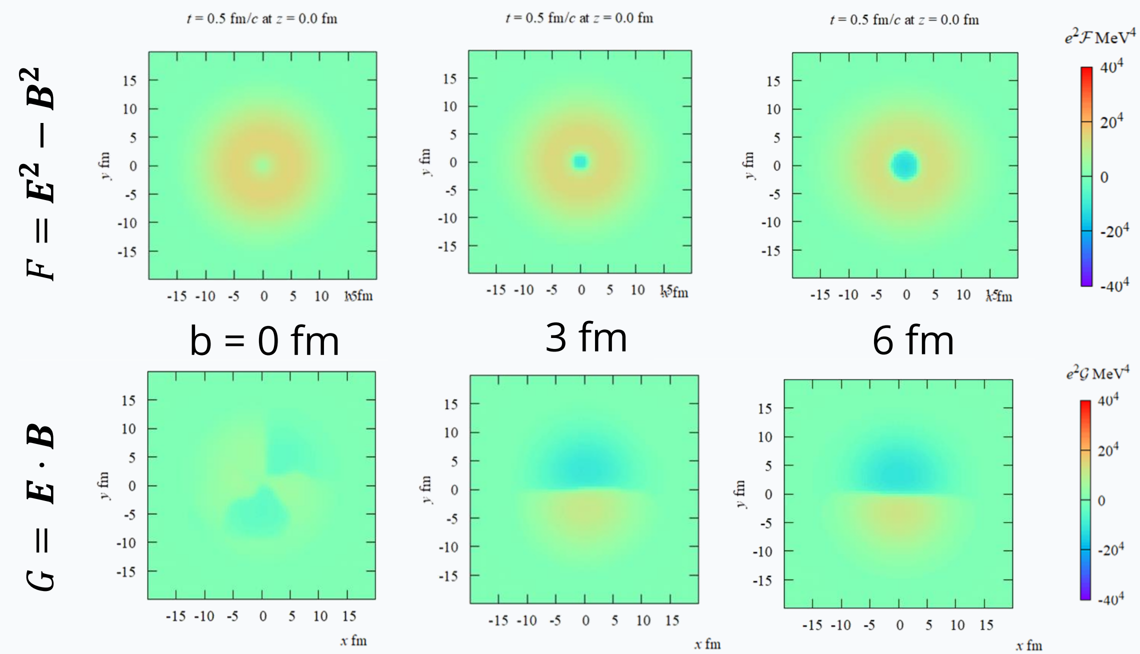
Low-energy ($\sqrt{s_{\mathrm{NN}}} = 2 - 10$) HIC is interesting among other strong-field systems

- Sales points: (1) the only system that has supercritical $F := \boldsymbol{E}^2 - \boldsymbol{B}^2 > 0,\ G := \boldsymbol{E}\cdot\boldsymbol{B} \neq 0$
  (2) sufficiently long lived ⇒ will discuss later

	high-power laser	magnetar	High-energy HIC	Low-energy HIC
Field profile	(usually) $F = G = 0$	$F < 0, G = 0$	$F < 0, G = 0$	$F > 0, G \neq 0$
strength	subcritical	supercritical	far-supercritical	supercritical
lifetime	super-long	super-long	super-short	long

- Numerical estimation of the EM profile in low-energy HIC (with JAM) @ $\sqrt{s_{\mathrm{NN}}}$ = 5.2 GeV





[HT, Nishimura, Ohnishi (2024)] [HT (in prep)]

(I'm interested in this and wanna study this further
⇒ Chiral XXX ? Axion electrodynamics? Novel QCD phase? Let's discuss if interested ☺)

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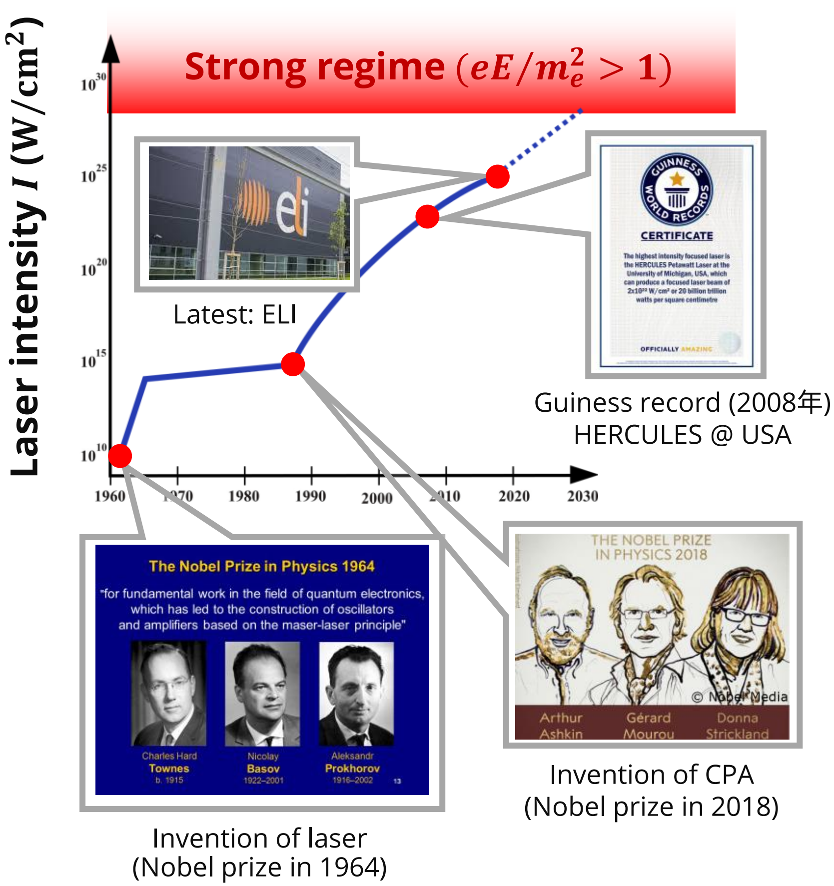

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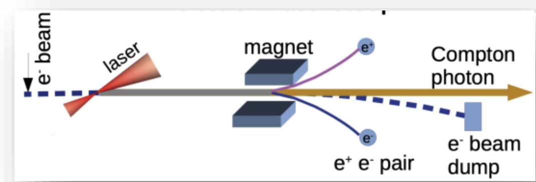

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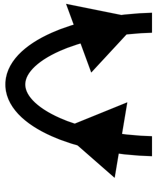
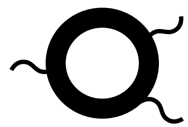
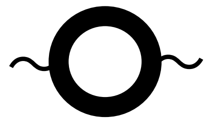

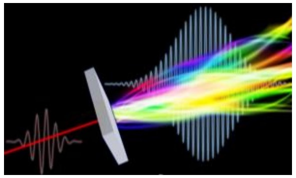

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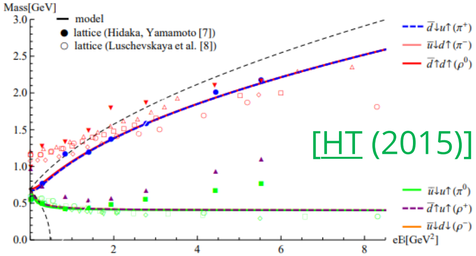

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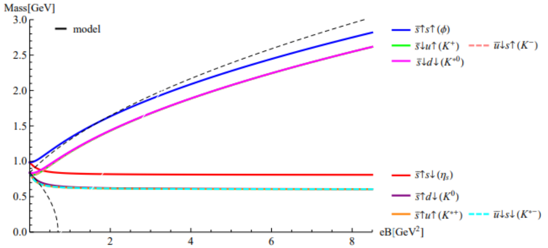

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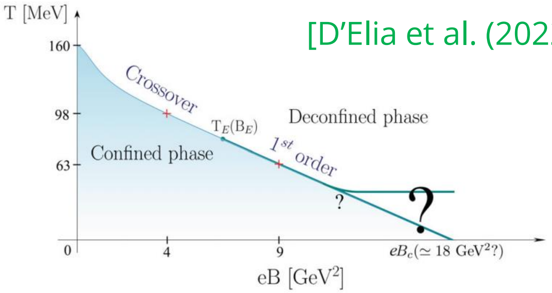

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✔ Vacuum pair production by quantum tunneling in electric field

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✔ Vacuum pair production by quantum tunneling in electric field

Why important/interesting ?

- Vacuum process

  ⇒ Fundamental, since all the physical processes occur on top of vacuum

- Non-perturbative

  ⇒ Interesting, since it is the unexplored region of QFT

- Interdisciplinarity

  ⇒ Similar phenomena appear in many other areas of physics
  e.g., Landau-Zener effect, Hawking radiation, broad resonance, ...

Basic of the Schwinger effect

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[Schwinger (1951)]

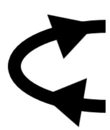
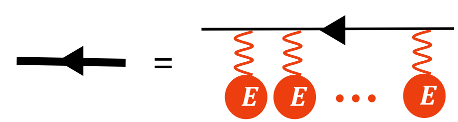
✔ Vacuum pair production by quantum tunneling in electric field

Well understood (only) for a constant E field (+ many assump.: weak coup., no backreac., ...)

Schwinger formula: $$N_{e^\pm} = \frac{(eE)^2 VT}{(2\pi)^3} \times \exp\left[-\pi \frac{m^2}{eE}\right] \sim \exp[-\# \times (\text{gap height}) \times (\text{gap length})]$$

[Schwinger (1951)] [Nikishov (1969)]

- Simple theory: Calculate scattering amplitude for $|0;\text{in}\rangle \to |e^- e^+;\text{out}\rangle$

⇒ Evaluate  w/ a dressed wavefunc (or propagator) : 

[Furry (1951)]

⇒ Sufficient to solve Dirac eq. w/ strong-field (classical c-number) potential: $0 = [i\partial - eA - m]\psi$

- Notice the strong exp suppression ⇒ the reason why Schwinger has never been verified in experiments

cf.) Guinness world record: $eE \approx (1\text{ keV})^2 \ll m_e^2 = O(1\text{ MeV})^2$

[Yanovsky et al (2008)]

Open problems

Strong-field guys are trying to go beyond the Schwinger formula

✔ Relax the strong assumptions of the Schwinger formula

- to find a formula applicable to realistic situations
- to find something new/interesting
  (not a splitting hair but rich physics appears beyond the Schwinger formula !)

✔ Examples

- Beyond the constant-E-field approximation
- Beyond the no-backreaction approximation
- Beyond the weak-coupling limit
- Observables other than N ?

Review: [Fedotov, Ilderton, Karbstein, King, Seipt, HT, Torgrimsson, 2203.00019]

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What happens E field is not constant in time ?

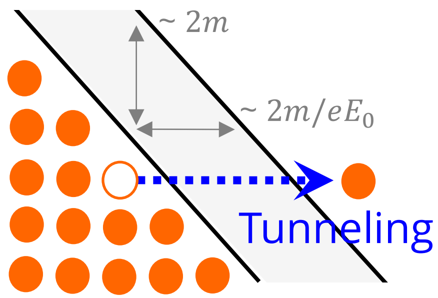
No Schwinger effect for "fast" E fields

✔ Time-dependent E field with strength eE_0 & frequency Ω (or lifetime $\tau = 1/\Omega$)

No Schwinger effect for "fast" E fields

✔ **Time-dependent E field with strength eE_0 & frequency Ω (or lifetime $\tau = 1/\Omega$)**

Slow = small Ω
⇒ Non-pert. tunneling $N \sim \exp[\#/eE_0]$



Tunneling time $\Delta t \sim \frac{2m}{eE_0}$

⇒ E field must be slower than Δt

⇒ $\Omega^{-1} \gtrsim \Delta t$

[Keldysh (1965)]

⇒ $1 \gtrsim \frac{\Delta t}{\Omega^{-1}} = \frac{\Omega m}{eE_0} \equiv \gamma$ (Keldysh parameter)

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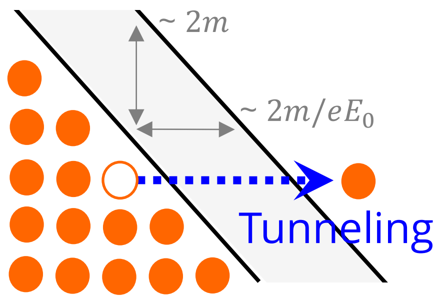

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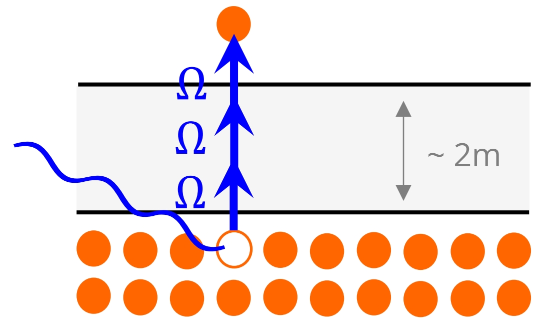
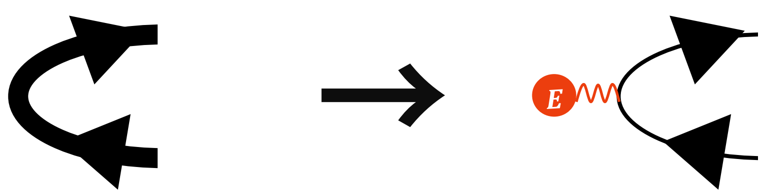
Fast = large Ω
⇒ Pert. photon scattering $N \sim eE_0^{2n}$



⇒ Incoherent photons, rather than coherent E field

⇒ pair prod. when $n\Omega > 2m$

⇒ QED version of the photoelectric effect in material



Development 1/5: A better understanding of non-pert Schwinger vs pert pair prod

✔ "Phase diagram" of the Schwinger effect

Theory: (1) Semi-classical approx. = Trans-series expansion in $\hbar$

$$N = \sum_{n,m} N_{n,m} \hbar^n \mathrm{e}^{-m\frac{S}{\hbar}} = (N_{0,1} + O(\hbar))\, \mathbf{e}^{-\frac{S}{\hbar}} + O(\mathrm{e}^{-\frac{2S}{\hbar}})$$

[Brezin, Itzykson (1970)] [Popov (1972)] [Berry (1989)] [Dunne, Shubert (2005)] [HT, Fujimori, Misumi, Nitta, Sakai (2020)]

(2) Compare with exactly solvable cases [HT, Fujiii, Itakura (2014)]

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- Two dim-less params control the interplay

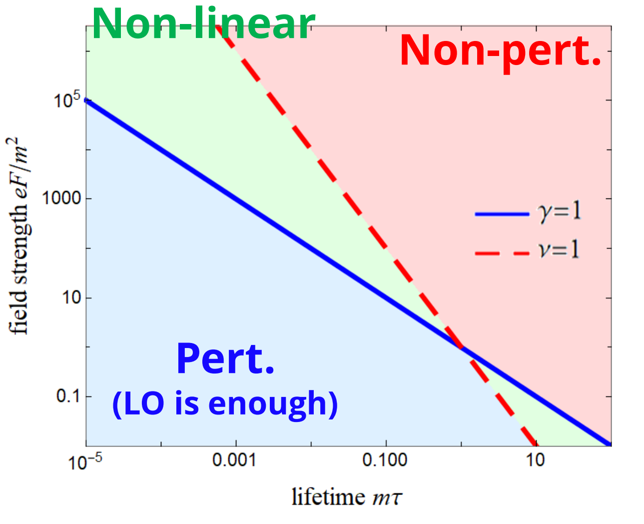
  ⇐ 3 dimensionfull params in the system: $eE, \tau \coloneqq 1/\Omega, m$

$$\gamma = \frac{m\Omega}{eE} \quad \text{: Keldysh parameter}$$

$$\nu = \frac{eE\tau}{\Omega} = \frac{\text{(Work done by field)}}{\text{(One photon energy)}} = \text{(number of photons)}$$

- Non-pert Schwinger if $\gamma \ll 1, \nu \gg 1$

  Pert pair production if $\gamma \gg 1, \nu \ll 1$



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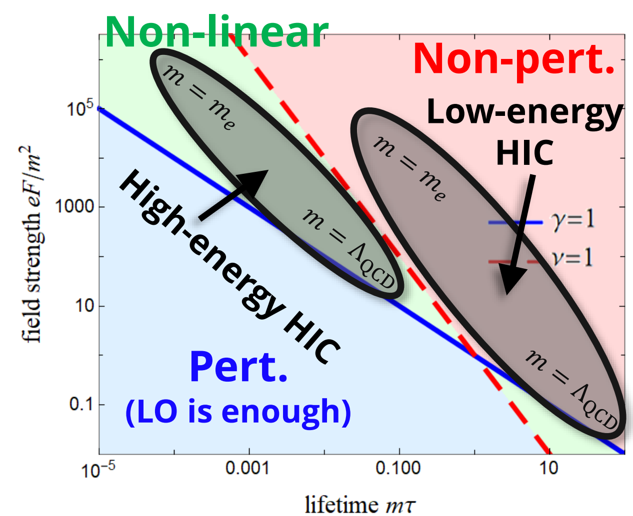

- Implication: "Strong field ⇒ Non-pert strong-field physics" is necessarily correct

⇒ Not only strength but also lifetime (& other dimful params, if any) is important

e.g., High-energy HIC is not non-pert.: $eF \sim (1\,\mathrm{GeV})^2$, $\tau \sim 0.1\,\mathrm{fm}/c \Rightarrow \gamma \sim \begin{cases} 10^{-3}\ (m = \Lambda_{\mathrm{QCD}}) \\ 10^{-5}\ (m = m_{\mathrm{e}}) \end{cases}$, $\nu \sim 0.1$
($\sqrt{s_{NN}} > O(100\,\mathrm{GeV} - 1\,\mathrm{TeV})$)

Low-energy HIC is non-pert.: $eF \sim (100\,\mathrm{MeV})^2$, $\tau \sim 10\,\mathrm{fm}/c \Rightarrow \gamma \sim \begin{cases} 10^{-1}\ (m = \Lambda_{\mathrm{QCD}}) \\ 10^{-4}\ (m = m_{\mathrm{e}}) \end{cases}$, $\nu \sim 10$
($\sqrt{s_{NN}} = O(1 - 10\,\mathrm{GeV})$)

[HT, Nishimura, Ohnishi, (2024)]

Development 2/5: Importance of pert pair production

✔ **Schwinger formula is inapplicable for fast E fields**

Slow ⇒ Non-pert ⇒ Strong exp suppression $N \sim \exp[-m^2/eE_0]$

Fast ⇒ Pert ⇒ Weak power suppression $N \sim (eE_0/m^2)^{2n}$

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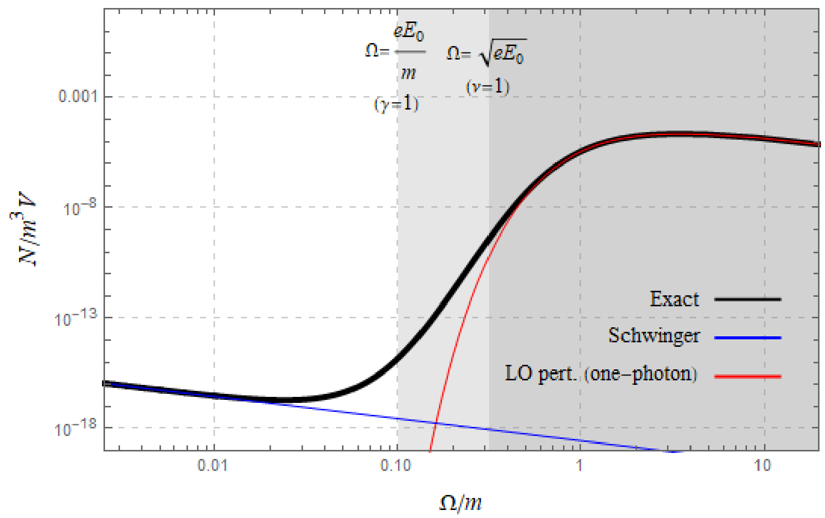

Demonstration:

Pair prod from pulsed E field w/lifetime $\tau = 1/\Omega$

(Sauter field $eE(t) = \frac{eE_0}{\cosh^2(\Omega t)}$)

[HT, Fujiii, Itakura (2014)]
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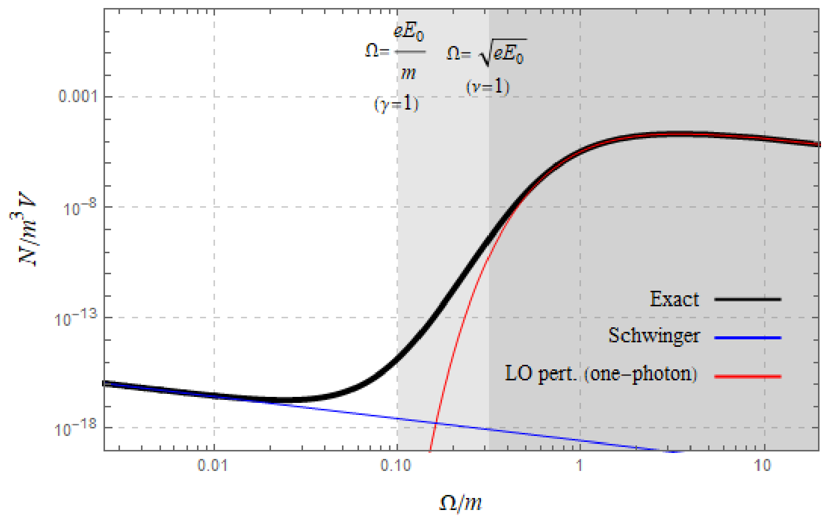


✔ Some application to actual physical problems

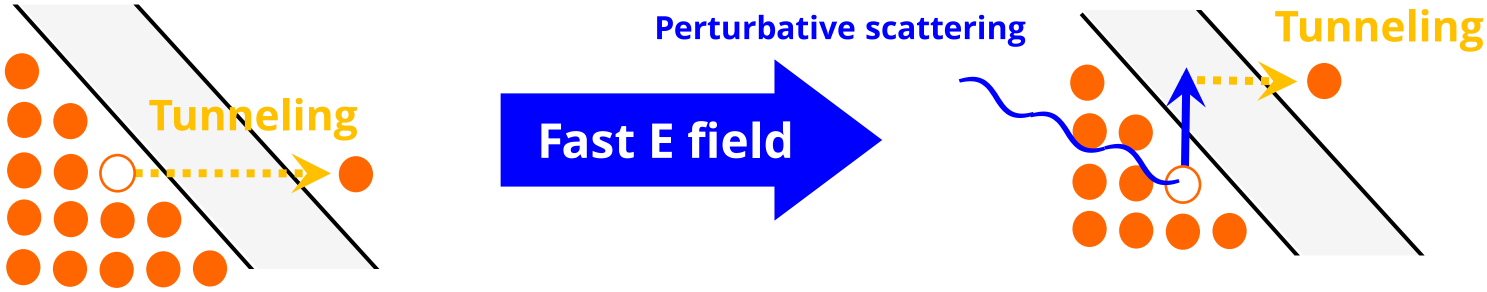
- Enhancement of heavy quark prod in heavy-ion collisions [Levai, Skokov (2010)]
- Use of fast fields to enhance the Schwinger effect in weak-field exp. (e.g., laser)

⇒ **Dynamically assisted Schwinger effect** (next slide)

Development 3/5: Dynamically assisted Schwinger effect (1/2)

[Dunne, Gies, Schutzhold (2008), (2009)]

✔ **Significant enhancement of the Schwinger effect by superimposing fast (weak) E fields**



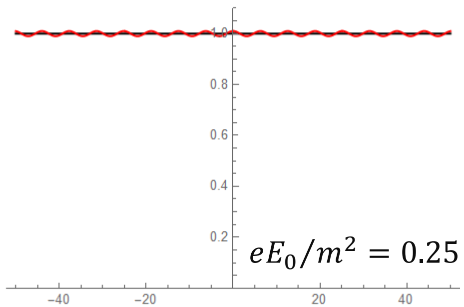
$$N \sim \exp[-\# \times (\text{gap height}) \times (\text{gap length})]$$ ⇒ **Enhancement of pair prod**

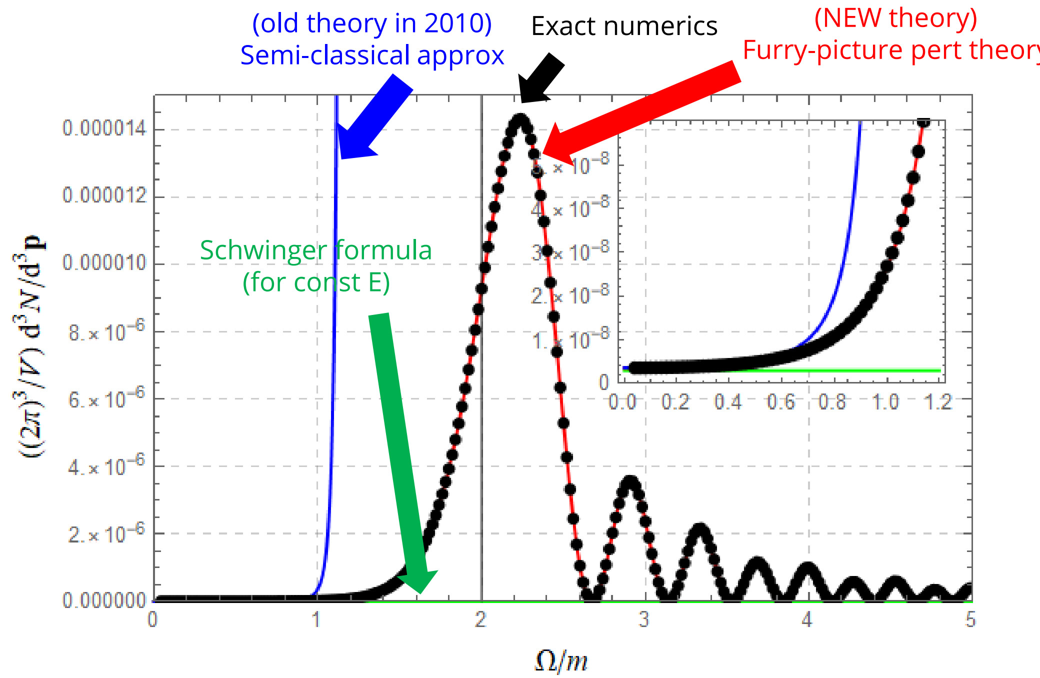
Reduced by the pert scattering

Development 3/5: Dynamically assisted Schwinger effect (1/2)

Field config

$$E = E_0\left(1 + \frac{1}{100}\cos\Omega t\right)$$





✔ Physics outcome

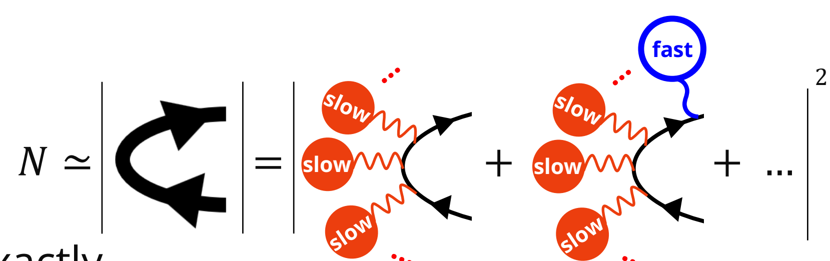
- Expected: Huge enhancement, even for very weak fast field
- Un-expected: Oscillating behavior above the mass gap

  ⇐ Related to the Dirac-sea structure in strong E field (next slide)

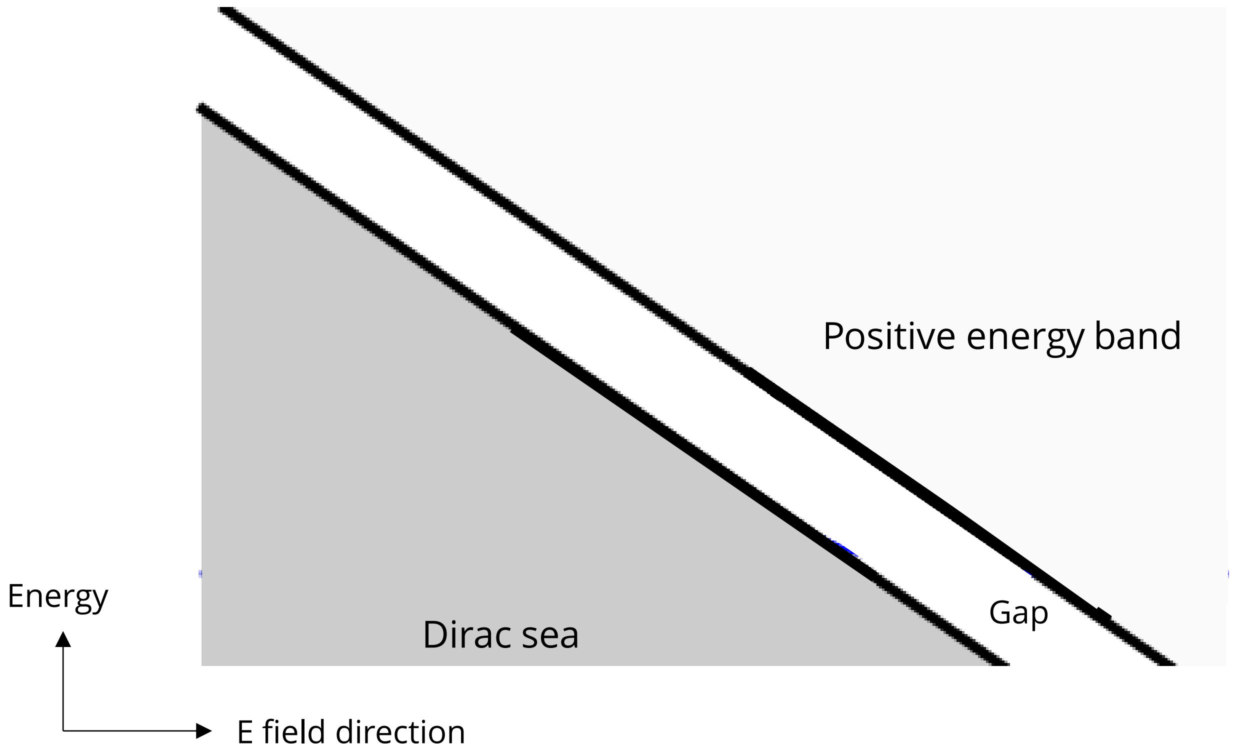
✔ Technical advancement

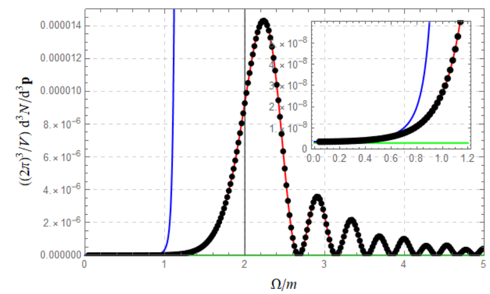
[HT, (2019)] [Huang, HT, (2019)]

- Dressed scattering theory w/ unstable vacuum
  ≈ Expand w/ fast field, while keeping slow field exactly



Development 4/5: Modified Dirac-sea structure by E field (1/2)





∴ The spectrum of the dynamically-assisted Schwinger effect
reflects the modified Dirac-sea structure in strong E field

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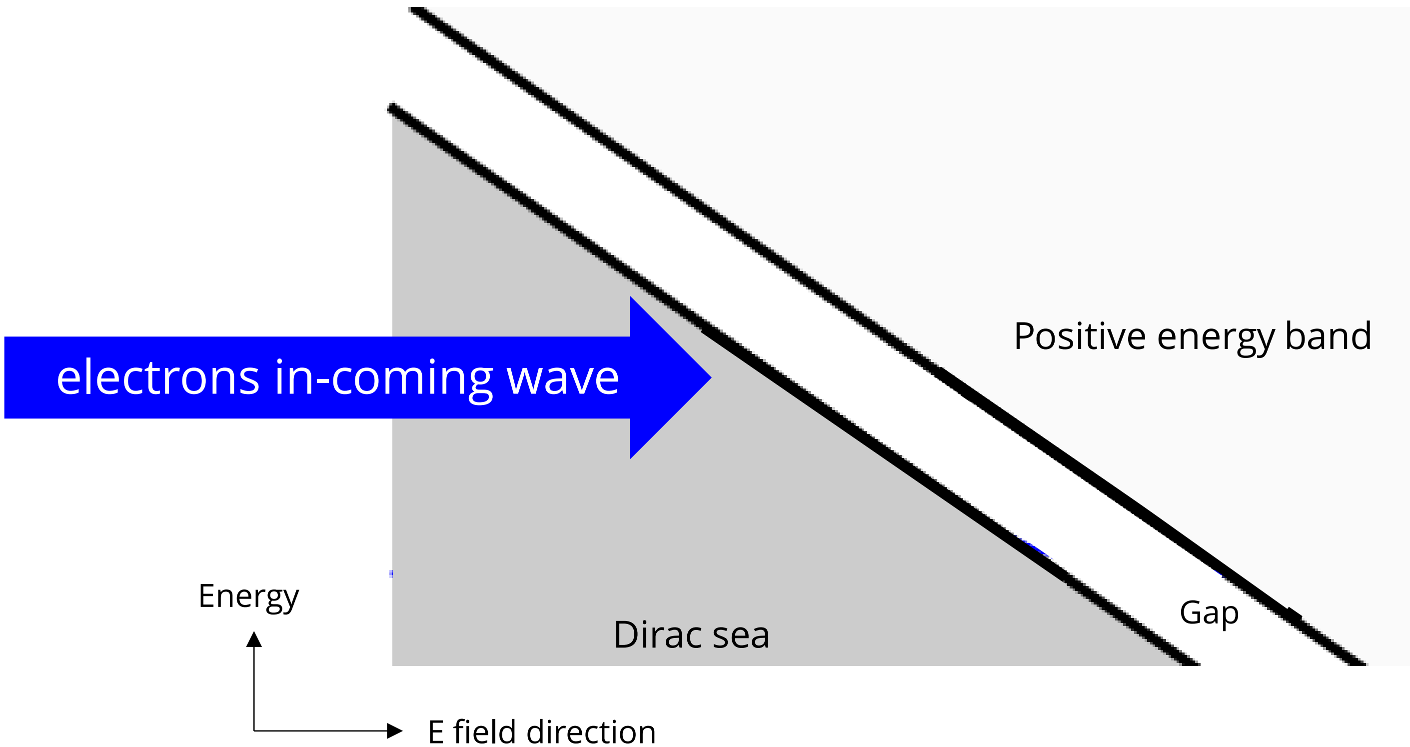

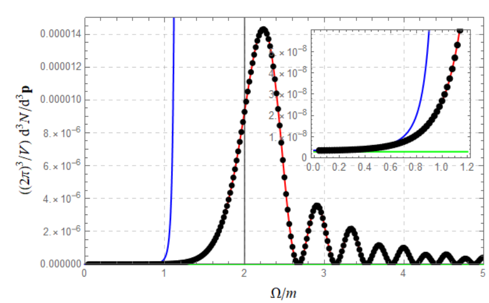

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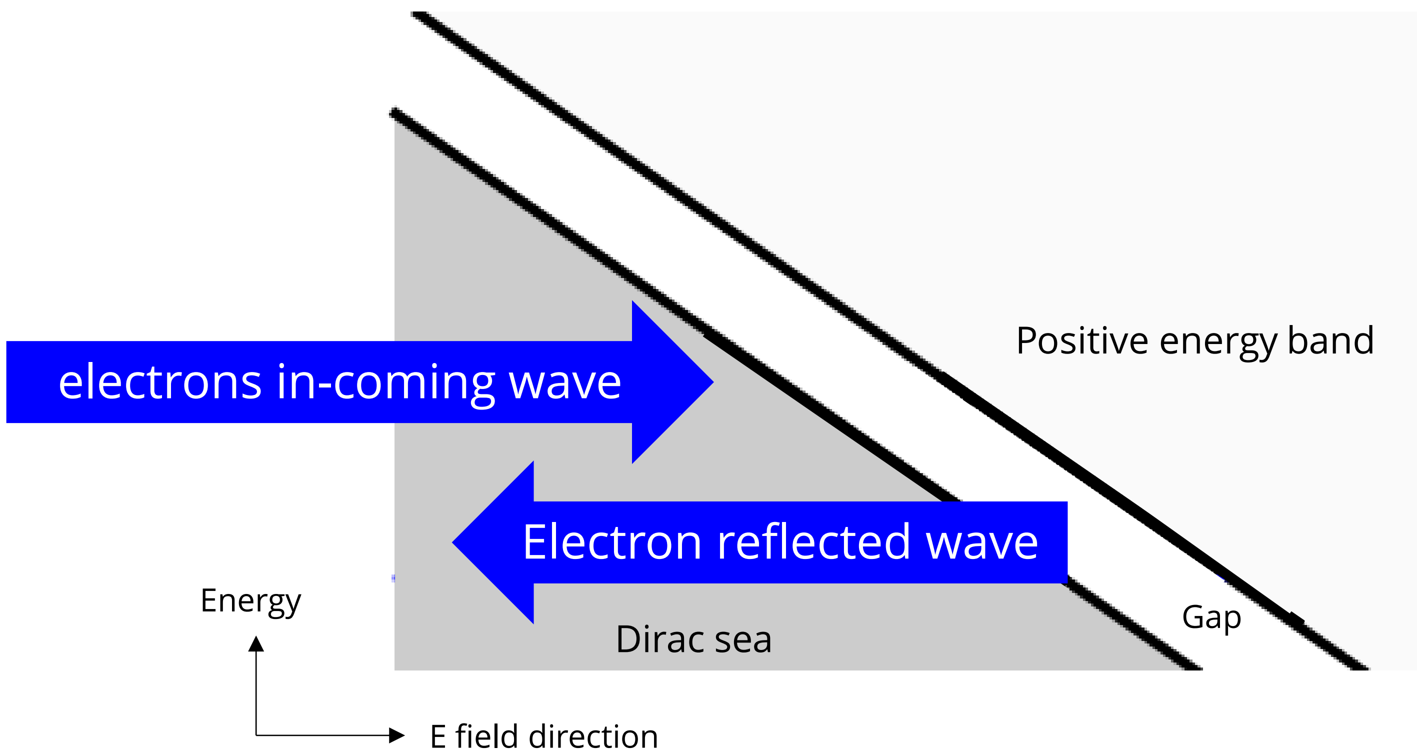

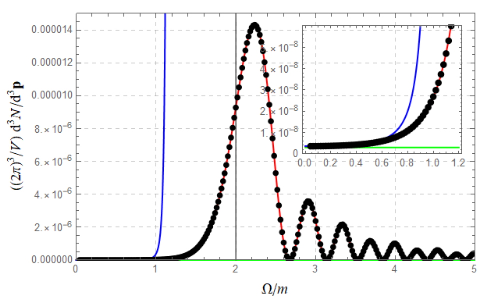

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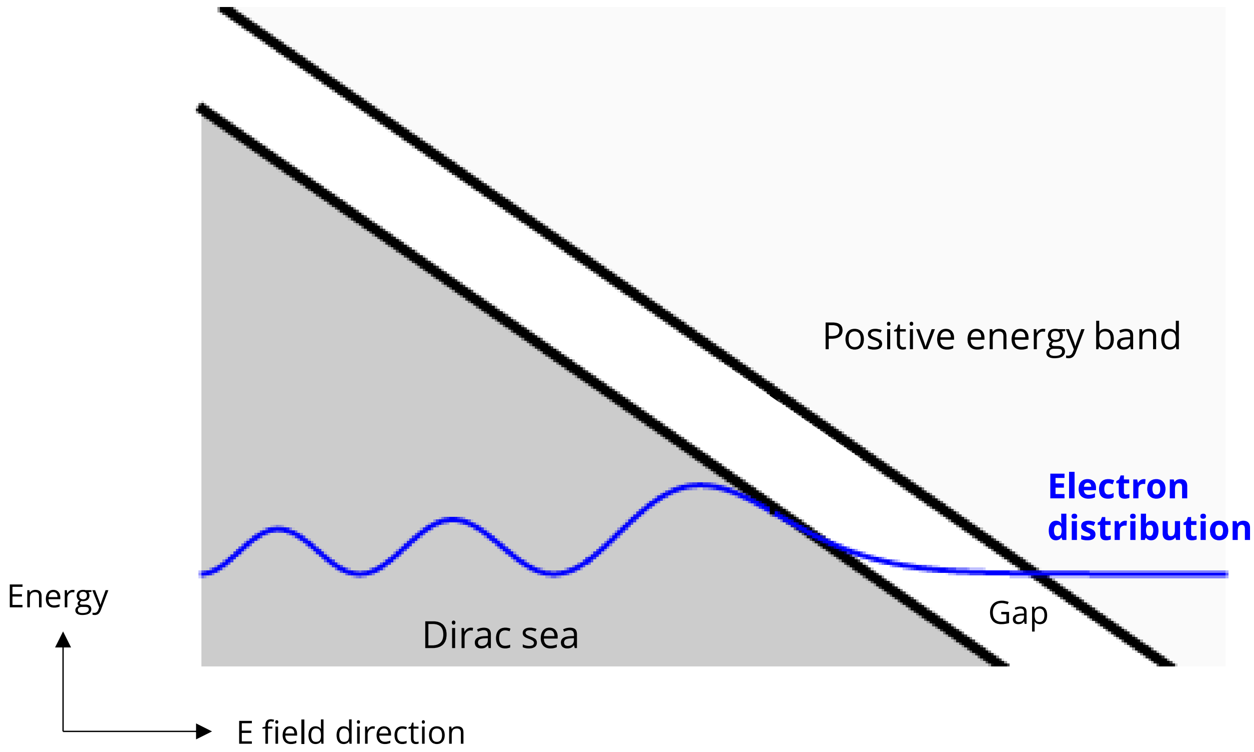

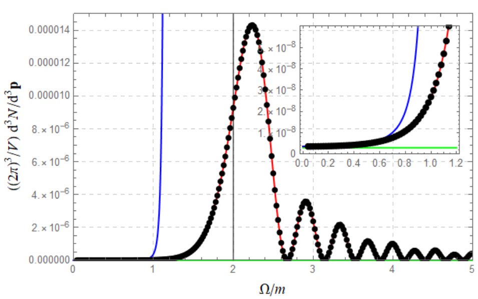

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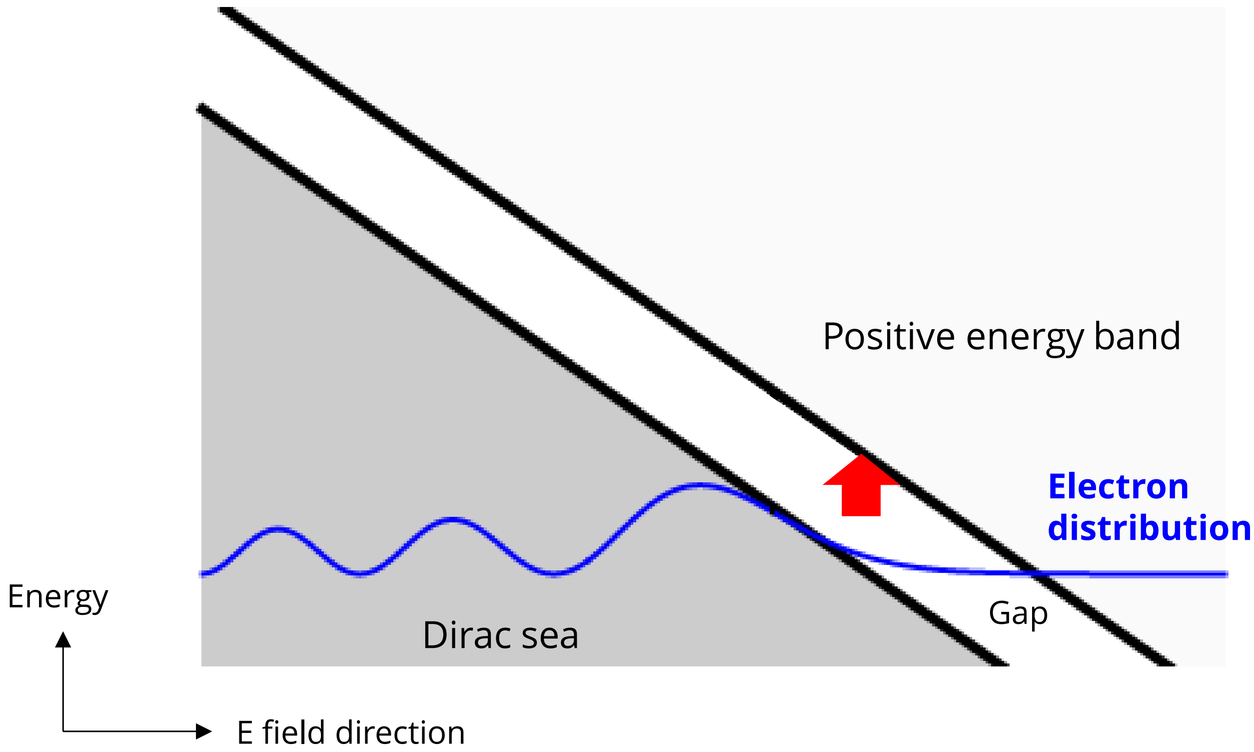
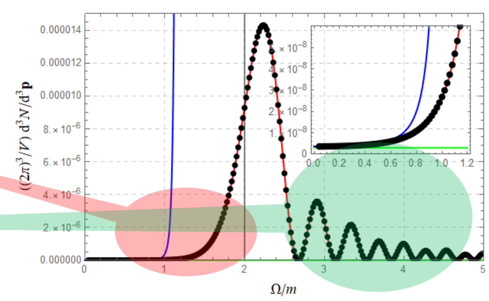
Development 4/5: Modified Dirac-sea structure by E field (1/2)

cf. similar argument in Franz-Keldysh effect in semi-conductor



- **Tunneling ⇒ Enhancement**



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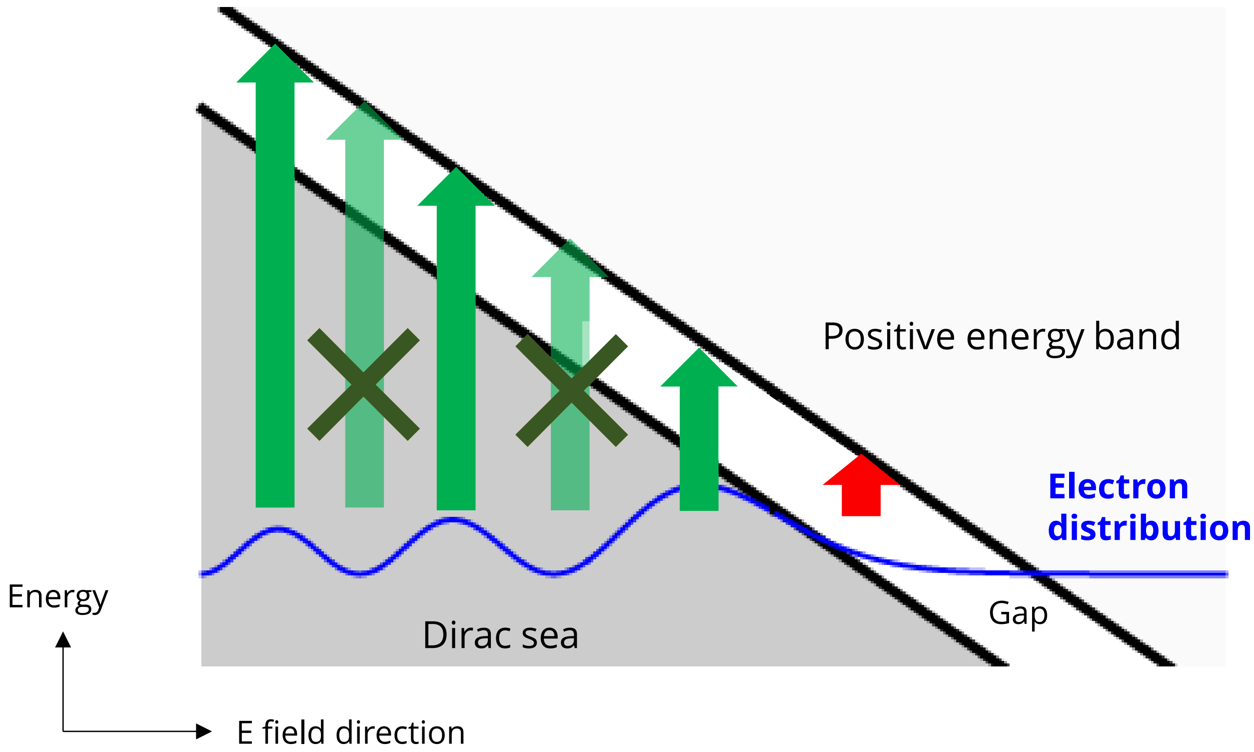
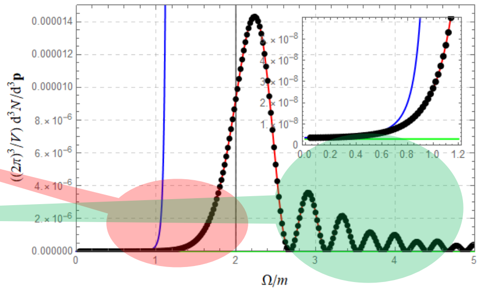


- **Tunneling ⇒ Enhancement**
- **Reflection ⇒ Oscillation**



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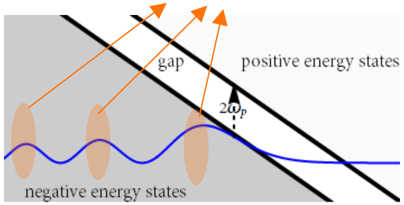
Development 4/5: Modified Dirac-sea structure by E field (2/2)

Larger electron density ⇒ affects more



The modified Dirac-sea struc affects everything

∵ Any process occurs on top of the vacuum

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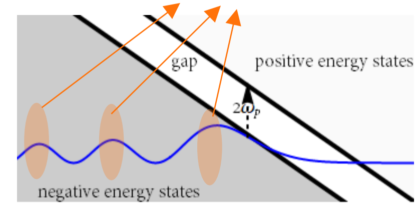

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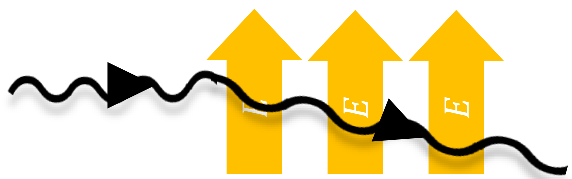
Example: Photon birefringence (electric permittivity) in strong E field

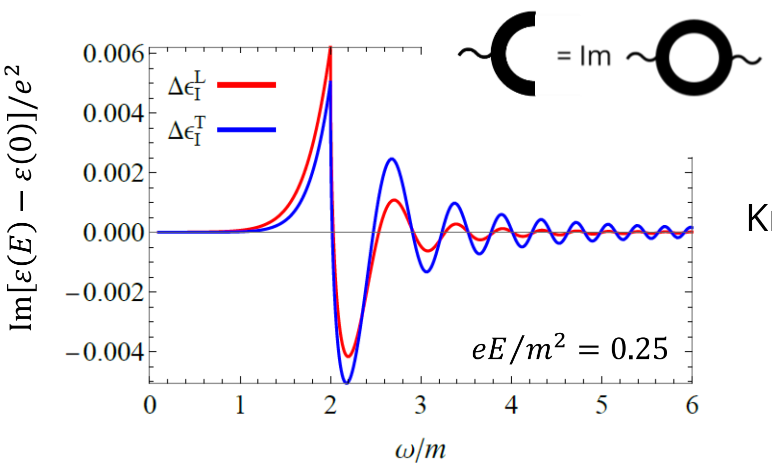
[HT, Ironside, (2024)]
cf. Birefringence in B field [Hattori, Itakura (2013)]

$\mathcal{E} \propto \cos \omega t$



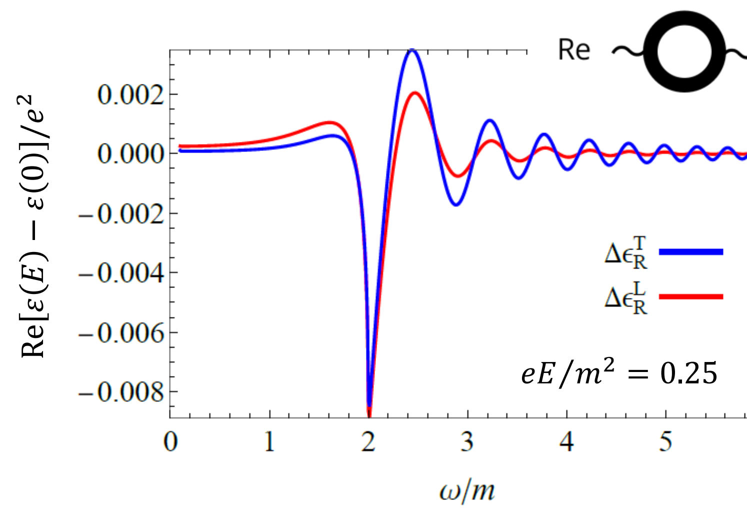
$\mathcal{D} = \varepsilon \mathcal{E} = n^2 \mathcal{E} = (1+\chi)\mathcal{E}$

Imaginary part



Kramers-Kronig rel.

Real part



⇒ Characteristic oscillation, as expected from the modified Dirac-sea structure !

Contents

1. Overview of strong-field physics

[Fedotov, Ilderton, Karbstein, King, Seipt, HT, Torgrimsson, 2203.00019]

2. Recent development of the Schwinger effect

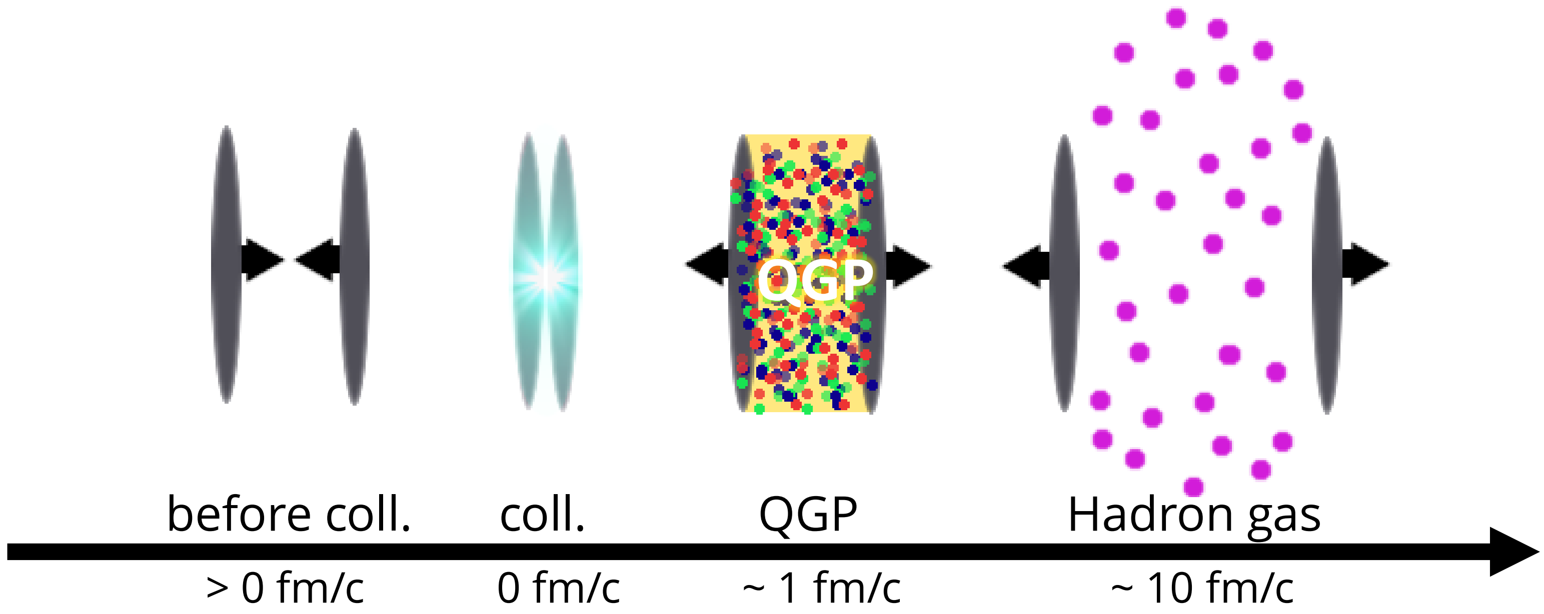
- focus on the Schwinger effect with time-dependent E fields

[HT, Itakura, Fujii, 1405.6182] [HT, 1812.03630] [HT, Fujimori, Misumi, Nitta, Sakai, 2010.16080]
[HT, Ironside, 2308.11248] [HT, Nishimura, Ohnishi, 2402.17136]

3. An application of the Schwinger effect to QCD: the early-time dynamics of heavy-ion collisions

- Quark production is very fast ! [HT, 1609.06189] [HT, Ph. D thesis]

Spacetime evolution of high-energy HIC (1/2)



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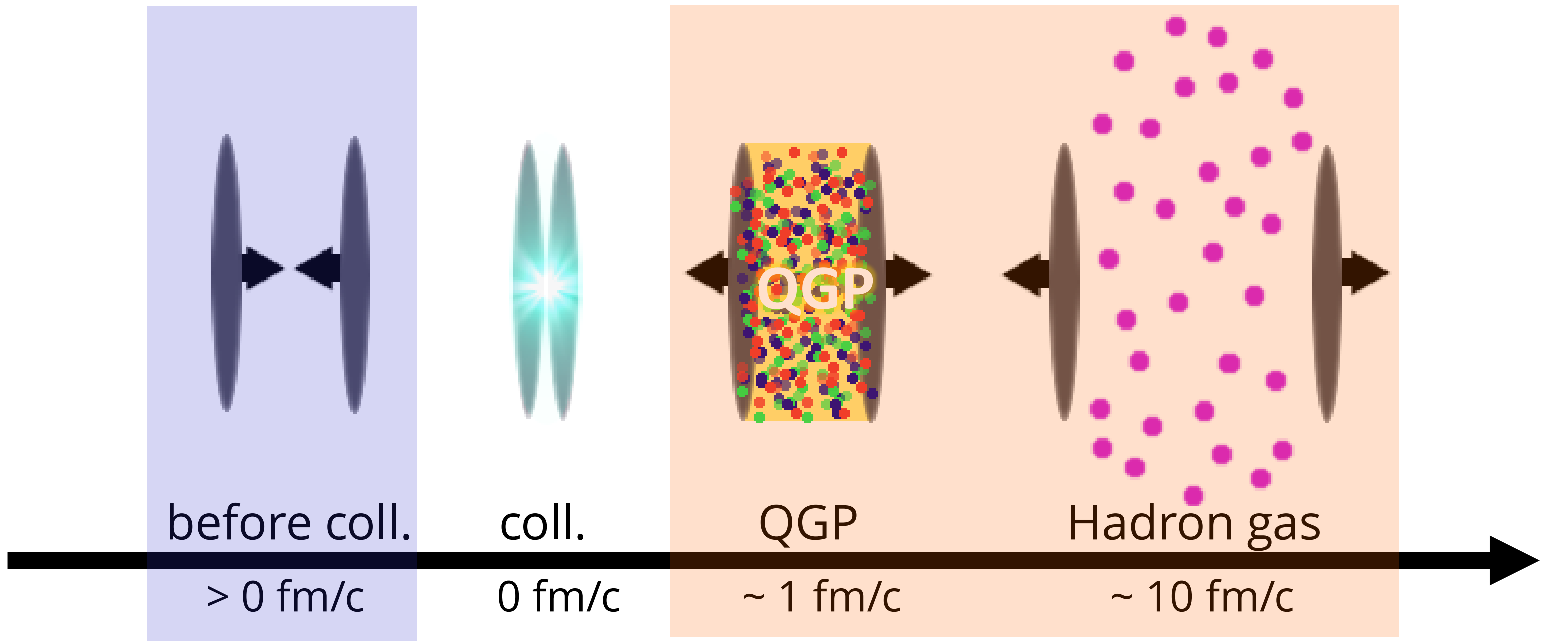

✔ ■ & ■ **are well understood**

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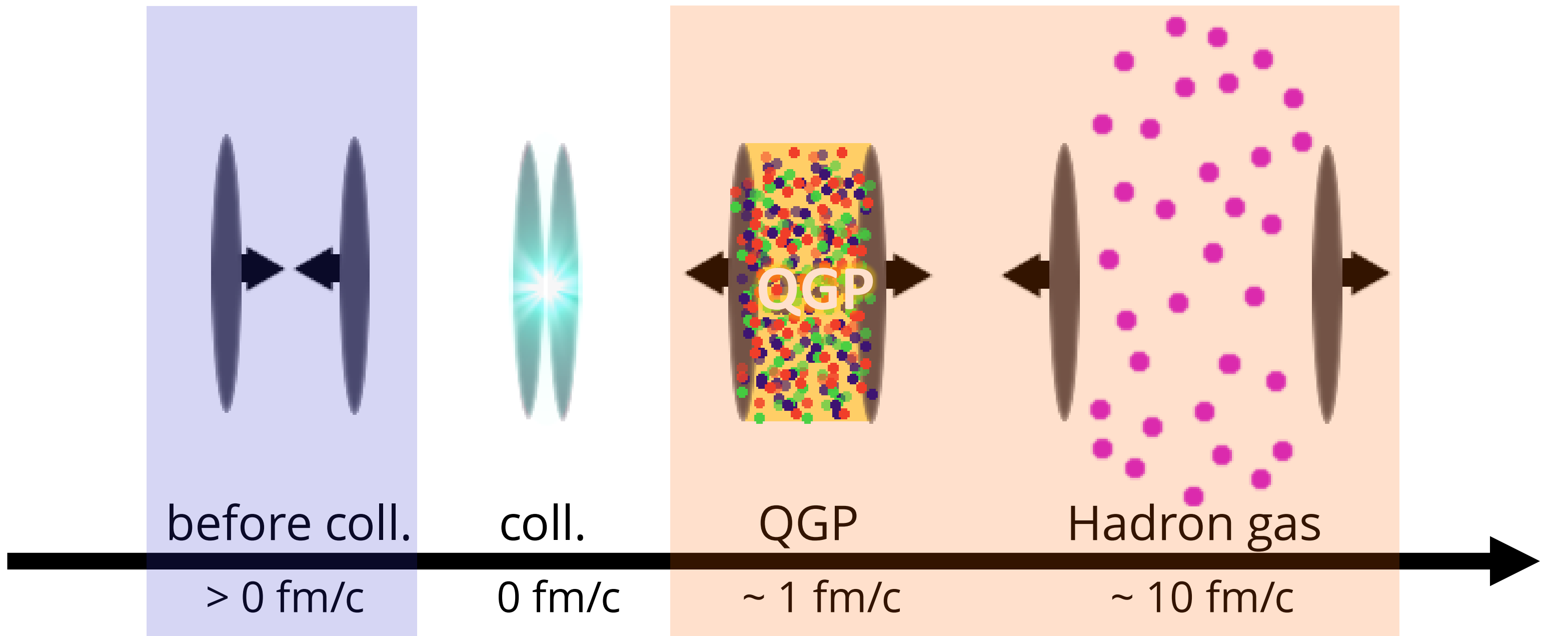
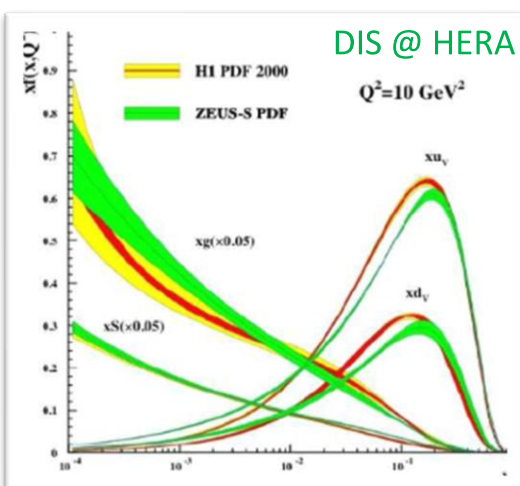


✔ [blue square] & [red square] **are well understood**



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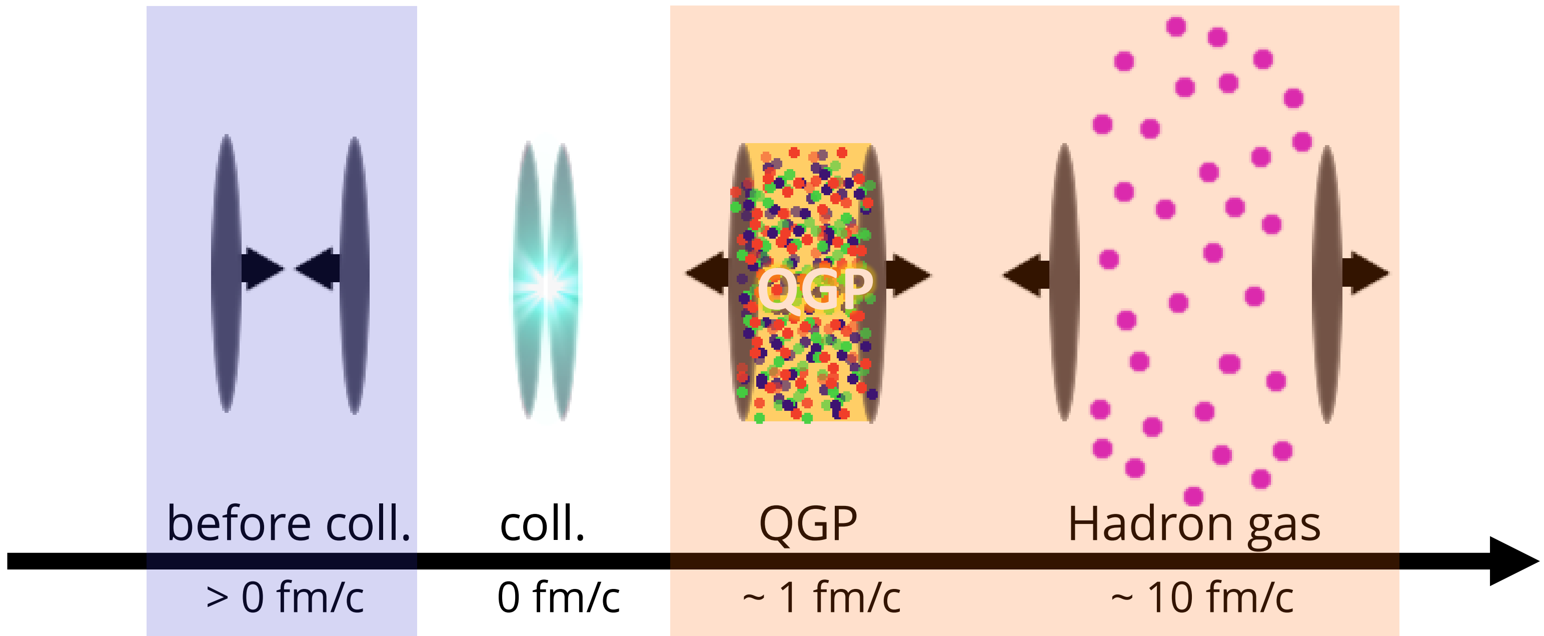

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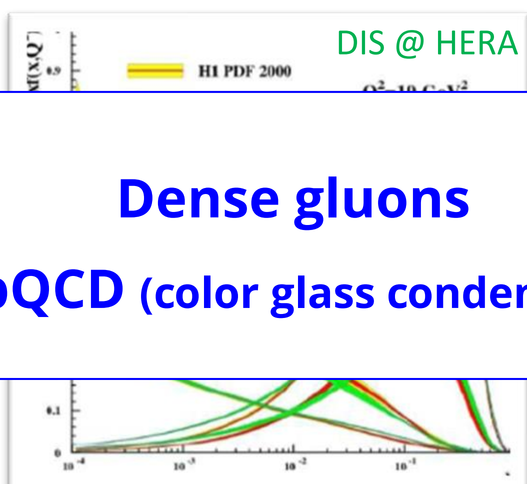

Dense gluons

⇒ pQCD (color glass condensate ?)

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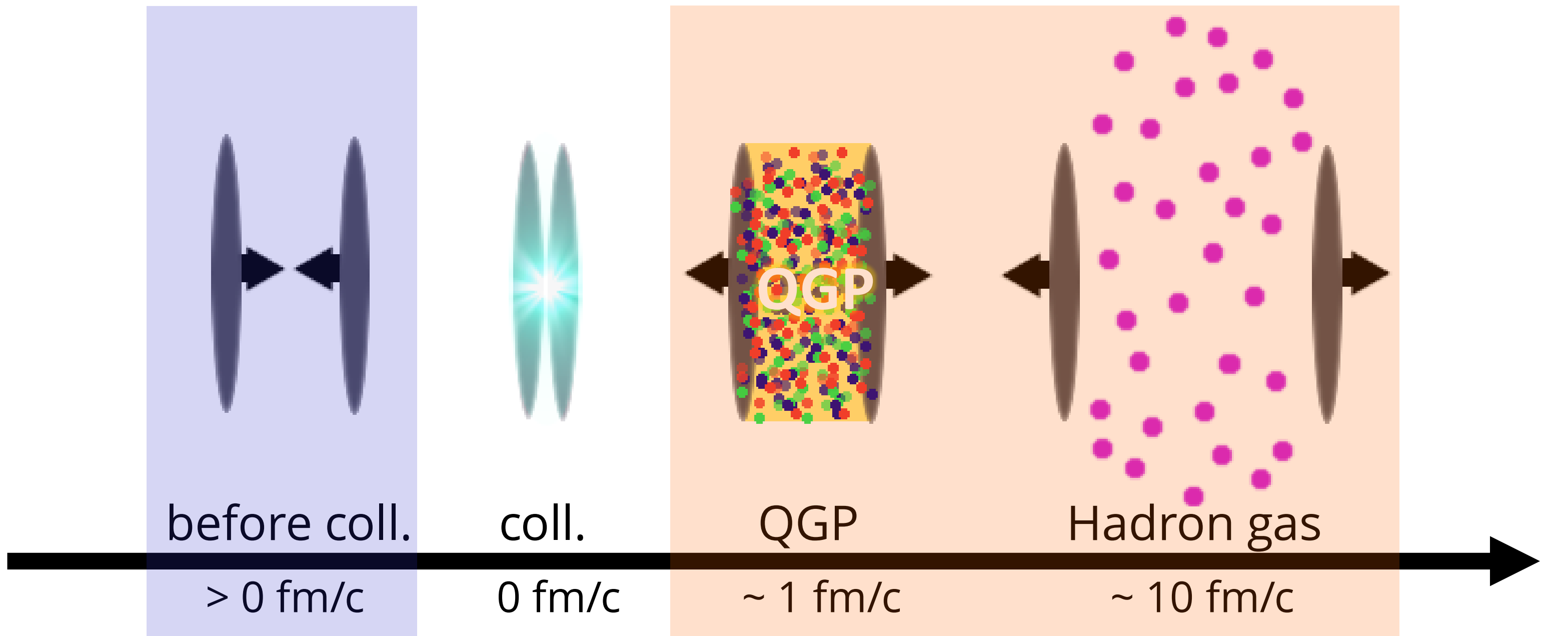

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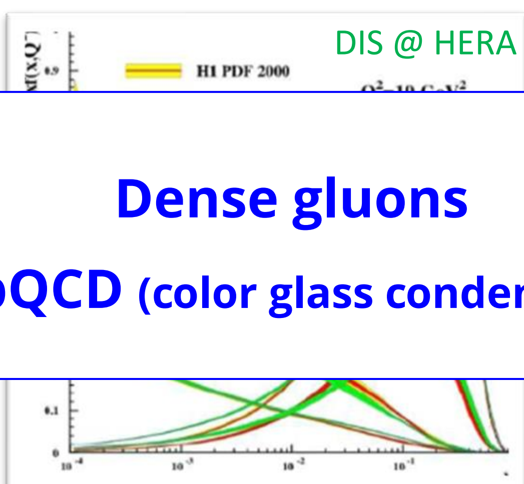

Dense gluons

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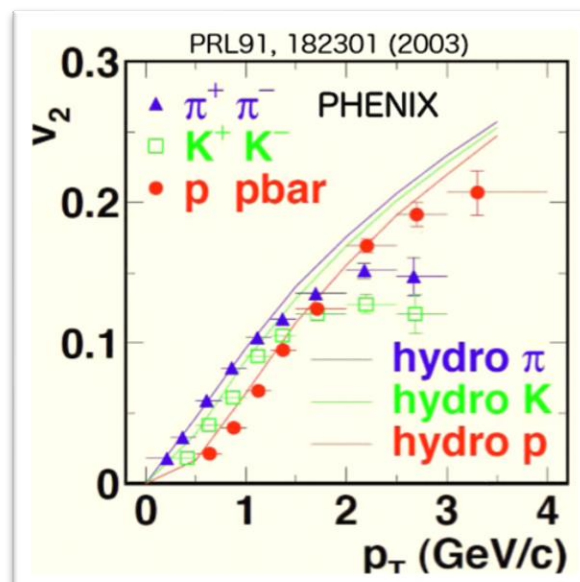
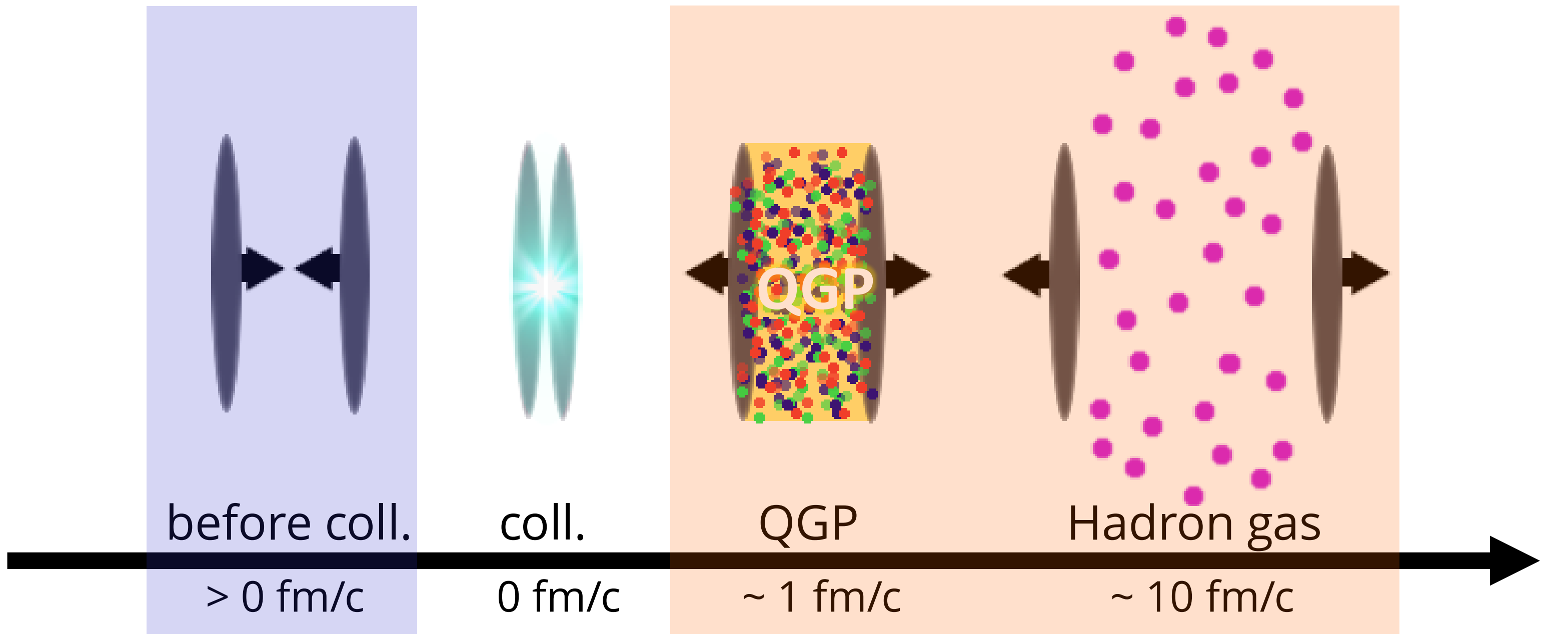
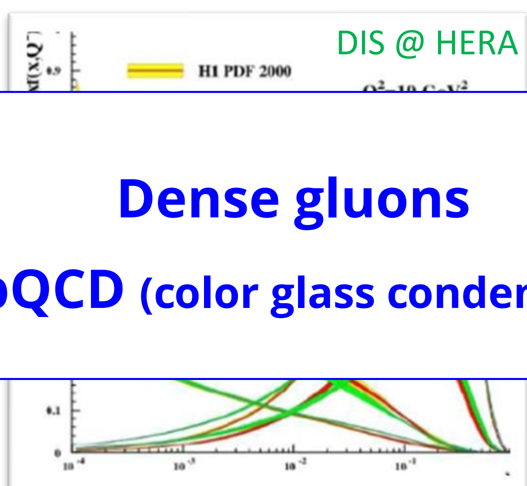


Spacetime evolution of high-energy HIC (1/2)



✔ ■ & ■ are well understood



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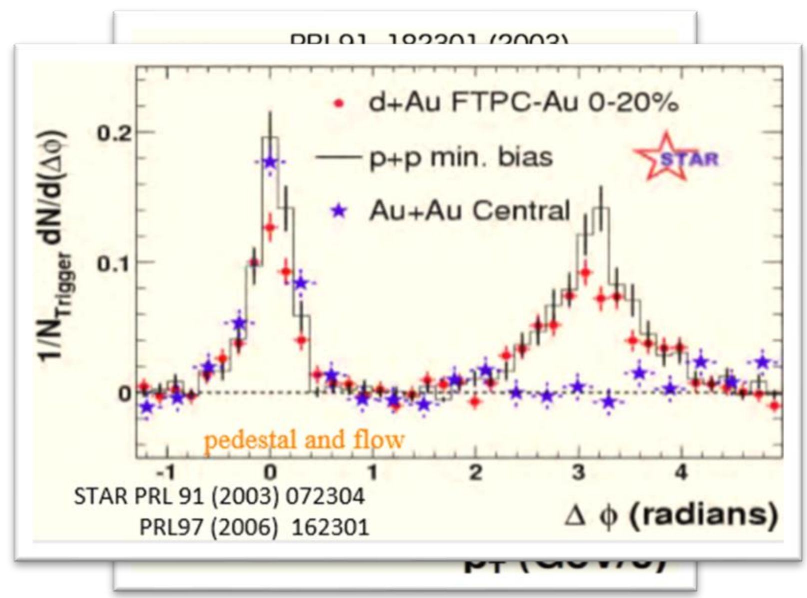

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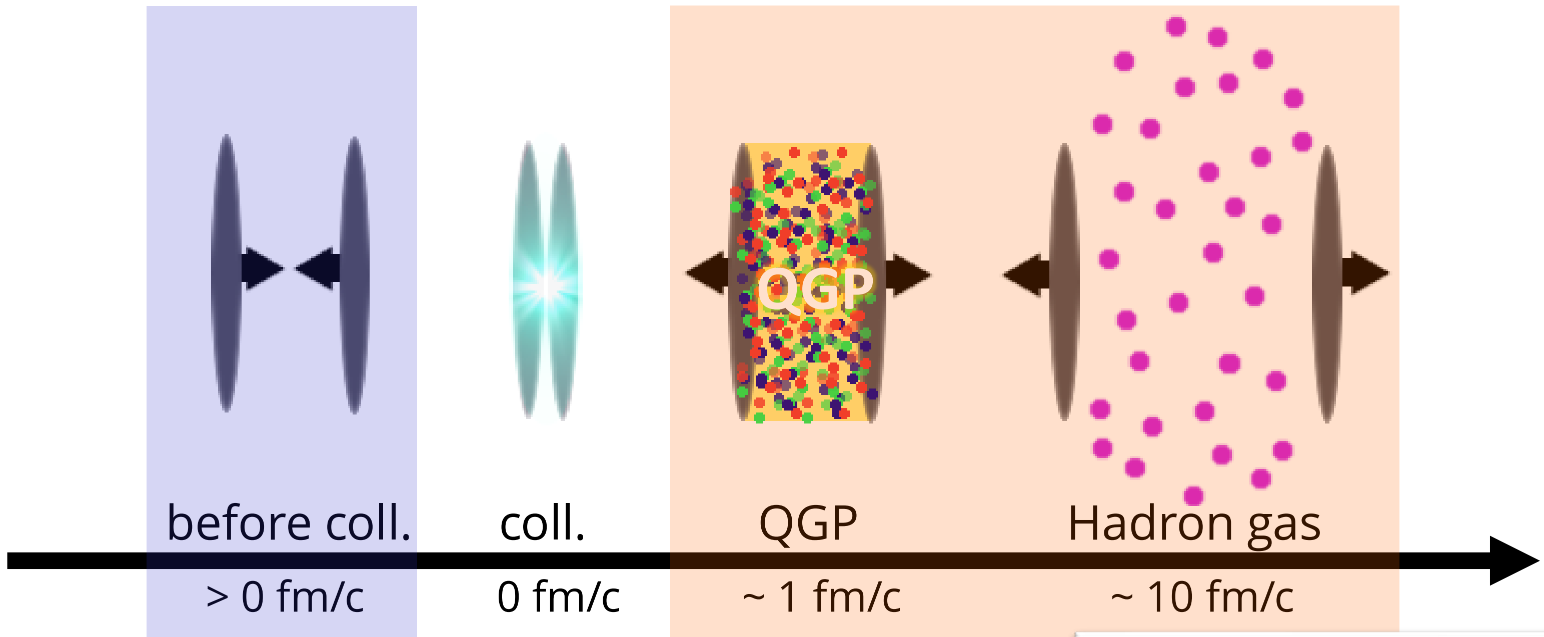

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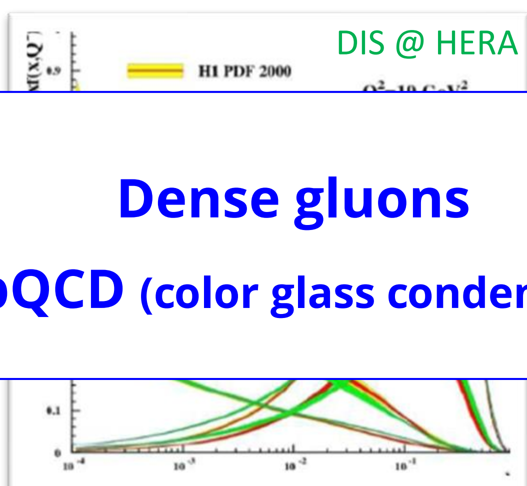

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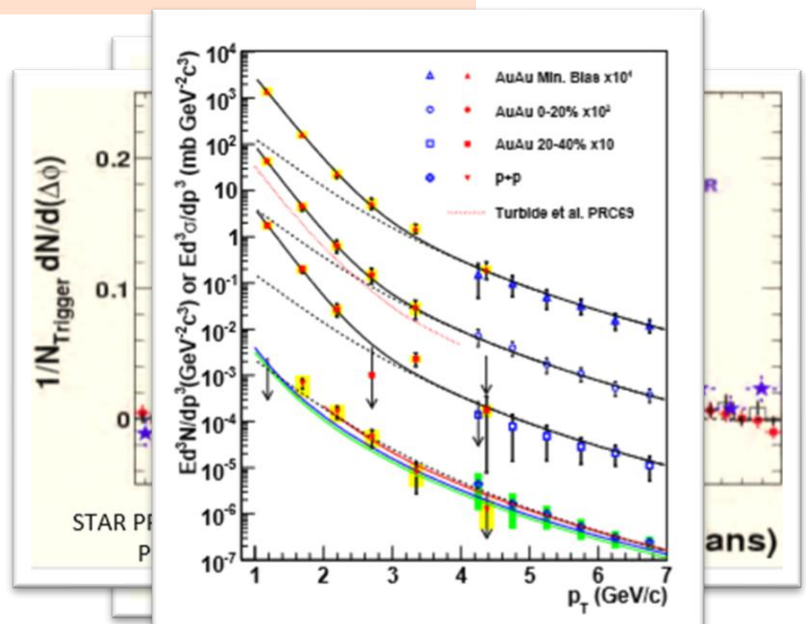

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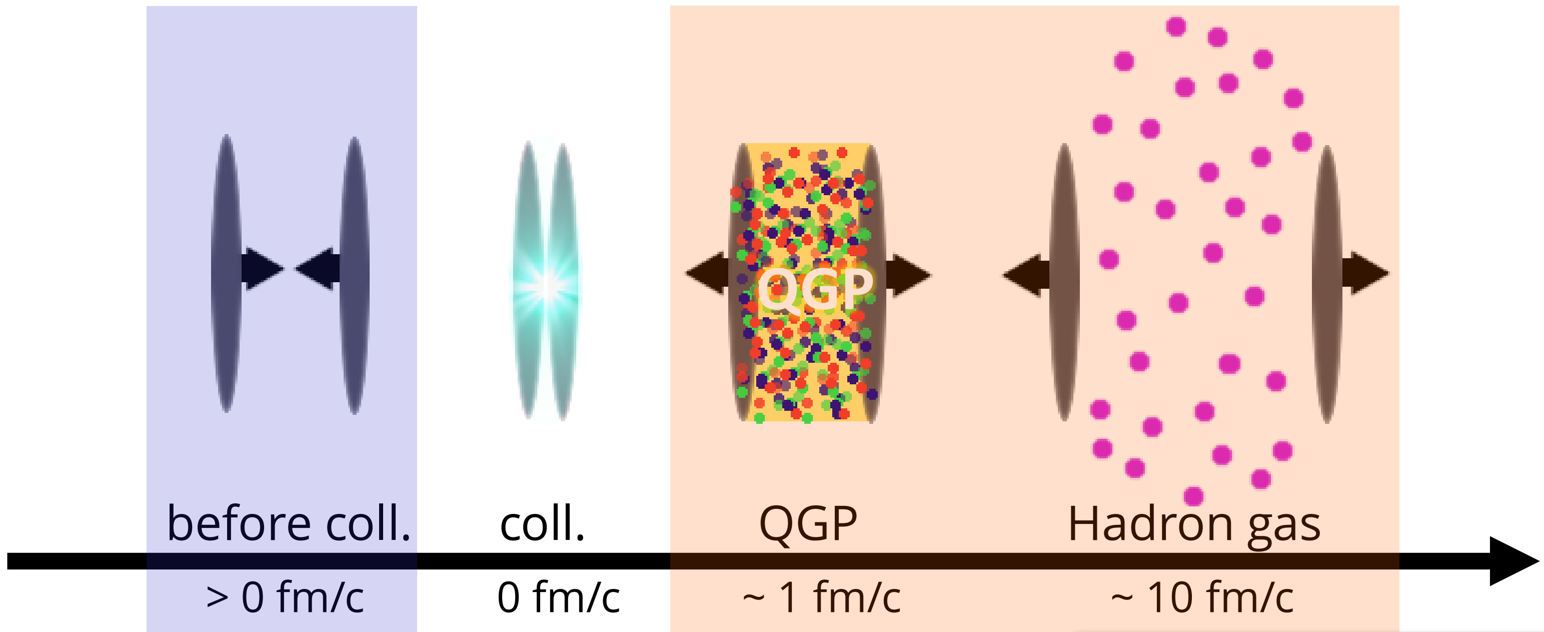

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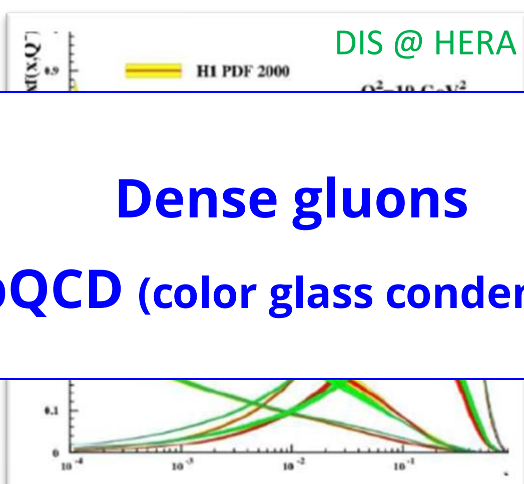

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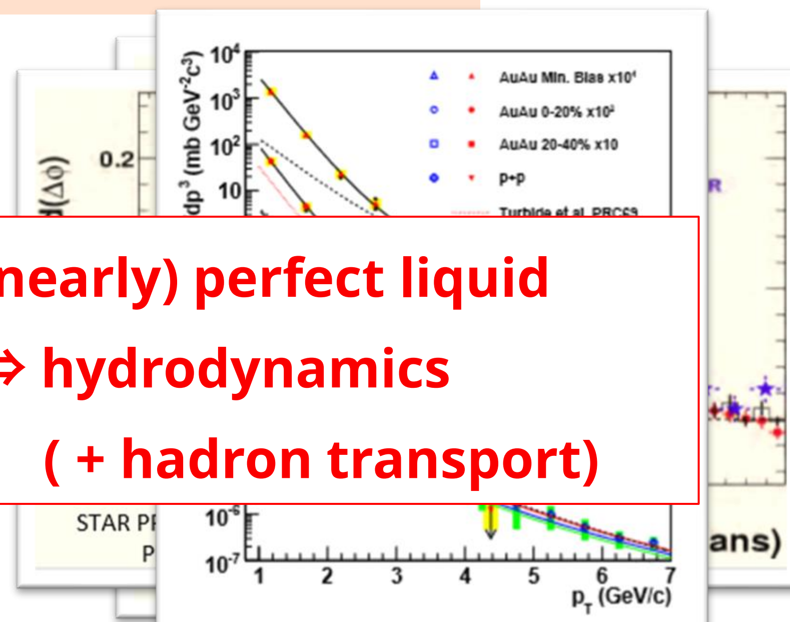
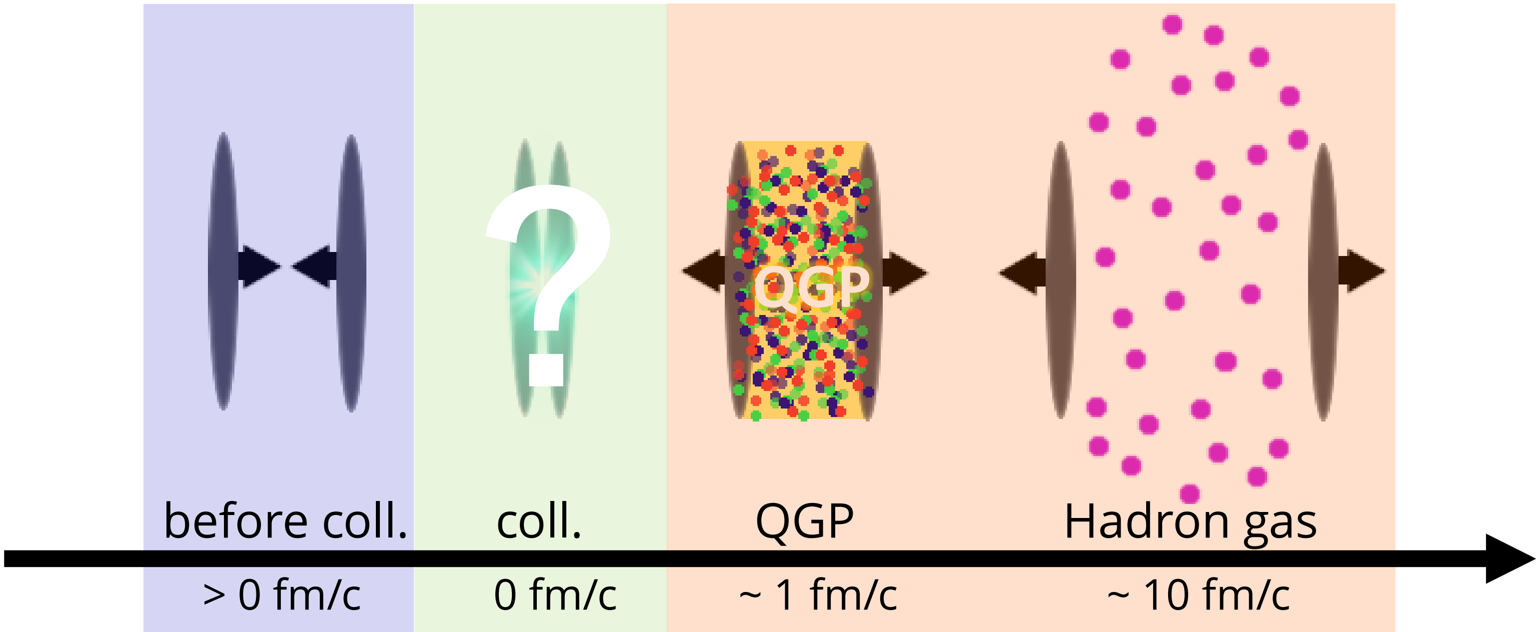


(nearly) perfect liquid

⇒ hydrodynamics

(+ hadron transport)

Spacetime evolution of high-energy HIC (2/2)



✔ ■ **is less understood**

cf. the weak-coupling kinetic picture from Heidelberg group

⇒ Formation dynamics of QGP is still an open issue

- How are the huge number of quarks & gluons produced dN/dy=O(1000) ?
- How do they thermalize (hydrodynamize) to form the liquid-like QGP ?
- How to explain the "early thermalization" O(1fm/c), indicated by exp data ?

✔ **Not only important for completing our spacetime picture of HIC but also for deepening our understanding of QGP**

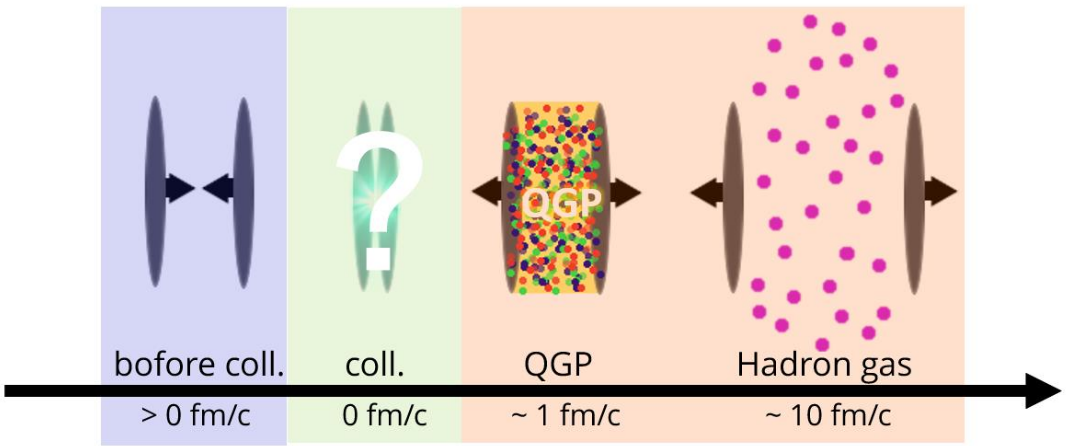
(e.g.: provide the initial cond. for hydro sim. ⇒ better determination of QGP properties)

Strong color EM field (glasma) (1/2)

Low-Nussinov model: Low, Nussinov, Casher, Neuberger (1970~80)
Glasma: McLerran, Lappi, Kovner, Weigert (~2005)

✔ **The key: Decay of the strong color EM field into particles**

⇒ **Schwinger effect !** [Kerman, Matsui, Gatoff (1987)]



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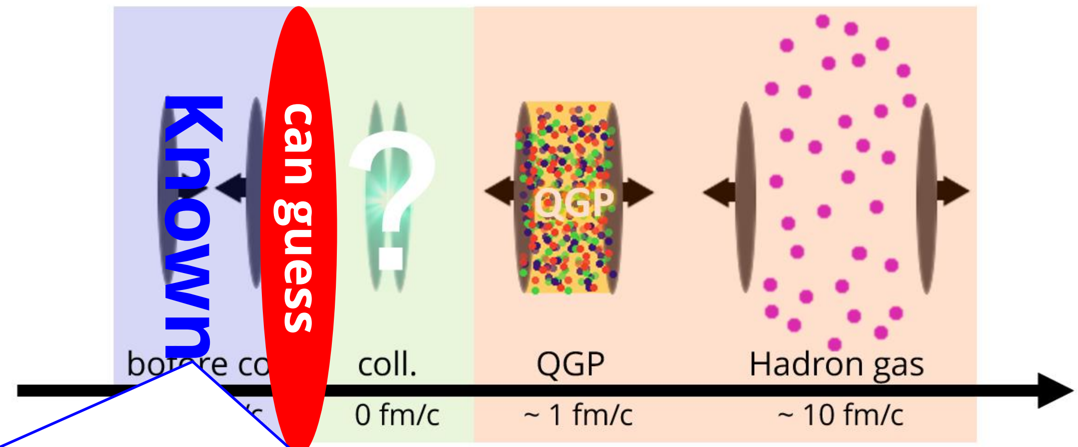
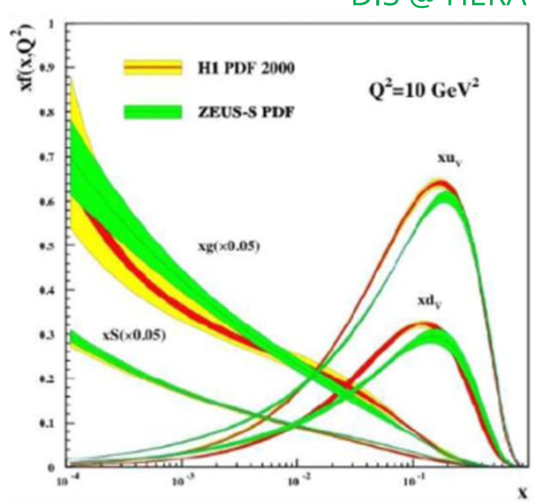
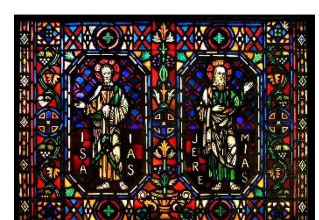


DIS @ HERA



$\propto 1/\sqrt{s_{\rm NN}}$

high-energy heavy ion
||
dense gluon state
≀
"color plate"
with huge color density
$(\sigma \sim Q_s^2 \sim O(1)\mathrm{GeV}^2)$

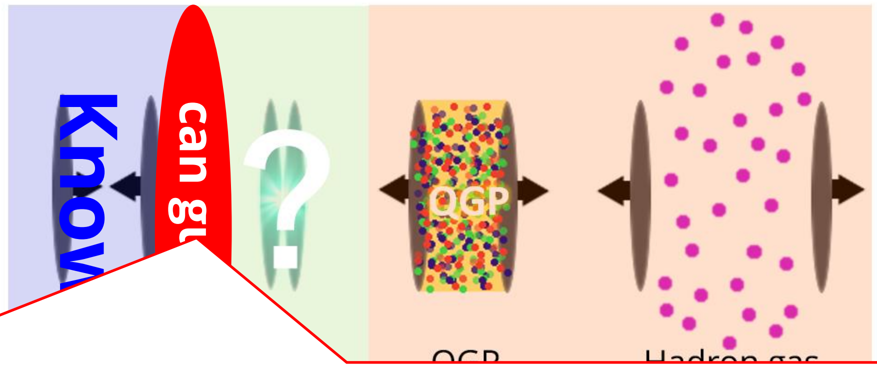


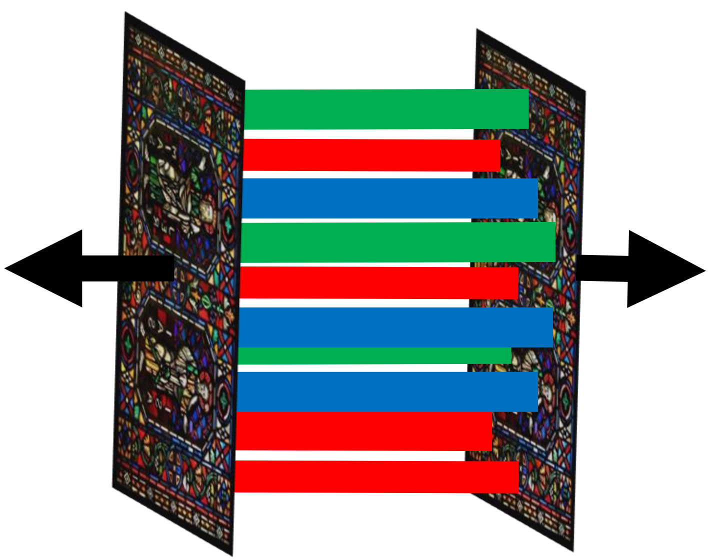
Strong color EM field (glasma) (2/2)

Low-Nussinov model: Low, Nussinov, Casher, Neuberger (1970~80)
Glasma: McLerran, Lappi, Kovner, Weigert (~2005)

✔ **The key: Decay of the strong color EM field into particles**

⇒ **Schwinger effect !** [Kerman, Matsui, Gatoff (1987)]





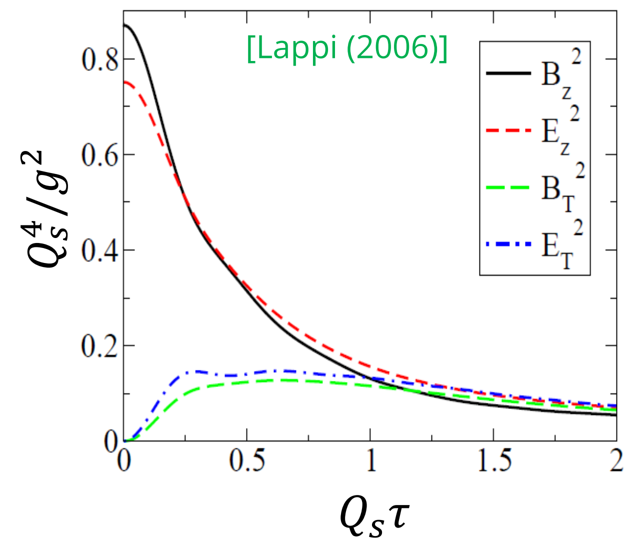
just after a collision

≀

an expanding
"color capacitor"
($\sigma \sim Q_s^2 \sim O(1)\mathrm{GeV}^2$)

⇓

strong color EM field
($gE, gB = O(1)\ \mathrm{GeV}^2$)



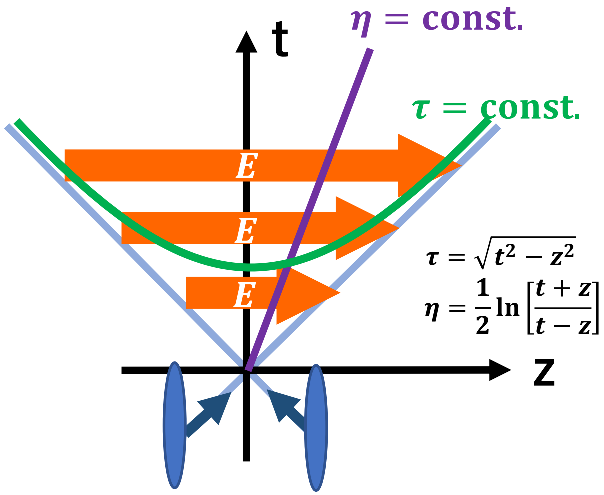
What I am going to do

✔ Purpose:

Study the quark & gluon production in the early-time dynamics of HIC
by applying the Schwinger-effect realtime technique developed in QED to QCD

✔ Setup:

- QCD with Nc=3 and Nf=6 (with actual quark masses)
- Boost-invariantly expanding color E field $E(\tau, \eta, \boldsymbol{x}_\perp) = E(\tau)$
- Neglect color magnetic field
  (⇐ problem due to gluon instability)
- Solve QCD within mean-field approx. (next slides)



Mean-field theory (1/2)

QED: [Kluger, Eisenberg, Svetitsky,
Cooper, Mottola (~1990)]
[Tanji (2008)]

Same as:
Bogoliubov-de Gennes (TD-BdG)
in cond-mat

Mean-field theory (1/2)

$$L_{\mathrm{QCD}} = -\frac{1}{2}\mathrm{tr}\left[F^{\mu\nu}F_{\mu\nu}\right] + L_{\mathrm{quark}} + L_{\mathrm{FP+GF}}$$

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$$L_{\rm QCD} = -\frac{1}{2}{\rm tr}\left[F^{\mu\nu}F_{\mu\nu}\right] + L_{\rm quark} + L_{\rm FP+GF}$$

STEP 1 Split the total gauge field A into classical (strong) part $\bar{A} = \langle A \rangle$ and quantum fluctuation on top of it a, i.e., $A = \bar{A} + a$

STEP 2 Expand $L_{\rm QCD}$ i.t.o a

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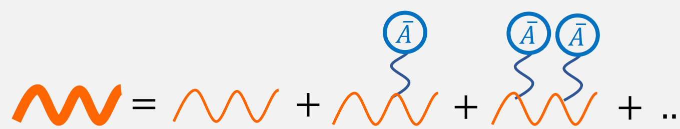
$$L_{\mathrm{QCD}} = -\frac{1}{2}\mathrm{tr}\left[\bar{F}^{\mu\nu}\bar{F}_{\mu\nu}\right] + O(a^1) + O(a^2) + g \times O(a^3) + g^2 \times O(a^4) + L_{\mathrm{quark}} + L_{FP+GF}$$

✔ a is couplted to $\bar{A}$ in non-perturbatively:



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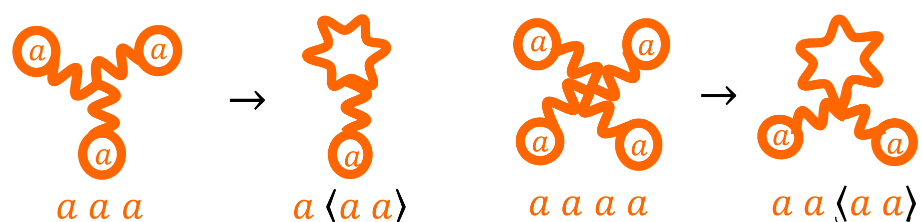
✔ a is couplted to $\bar{A}$ in non-perturbatively:

STEP 3 Apply mean-field approx. to the non-linear terms $O(a^3, a^4)$



$L_{\rm QCD}$ = (up to the second orer in a)

Mean-field theory (2/2)

QED: [Kluger, Eisenberg, Svetitsky, Cooper, Mottola (~1990)] [Tanji (2008)]

Same as: Bogoliubov-de Gennes (TD-BdG) in cond-mat

STEP 4 Get equation of motion

Coupled linear EoMs

EoM for quantum fluct. a

$$0 = [(\partial + ig\bar{A})^2 g^{\mu\nu} + \langle M^{\mu\nu} \rangle] a_\nu$$

EoM for classical field $\bar{F}$

$$\langle j^\mu \rangle = \partial_\nu [\bar{F}^{\nu\mu} + \langle f^{\nu\mu} \rangle]$$

Mean-field theory (2/2)

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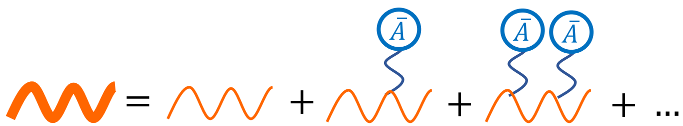
Multiple scattering b/w $\bar{A}$ and a

⇒ **Particle production of a from $\bar{A}$**



Current $\langle j^\mu \rangle$ produced by a screens out $\bar{F}$

⇒ **Backreaction to $\bar{F}$ by a**

Mean-field theory (2/2)

QED: [Kluger, Eisenberg, Svetitsky, Cooper, Mottola (~1990)]
[Tanji (2008)]

Same as:
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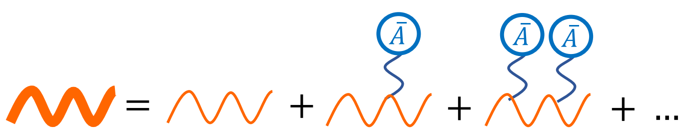


EoM for classical field $\bar{F}$

$$\langle j^\mu \rangle = \partial_\nu [\bar{F}^{\nu\mu} + \langle f^{\nu\mu} \rangle]$$

Current $\langle j^\mu \rangle$ produced by a screens out $\bar{F}$
⇒ **Backreaction to $\bar{F}$ by a**

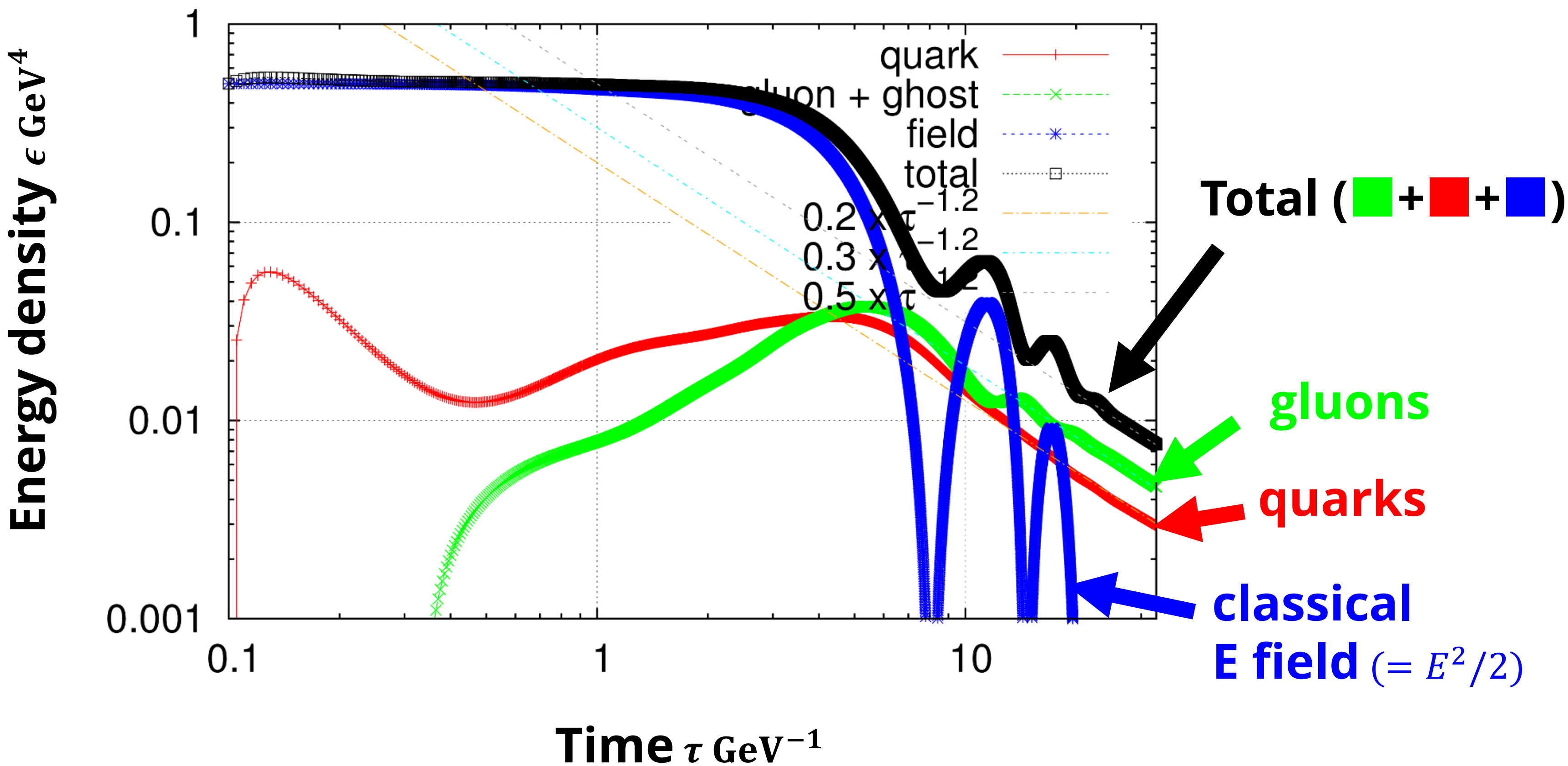
STEP 5 Solve EoM !

(1) assume $O(a^2)$ terms ($M^{\mu\nu}$ and j^μ) are negligible (≈ no backreaction)
⇒ analytically solvable ⇒ gives essentially the same to the Schwinger formula

(2) don't neglect $O(a^2)$ terms (≈ w/ backreaction) ⇒ numerically doable (⇒ this talk)

Results

Results (1/4): Energy balance



✔ Due to the particle prod (+ the Bjorken exp), the initial classical field decays into quark & gluon particles rapidly $\tau \sim 10\ \mathrm{GeV}^{-1} \sim 2\ \mathrm{fm}/c$

cf. Non-expanding QED: [Kluger et al. (~1990)] [Tanji (2008)]

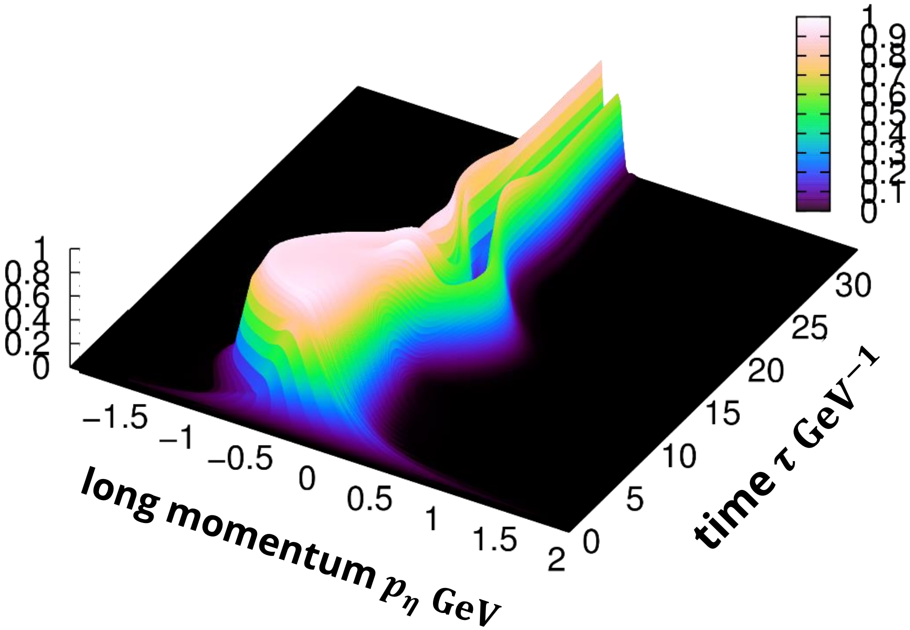
✔ Decay with oscillation (plasma oscillation)

$\because\ \dot{E} = -J \propto (\text{particles' velocity})$, but particles do not stop immediately at $E = 0$

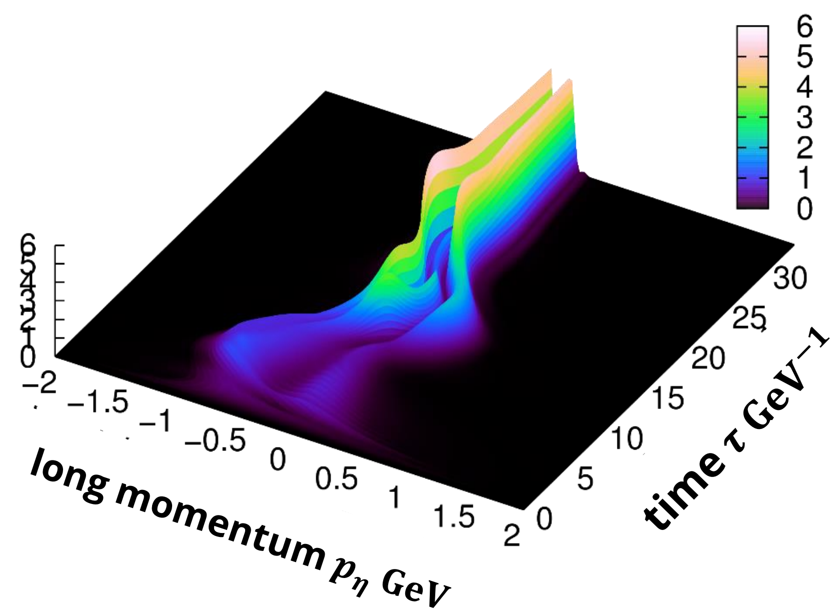
Results (2/4): (longitudinal) momentum dist. dN/dp

(longitudinal) momentum dist $\dfrac{\mathrm{d}^6 N}{\mathrm{d}x_T^2 \mathrm{d}\eta \, \mathrm{d}p_T^2 \mathrm{d}p_\eta}$ **at** $p_T \sim 0$

Quark



Gluon

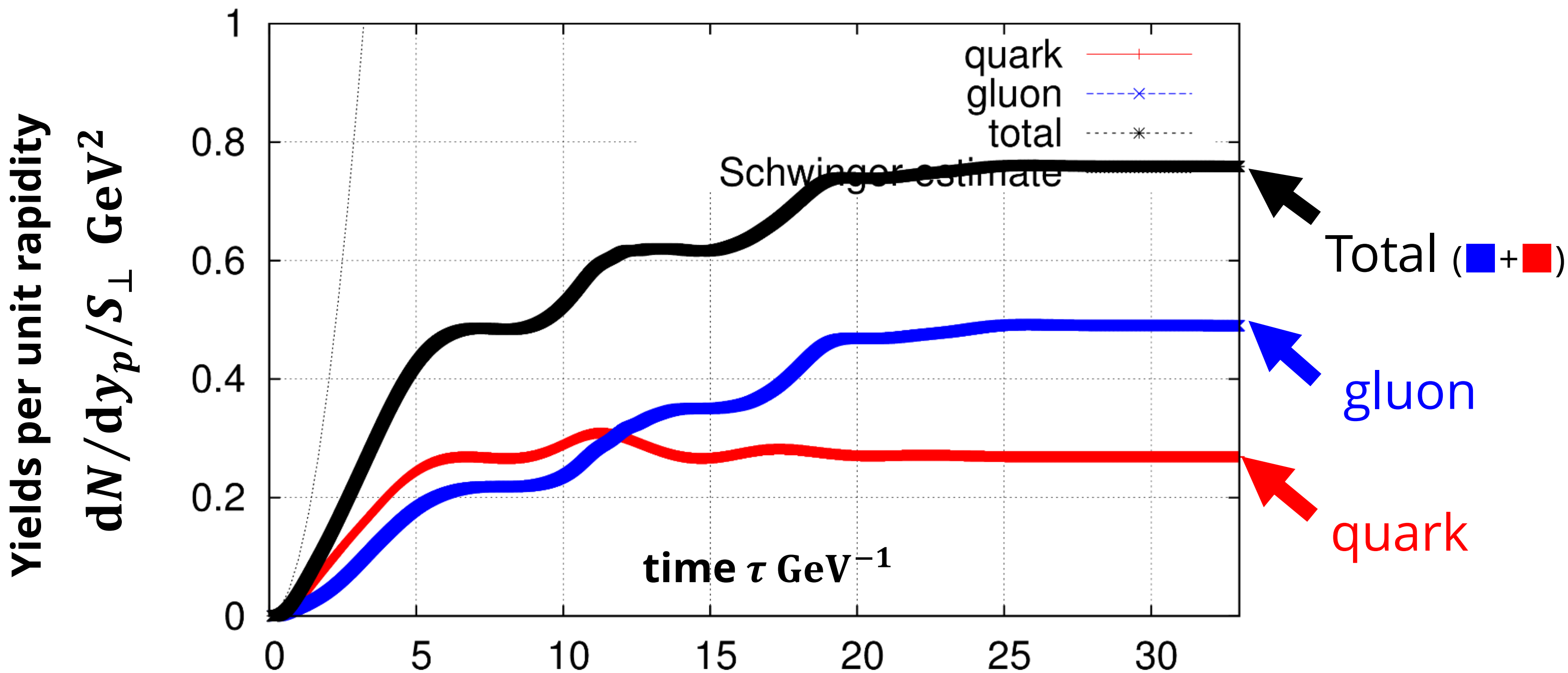


✔ Plasma oscillation = particles are going back and forth

✔ When comes back to p=0, where new particles are being produced, quantum interference occurs (known as stuckelberg interference in cond-mat)

- gluon: Bose enhancement ⇒ increase of the production
- quark: Pauli blocking ⇒ saturation behavior

Results (3/4): yields N per unit rapidity



(cf. $S_\perp \sim O(4000)$ GeV^{-2})

✔ A huge number of particles O(1000) can actually be produced at early time τ=O(1fm/c)

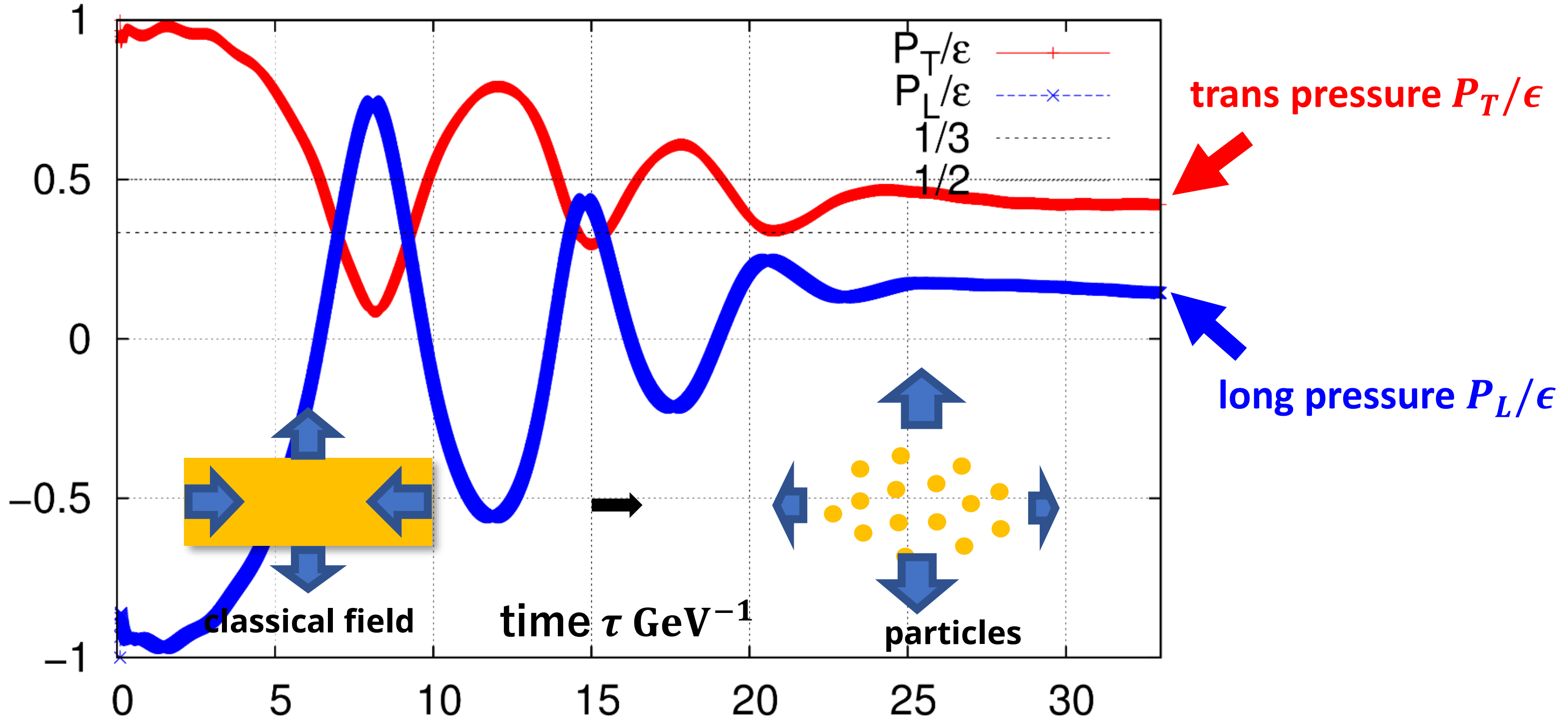
⇒ Strong-field physics (= decay of strong field into particles) actually plays an important role in the early-stage dynamics of HIC

✔ Quark production is fast and abundant

⇒ Quark DoG is non-negligible

- should affect the early-stage dynamics …
- good news for CME search (∵ U(1) B field decays very rapidly)

Results (4/4): anisotropy as a measure of thermalization



- ✔ Even within mean-field approx. (i.e., no interaction), anisotropy gets relaxed significantly ($P_T/P_L \sim 0.5$)

  (∵ non-zero long pressure due to acceleration by E field ⇒ don't simply go to the free streaming $P_L = 0$)

- ✔ Outlook: Need to go beyond mean-field approx. to really discuss thermalization (hydrodynamization)

  (cf. go beyond MFA is new not only in QCD but also in QED, so should be interesting ...)

Contents

1. Overview of strong-field physics

[Fedotov, Ilderton, Karbstein, King, Seipt, HT, Torgrimsson, 2203.00019]

2. Recent development of the Schwinger effect

- focus on the Schwinger effect with time-dependent E fields

[HT, Itakura, Fujii, 1405.6182] [HT, 1812.03630] [HT, Fujimori, Misumi, Nitta, Sakai, 2010.16080]
[HT, Ironside, 2308.11248] [HT, Nishimura, Ohnishi, 2402.17136]

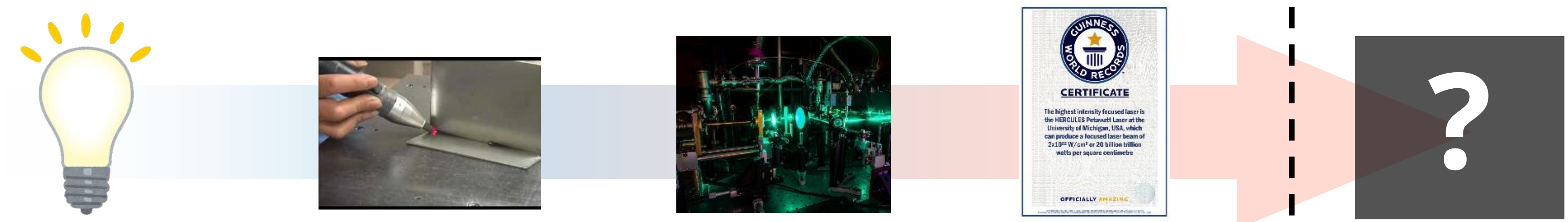
3. An application of the Schwinger effect to QCD: the early-time dynamics of heavy-ion collisions

- Quark production is very fast ! [HT, 1609.06189] [HT, Ph. D thesis]

Today's talk

Q: What happens if we make light (or "field" in general) stronger and stronger ?

LED	Laser welding	Cond.-mat (THz laser)	Guiness record	???



HERCULES @ USA (2008)

LED	Laser welding	Cond.-mat (THz laser)	Guiness record	
$I \sim 10^{-5}\ \mathrm{W/cm^2}$	$10^6\ \mathrm{W/cm^2}$	$10^{10}\ \mathrm{W/cm^2}$	$10^{22}\ \mathrm{W/cm^2}$	
or $eE \sim 10^{-1}\ \mathrm{eV/cm}$	$10^4\ \mathrm{eV/cm}$	$10^6\ \mathrm{eV/cm}$	$10^{12}\ \mathrm{eV/cm}$	**Our limit at the present**
$\sim (10^{-3}\mathrm{eV})^2$	$(10^{-1}\mathrm{eV})^2$	$(1\ \mathrm{eV})^2$	$(1\ \mathrm{keV})^2$	

Daily life → Industry → Science →

Purpose: Review physics of such strong field

Take-home messages:

(1) Once $eE >$ (typical energy scale), sthg extremely non-trivial occur (e.g., Schwinger effect ≈ "something" from "nothing")

(2) Such strong fields are now (or soon will be) within the exp. reach

(3) Of relevance to hadron/QCD physics, in particular, heavy-ion collisions

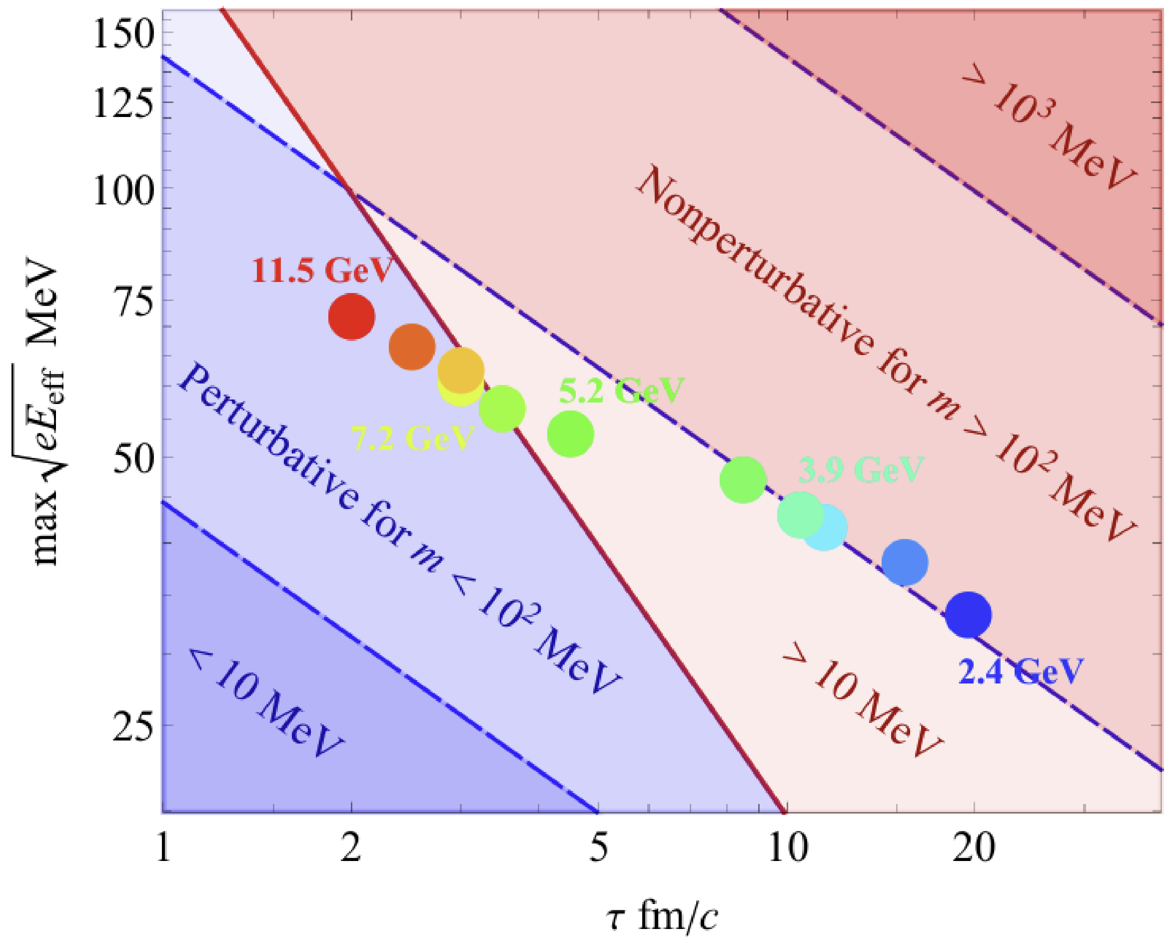


FIG. 4. Sensitivity plot for nonperturbativity of the produced electric field in intermediate-energy heavy-ion collisions. The dots represent the characteristics of the field $(\tau, \max\sqrt{eE_{\rm eff}})$ extracted from Fig. 3 at each collision energy $\sqrt{s_{\rm NN}}$, ranging from $\sqrt{s_{\rm NN}} = 2.4$ GeV (blue), 3.0, 3.5, 3.9, 4.5, 5.2, 6.2, 7.2, 7.7, 9.2, to 11.5 GeV (red). The lines represent the nonperturbativity parameters (1): $1 = \xi(10\text{ MeV})$ (bottom blue dashed), $1 = \xi(10^2\text{ MeV})$ (middle blue dashed), $1 = \xi(10^3\text{ MeV})$ (top blue dashed), and $1 = \nu$ (red). Those lines set "phase boundaries" of the nonperturbativity (of the vacuum pair production). The red regions $\xi(m), \nu > 1$ are nonperturbative (for mass scales m), while it is perturbative in the blue regions $\xi(m), \nu < 1$.