Electric permittivity of the vacuum in a strong constant electric field

Hidetoshi Taya

(Keio U.)

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This talk

Discuss the electric permittivity ϵ of the vacuum in a strong constant electric field

In the pure vacuum $\mathcal{D} = \mathcal{E}$

In an EM field $\mathcal{D} = \epsilon\mathcal{E} \neq \mathcal{E}$

- ϵ is no longer a const. $\epsilon = \epsilon(\bar{E})$ due to the vacuum polarization
- many studies since the early days, [Heisenberg-Euler (1936)] [Toll 1952] [Klein-Nigam (1964)] [Baier-Breitenlohner (1967)] … Review: [King-Heinzl (2015)] [Ejlli et al. (PVLAS) (2020)] [Fedotov et al. (2023)] …

  but is still worthwhile to be investigated $\Rightarrow$ 3 motivations

Motivations (1/3)

(1) The current understanding is limited to weak/slow regime

The most famous formula = based on Euler-Heisenberg Lagrangian

$$\mathcal{D} = -\frac{\partial \mathcal{L}_{\mathrm{EH}}}{\partial \mathcal{E}} \quad \Rightarrow \quad \epsilon = \frac{\alpha}{45\pi}\left(\frac{e\bar{E}}{m^2}\right)^2 \times \begin{cases} 6 & (\parallel) \\ 2 & (\perp) \end{cases}$$

[Baier-Breitenlohner (1967)]

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[Baier-Breitenlohner (1967)]

Problem 1: Valid only in the weak limit $e\bar{E} \ll m^2$

power corrections could be included $(e\bar{E})^n$, but

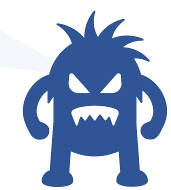
- is factorially divergent ⇒ does not necessarily improve the formula
- non-pert. factor like $e^{1/e\bar{E}}$ can never be included

cf. [Heinzl-Schroder (2006)]

Problem 2: Valid only in the slow limit $\omega \ll m$

- not possible to discuss ω dependence
- the physics must be different above the pair-production threshold $\omega > 2m$

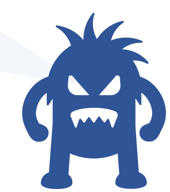
cf. [King-Heinzl-Blackburn (2023)]



Problem 3: Neglecting the imaginary part $\mathrm{Im}\,\mathcal{L}_{\mathrm{EH}}$

ϵ (in the coordinate space) must be real, so one must set $\mathrm{Im}\,\mathcal{L}_{\mathrm{Euler-Heisenberg}}$

⇒ pair production and "non-equilibrium-ness" of E field are completely dismissed



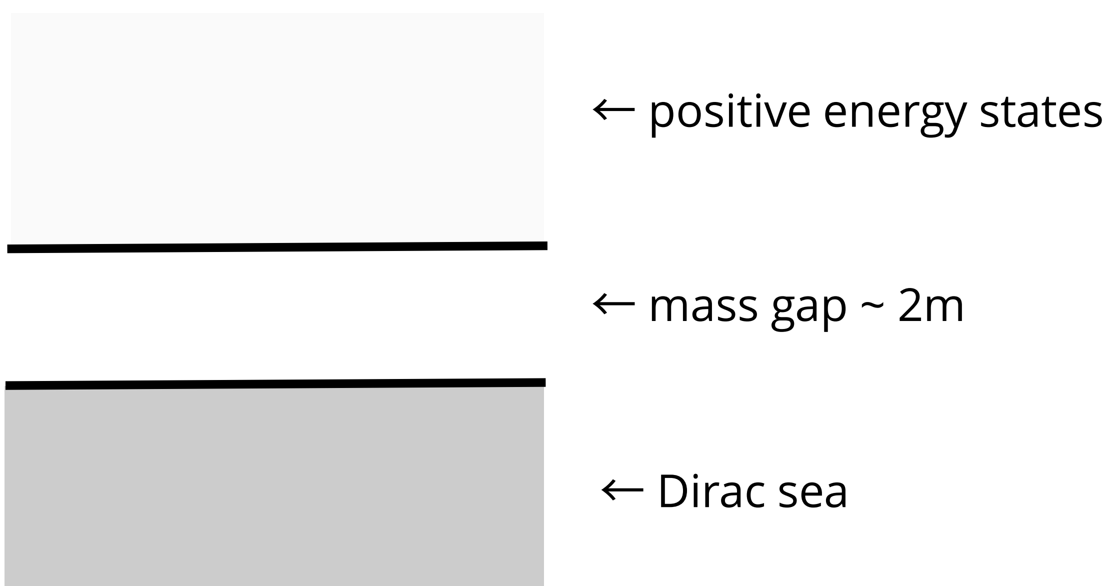
Q: What happens if I go beyond those limitations ?

Motivations (2/3)

(2) As a signature of non-trivial QED vacuum structure in E field

The QED vacuum (= the Dirac sea) has a non-trivial electron dist. in an E field, which can leave observable imprints

QED vacuum at $\overline{E} = 0$



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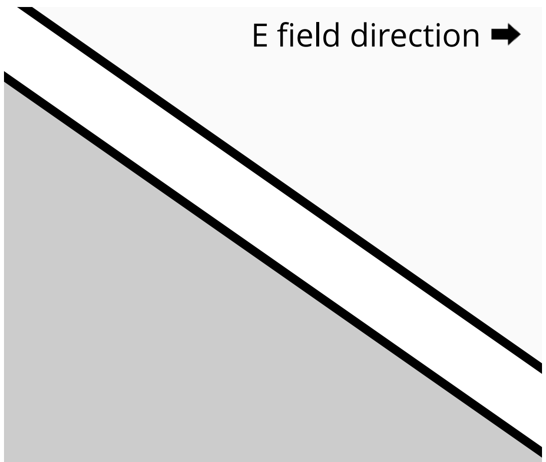

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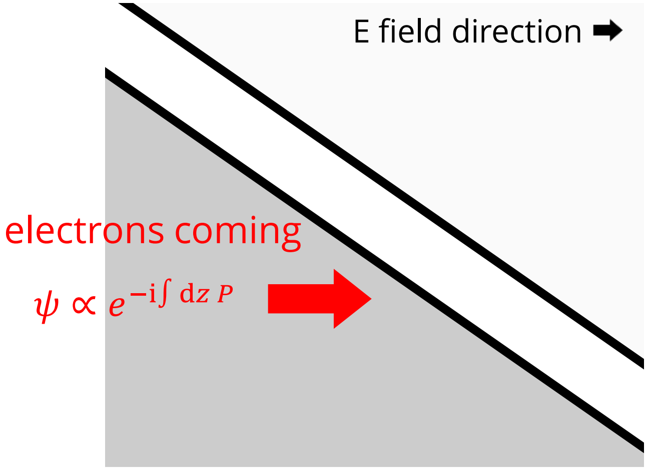

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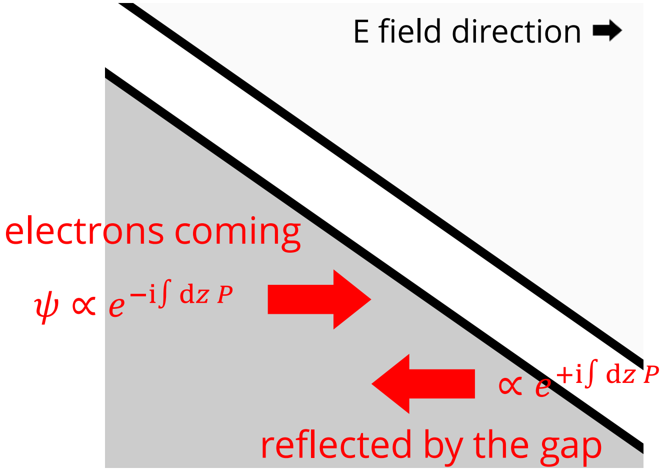

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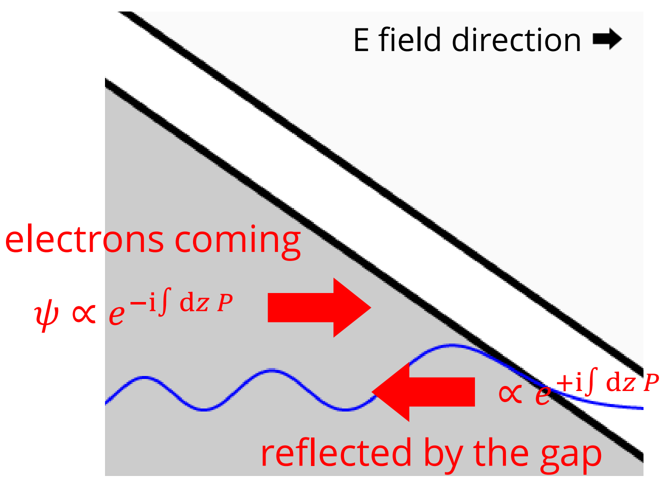

$$\propto \left|\#\, e^{+\mathrm{i}\int \mathrm{d}z\, P} + \#\, e^{-\mathrm{i}\int \mathrm{d}z\, P}\right|^2 \propto \cos \int \mathrm{d}z\, P$$

interfered electron distribution

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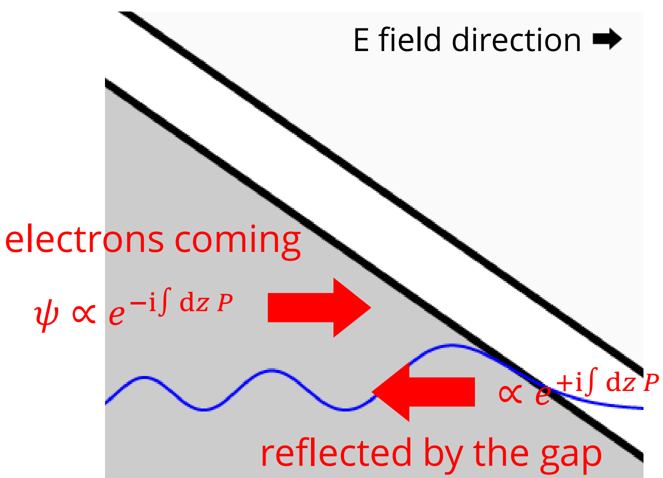
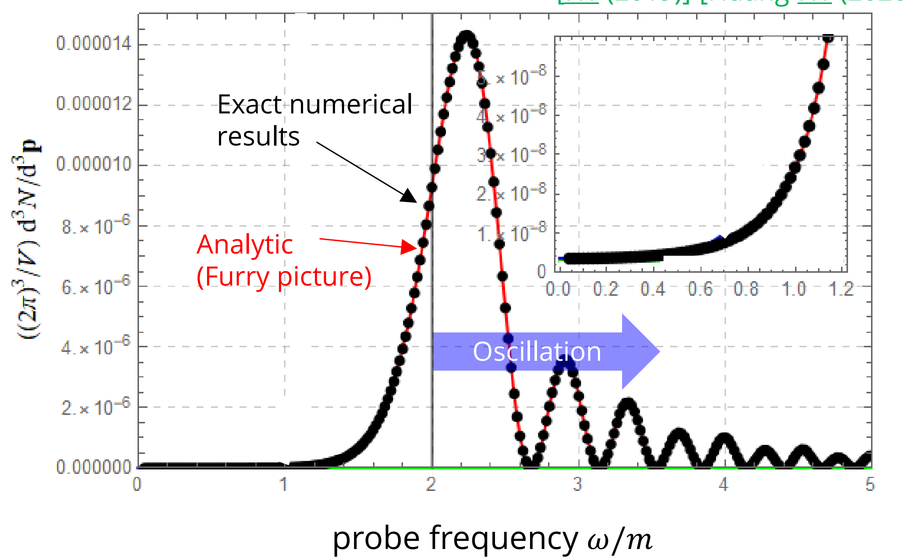


$\propto \left|\# e^{+i\int dz\, P} + \# e^{-i\int dz\, P}\right|^2 \propto \cos \int dz\, P$

interfered electron distribution

Observable imprint

Ex.) spectrum of the dynamically-assisted Schwinger
(= pair prod. in const $\bar{E}$ + fast probe $\mathcal{E}$)
[HT (2019)] [Huang-HT (2020)]



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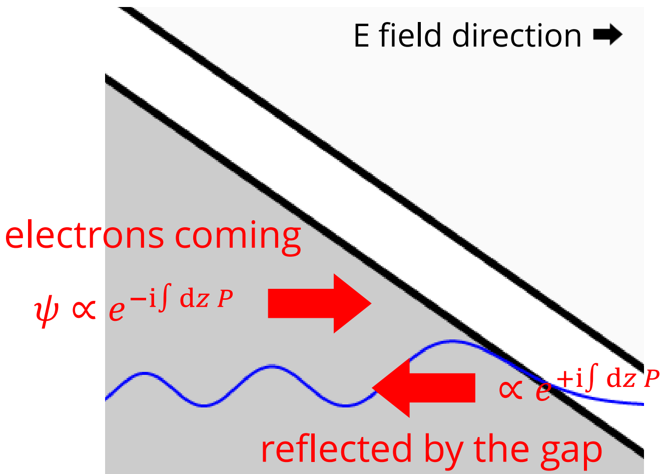

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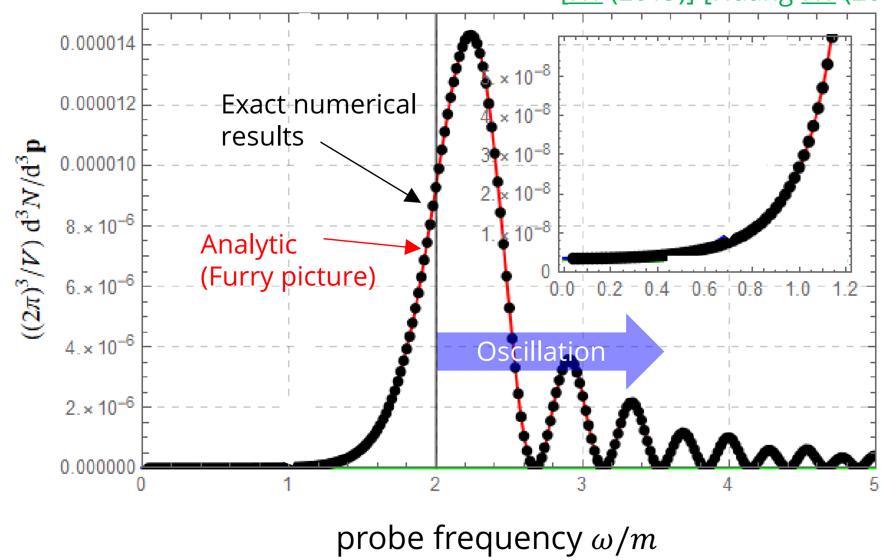

Q: What happens to the electric permittivity ?

Note: The motivation (1) (in particular, going beyond $\omega \ll m$) is important to achieve this

Motivations (3/3)

(3) Pursue analogy between strong-field QED and semicond. phys

- Ground-state structure of semicond. = The QED vacuum

  ⇒ the QED vacuum should response against external field in a similar way to a semiconductor and vice versa

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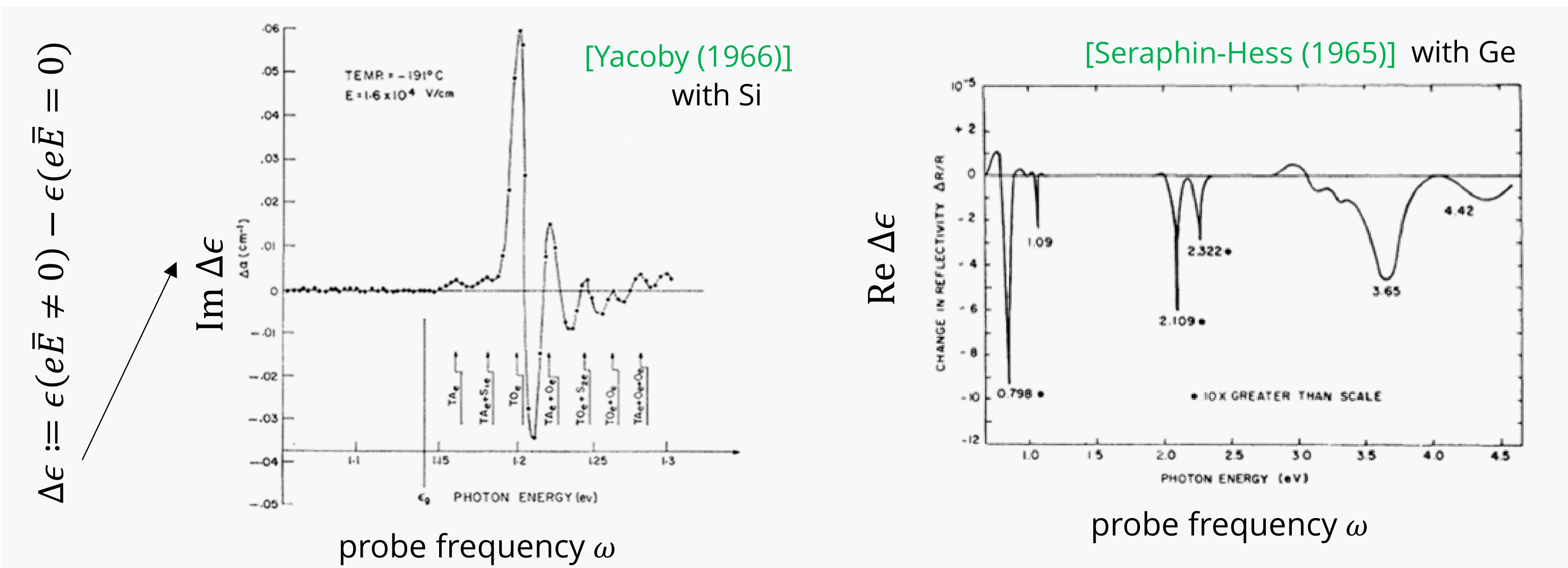
- Nontrivial oscillating change in ϵ (i.e., motivation (2)) has already been observed more than 50 yrs ago in semicond. physics !
  (⇒ the Franz-Keldysh effect and electroreflectance) [Franz (1958)] [Keldysh (1958)]



Q: Natural to expect this change in QED. Is this analogy true ?

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II. Theory

Linear response theory based on in-in formalism of QFT

III. Results

will give positive statements to the motivations (1) - (3)

IV. Summary

Theory (1/2)

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Setup: QED under a constant strong field $\bar{E}$ + a weak spatially homo. probe $\mathcal{E}(t)$

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Step 1: Definitions of $\mathcal{D}$ and ϵ

- Total flux $D = \underset{\bar{E}+\mathcal{E}}{E} + P(\bar{E},\mathcal{E}) = E + P_0(\bar{E}) + P_1(\bar{E})\mathcal{E} + \cdots$

$$= \bar{E} + P_0(\bar{E}) + \boxed{(1 + P_1(\bar{E}))\mathcal{E}} + \cdots \Rightarrow \epsilon = 1 + P_1(\bar{E})$$

$\mathcal{D}$

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$$= \bar{E} + P_0(\bar{E}) + \underbrace{(1 + P_1(\bar{E}))\mathcal{E}}_{\mathcal{D}} + \cdots \quad \Rightarrow \quad \epsilon = 1 + P_1(\bar{E})$$

Step 2: Calculate P_1

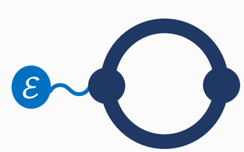
- Ampere law: $-\dot{P} = J = \langle 0;\mathrm{in}|\bar{\psi}(\bar{E},\mathcal{E})\gamma^\mu\psi(\bar{E},\mathcal{E})|0;\mathrm{in}\rangle$

$$= \langle 0;\mathrm{in}|\bar{\psi}_0(\bar{E})\gamma^\mu\psi_0(\bar{E})|0;\mathrm{in}\rangle$$

$$+ \underbrace{\langle 0;\mathrm{in}|\bar{\psi}_1(\bar{E})\gamma^\mu\psi_0(\bar{E}) + \bar{\psi}_0(\bar{E})\gamma^\mu\psi_1(\bar{E})|0;\mathrm{in}\rangle}_{P_1} \times \mathcal{E} + \mathcal{O}(\mathcal{E}^2)$$

- Not in-out amplitude, but in-in ! cf. [Copinger-Fukushima-Shi (2018)]

  ⇒ crucial when pair creating (or in non-equil.): $|0;\mathrm{out}\rangle = |0;\mathrm{in}\rangle + (\text{pair states like } |e^+e^-;\mathrm{in}\rangle)$

- Diagrammatically, evaluate 

Theory (2/2)

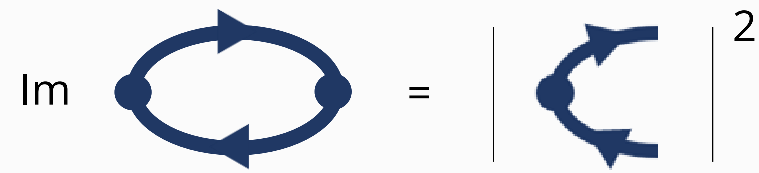
Detail 1: Loop diagram ⇒ UV divergent ⇒ use Kramers-Kronig relation

[Toll (1960)] [Heinzl, Schroeder (2006)] [Borysov et al. (2022)]

Causality ⇒ $\mathrm{Re}\,\epsilon(\omega) = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{+\infty} \mathrm{d}\omega' \frac{1}{\omega' - \omega}\,\mathrm{Im}\,\epsilon(\omega')$

⇒ sufficient to calculate the imaginary part
(same approach has been adopted in semicond.)

[Aspnes(1967)]



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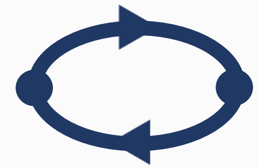
Im  = |  | ^{2}

Detail 2: $\mathrm{Im}\,\epsilon$ is directly related to the (dynamically-assisted) Schwinger effect

- Electromagnetism tells us:
  $\mathrm{Im}\,\epsilon$ is related to the dielectric energy loss
  See, e.g., textbook by Landau-Lefshitz

$$\frac{\mathrm{d}U_1}{\mathrm{d}t} = \mathcal{E}\frac{\mathrm{d}\mathcal{D}}{\mathrm{d}t} = \frac{1}{2}\omega\mathcal{E}^2\,\mathrm{Im}\,\epsilon$$

- Energy loss of probe due to the pair prod.

$$\frac{\mathrm{d}U_2}{\mathrm{d}t} = \omega\frac{N(\mathcal{E}\neq 0) - N(\mathcal{E}=0)}{VT}$$

- Microscopically, the dielectric energy loss should be caused by the pair prod.

$$U_1 = U_2 \;\Rightarrow\; \frac{N(\mathcal{E}\neq 0) - N(\mathcal{E}=0)}{VT} = \frac{1}{2}\mathcal{E}^2\,\mathrm{Im}\,\epsilon$$

$$\boxed{\therefore\; \textbf{Schwinger} \leftrightarrow \mathrm{Im}\,\epsilon \overset{\text{KK}}{\leftrightarrow} \mathrm{Re}\,\epsilon \;\Rightarrow\; \epsilon \textbf{ as an indirect sign. of Schwinger \& vice versa}}$$

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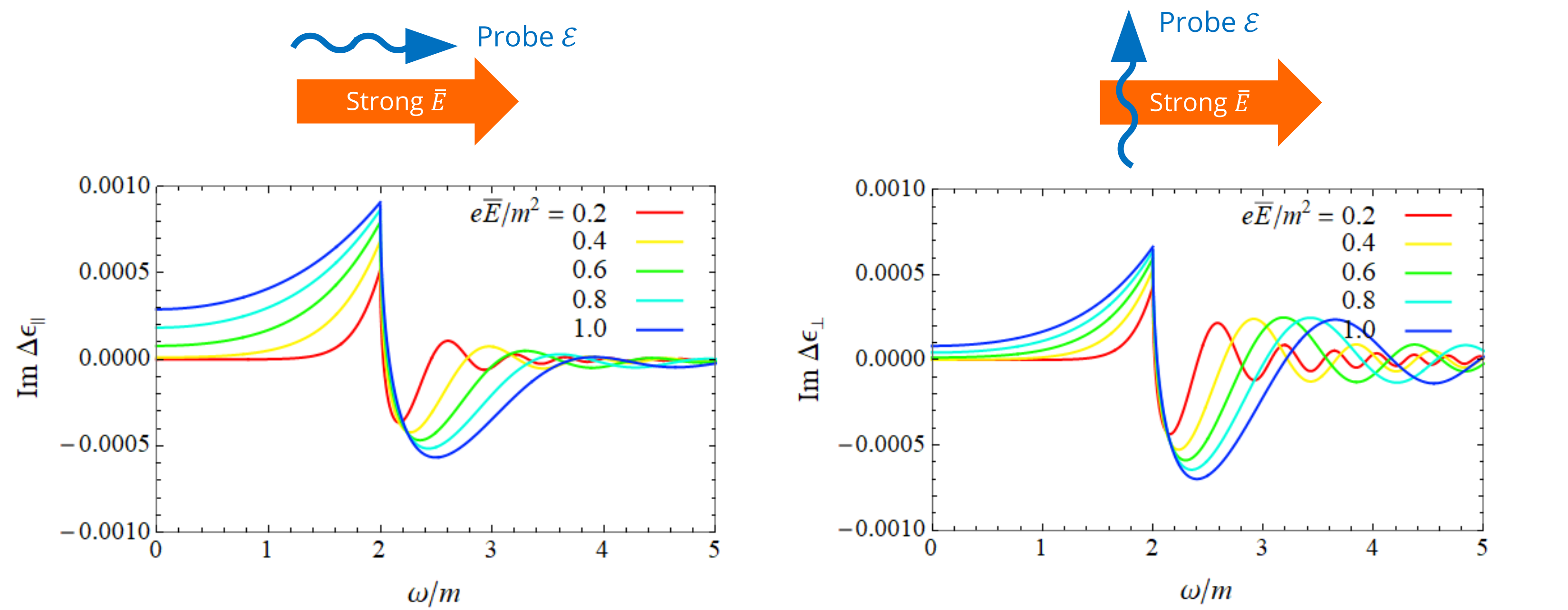
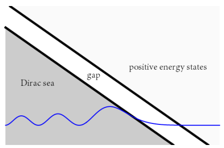
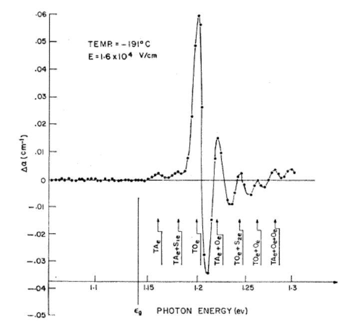
Results (1/3): Imaginary part of $\Delta\epsilon$

Imaginary part of the change $\Delta\epsilon = \epsilon(\bar{E} \neq 0) - \epsilon(\bar{E} = 0)$



- Oscillation, as expected from the tilted vacuum





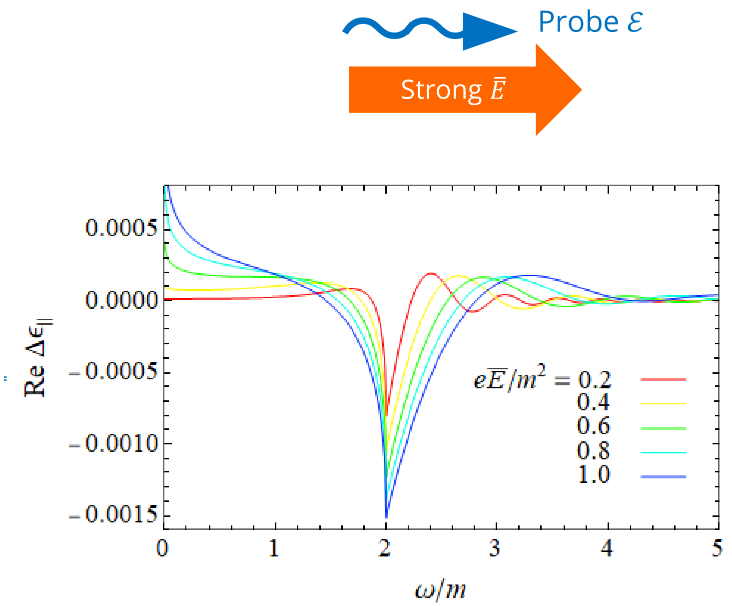
- Essentially the same pattern as semi-conductor observation
- Birefringent (Im $\Delta\epsilon_{\parallel} \neq$ Im $\Delta\epsilon_{\perp}$) but the basically the same
- Non-vanishing even at $\omega \to 0$ due to the strong-field non-perturbative effect

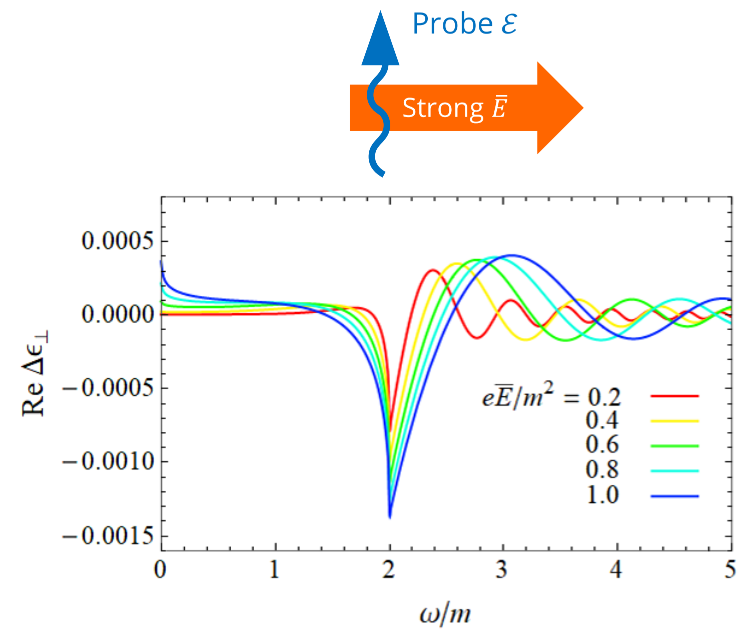
A simple explanation: In the slow limit, the standard Schwinger formula is valid

$$\Rightarrow \mathrm{Im}\,\epsilon \propto (N_{\mathrm{Schwinger}}(\bar{E} + \mathcal{E}) - N_{\mathrm{Schwinger}}(\bar{E})) \propto \left(\exp\left[-\pi \frac{m^2}{e(\bar{E}+\mathcal{E})}\right] - \exp\left[-\pi \frac{m^2}{e\bar{E}}\right]\right) = (\mathrm{finite}) \times \exp\left[-\pi \frac{m^2}{e\bar{E}}\right]$$

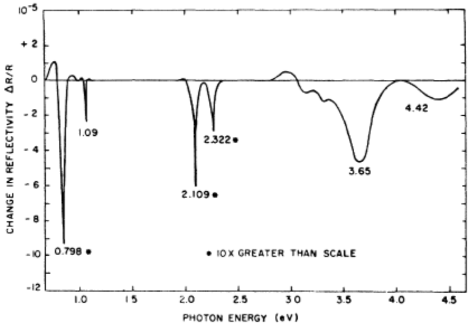
Results (2/3): Real part of $\Delta\epsilon$

Real part of the change $\Delta\epsilon = \epsilon(\bar{E} \neq 0) - \epsilon(\bar{E} = 0)$





- Oscillation, consistent with semi-conductor



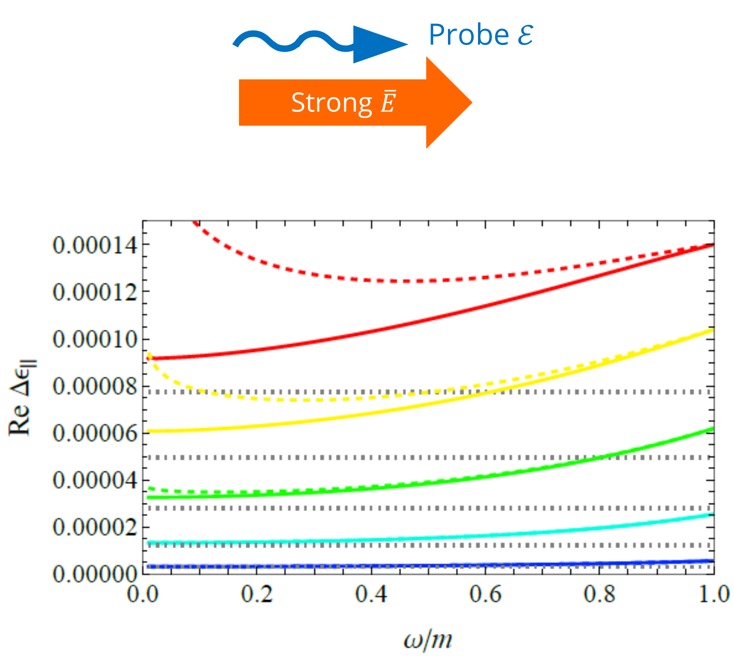
- Logarithmically divergent at $\omega \to 0$ due to the non-perturbative effect

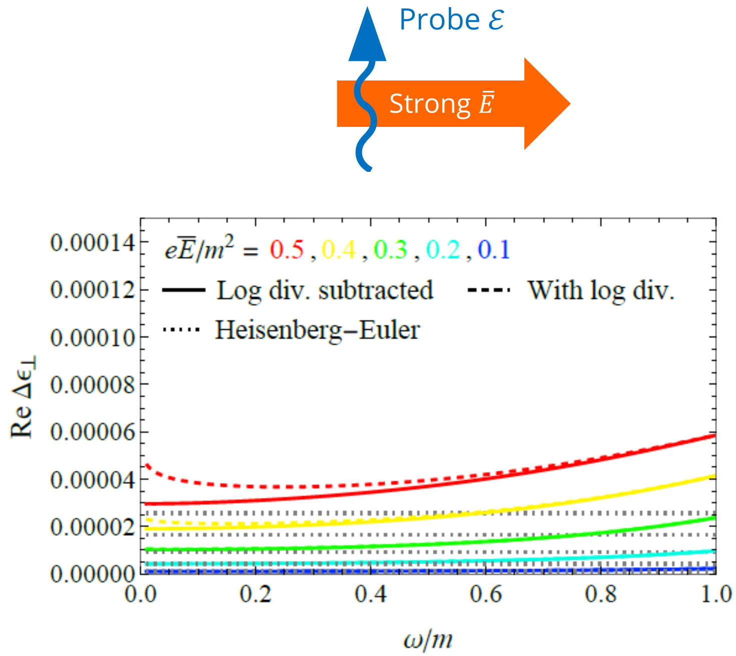
$$\because \quad \mathrm{Re}\,\epsilon(0) = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{+\infty} d\omega' \frac{1}{\omega'}\mathrm{Im}\,\epsilon(\omega')$$

$$\sim \frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega' \frac{1}{\omega'}\mathrm{Im}\,\epsilon(0) \sim (\text{log div.}) \times \exp\left[-\pi\frac{m^2}{e\bar{E}}\right]$$

Results (3/3): Comparison w/ EH

Comparison w/ Euler-Heisenberg around $\omega \approx 0$





- After the log subtraction, the result is consistent with the EH result $\Delta\epsilon_{\mathrm{EH}} = \frac{\alpha}{45\pi}\left(\frac{e\bar{E}}{m^2}\right)^2 \times \begin{cases} 6 & (\parallel) \\ 2 & (\perp) \end{cases}$
- Significant deviation for finite ω and/or stronger $\bar{E}$

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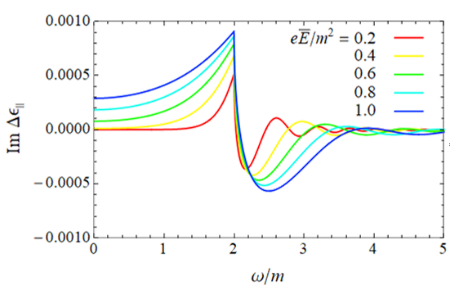
Summary

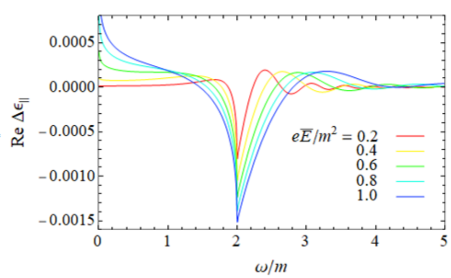
Discussed the electric permittivity ϵ of the vacuum in a strong constant electric field

Three motivations and my answers to them:

Motivations: (1) The current understanding is limited to weak/slow regime
(2) As a signature of non-trivial QED vacuum structure in E field
(3) Pursue analogy between strong-field QED and semicond. phys

Answers: (1): Succeeded with a linear response theory based on in-in formalism
(2)&(3): Yes, a characteristic oscillating structure in ϵ, similarly to semicond.





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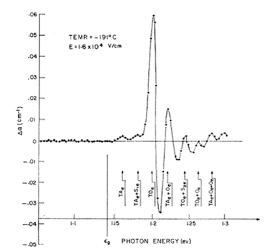
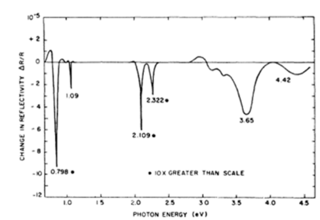




Other lessons (and further comments)

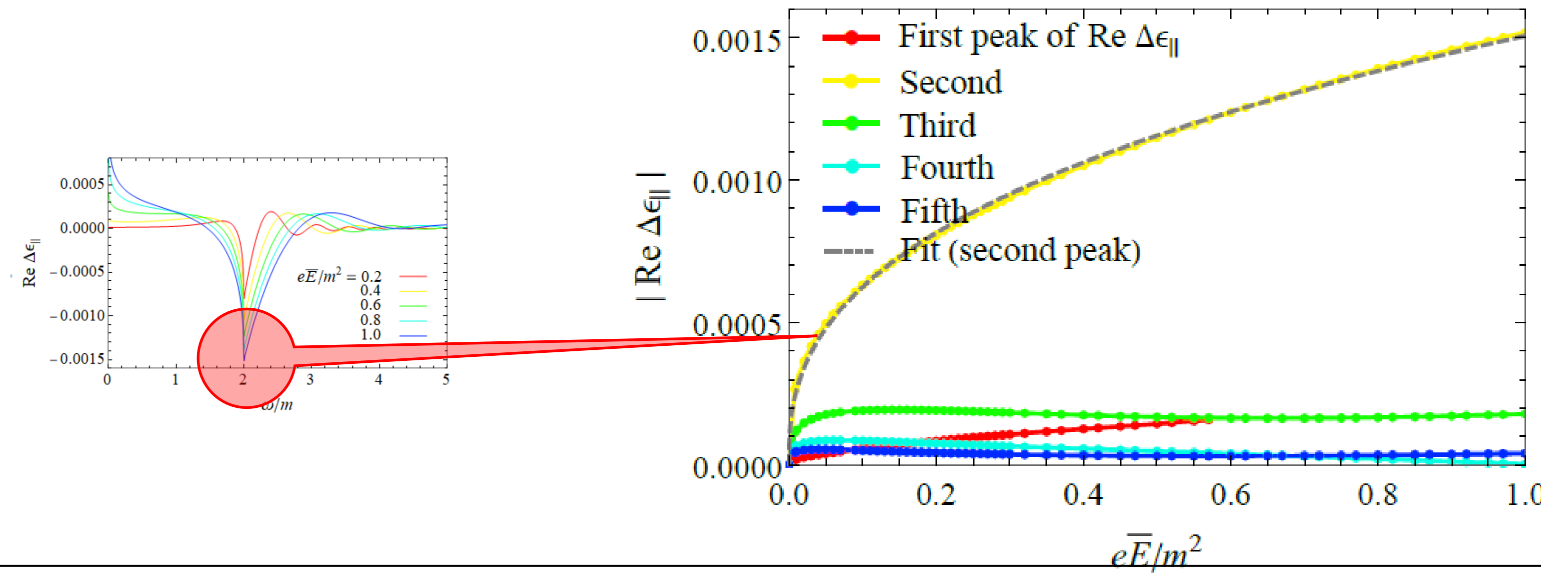
- Higher frequency gives a bigger signal of ϵ (or vacuum birefringence, in general)

  e.g. peak at $\omega \sim 2m$: $\mathrm{Re}\,\Delta\epsilon_{\parallel} \approx (-1.4\times 10^{-4}) \times \left(\frac{I}{1\times 10^{23}\ \mathrm{W/cm^2}}\right)^{0.19} \gg \Delta\epsilon_{\mathrm{EH}} = O(10^{-7})$ for $I = O(1\times 10^{23}\ \mathrm{W/cm^2})$ (PW laser)

- Need to go beyond EH for large ω or strong $\bar{E} \Rightarrow$ implications to heavy-ion coll., magnetar, … ?
- Schwinger $\leftrightarrow$ $\mathrm{Im}\,\epsilon \overset{\mathrm{KK}}{\leftrightarrow} \mathrm{Re}\,\epsilon \Rightarrow \epsilon$ as an indirect signature of Schwinger & vice versa

Peak value

Behaviors of the peaks



Numerical fit says (similar numbers for $\mathrm{Im}\,\epsilon_{\perp}$ and $\mathrm{Re}\,\epsilon_{\parallel,\perp}$)

$$\mathrm{Re}\,\Delta\epsilon_{\parallel} \approx (-1.4\times10^{-4}) \times \left(\frac{I}{1\times10^{23}\ \mathrm{W/cm^2}}\right)^{0.19}$$

where $I = \bar{E}^2/2$ is the focused intensity

- much larger than EH e.g., $\Delta\epsilon_{\mathrm{EH}} = O(10^{-7})$ for $I = O(1\times10^{23}\ \mathrm{W/cm^2})$ (PW laser)

 $\Rightarrow$ High-frequency probe is useful to study ϵ (or vacuum birefringence, in general)

- weak E dependence $\Delta\epsilon \propto I^{\frac{1}{5}\sim\frac{1}{6}}$ $\Rightarrow$ the peaks of $\Delta\epsilon$ can still be large for subcritical fields

e.g., only one-order smaller $\Delta\epsilon = O(1\times10^{-5}) \gg \Delta\epsilon_{\mathrm{EH}}$ at GW scale