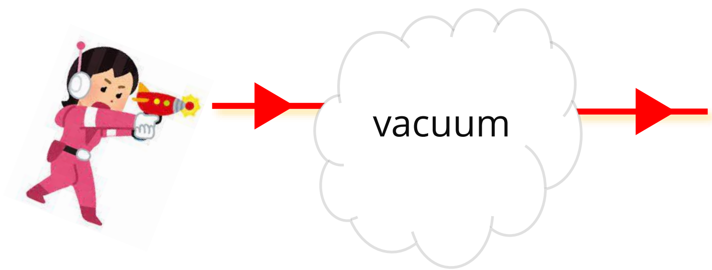
Vacuum HHG in strong-field QED

Hidetoshi Taya (Keio U.)

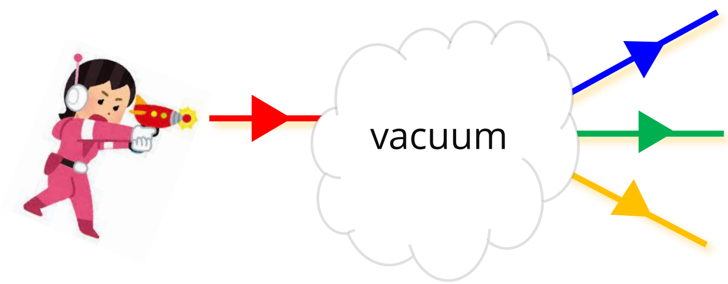
This talk

Discuss vacuum high-harmonic generation (HHG) in strong-field QED

Naively …



Vacuum HHG



Take-home messages:

- Vacuum HHG surely occurs and semi-classical analysis can explain the basic features (e.g., cutoff law)
- Physics: Interband current induced by the Schwinger effect and quantum interference of the Schwinger effect are the essence

NB: Very preliminary, so comments/criticisms are welcome !

NB: This work is an extension of my previous work for 2d Dirac material [HT, Hongo, Ikeda (2021)]

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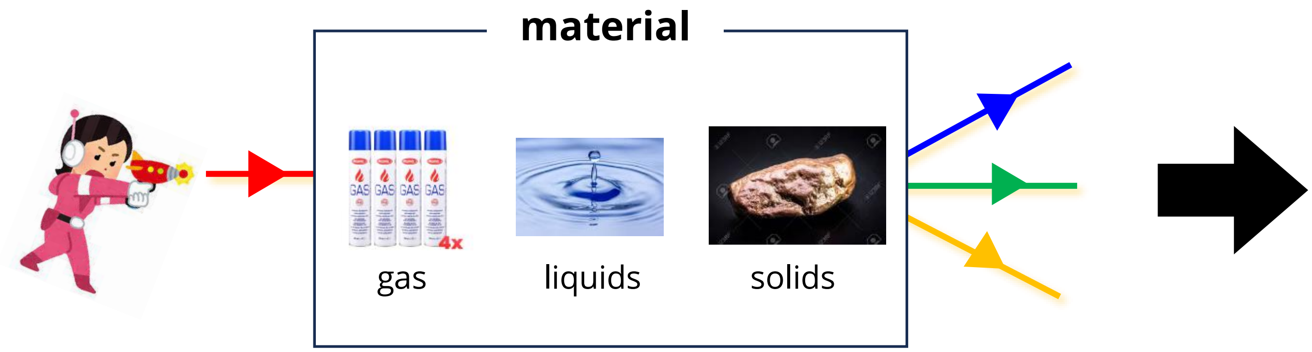
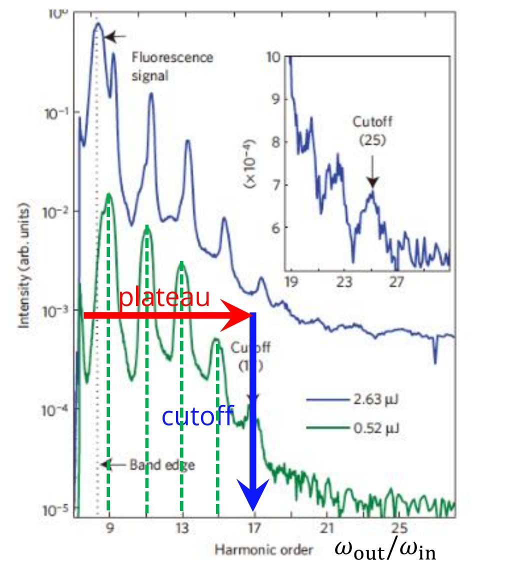
Introduction to HHG



Actual HHG spectrum in a solid



[Ghimire et al., (2011)]

Originally developed in optics and solid-state physics

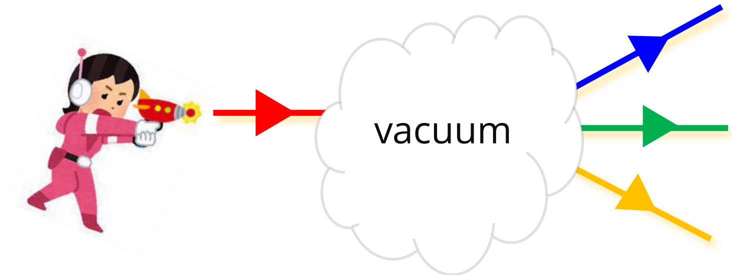
- First observation with gas (= one-band material) in 1988

  ⇒ Many applications: laser pointer, attosecond light source, ...
  Nobel prize in 2023

- Recent development: observation with solids (= band materials) in 2011

  ⇒ Key features: plateau + cutoff + discretized peaks

QED analog ?



Very natural, if we admit the analogy b/w band-materials & QED vacuum

- Many successful analogies: Schwinger effect ↔ Landau-Zener transition
  Dynamically assisted Schwinger ↔ Franz-Keldysh effect
  Quantum interference of Schwinger ↔ Stuckelberg-phase effect …

Not only interesting but also useful

[T. Gaumnitz et al., (2017)]

- The fastest light source of the zs (zepto second ~ $1/m_e$) order (cf. current world record: 43 as ~ $(100\ \mathrm{eV})^{-1}$)
- Application to physics near/beyond the pair creation threshold (e.g., dynamically assisted Schwinger)

Previous studies

- This is not the first study: [Piazza, Hatsagortsyan, Keitel (2005)], [Fedotov, Narozhny (2006)], [Bohl, King, Ruhl (2015)] …
- But, previous studies are based on constant-field (low freq.) approx. (e.g., EH Lagrangian)

  ⇒ Problematic, since time dependence can change physics drastically !

Sales point: Proper inclusion of the time dependence of the input driving field

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Formulation

Setup: QED in a spatially-homogeneous AC field $\bar{E}(t) = \bar{E}_0 \cos(\Omega t)$

$$\mathcal{L} = \bar{\hat{\psi}}(\mathrm{i}\not{\partial} - e\hat{\not{A}} - m)\hat{\psi} - \frac{1}{4}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} - \bar{J}\hat{A} \quad \text{where} \quad \hat{A} = \bar{A} + \hat{\mathcal{A}}$$

classical driving field $\bar{A} = \langle \bar{A} \rangle$ sourced by J

response

Observable: Harmonic spectrum of the response EM field

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Step 1: Write down EoMs

Electron: $(\mathrm{i}\partial\!\!\!/ - e\bar{A}\!\!\!/ - m)\hat{\psi} = e\hat{\mathcal{A}}\!\!\!/\hat{\psi}$ Driving field: $\partial^2 \bar{A}^\mu = \bar{J}^\mu$ Response: $\partial^2 \hat{\mathcal{A}}^\mu = e\hat{\bar{\psi}}\gamma^\mu\hat{\psi}$

NB: The response can be macroscopically large $\langle \hat{\mathcal{A}}^\mu \rangle \neq 0$ since $\left\langle e\hat{\bar{\psi}}\gamma^\mu\hat{\psi} \right\rangle \neq 0$

$\Rightarrow$ focus on $\langle \hat{\mathcal{A}}^\mu \rangle = O(\hbar^0)$ and neglect $\hat{a} \coloneqq \hat{\mathcal{A}} - \langle \hat{\mathcal{A}} \rangle = O(\hbar)$

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Step 2: Solve the EoMs with the Green function method

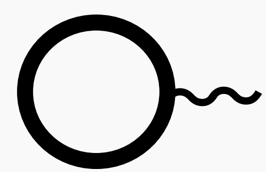
- Electron EoM $\Rightarrow \hat{\psi}(x) = \hat{\psi}_0(x) + \int \mathrm{d}^4x' S(x,x') e\mathcal{A}\!\!\!/(x')\hat{\psi}(x') = \hat{\psi}_0 + O(e)$

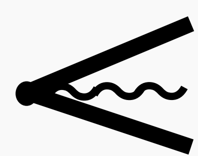
  where $(\mathrm{i}\partial\!\!\!/ - e\bar{A}\!\!\!/ - m)S(x,x') = \delta^4(x-x')$ and $(\mathrm{i}\partial\!\!\!/ - e\bar{A}\!\!\!/ - m)\hat{\psi}_0 = 0$

- Response EoM $\Rightarrow \hat{\mathcal{A}}(x) = \hat{\mathcal{A}}_0(x) + \int \mathrm{d}^4x' G(x,x') e\hat{\bar{\psi}}(x')\gamma^\mu\hat{\psi}(x')$

  where $\partial^2 G(x,x') = \delta^4(x-x')$ and $\partial^2 \hat{\mathcal{A}}_0 = 0$

$$\Rightarrow \langle \hat{\mathcal{A}} \rangle = \int \mathrm{d}^4x' G(x,x') \left\langle e\hat{\bar{\psi}}_0(x')\gamma^\mu\hat{\psi}_0(x') \right\rangle \approx$$



NB: The quantum part $\hat{a} = \hat{\mathcal{A}} - \langle \hat{\mathcal{A}} \rangle$ corresponds to , which is kinematically disfavored

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Step 3: Take the Fourier transform

$$\langle \hat{\mathcal{A}}(x) \rangle = \int \mathrm{d}^4 x' G(x, x') \left\langle e\hat{\bar{\psi}}_0(x')\gamma^\mu \hat{\psi}_0(x') \right\rangle$$

F.T.

Harmonic spectrum: $\tilde{\mathcal{E}}(\omega) := -i\omega \langle \widetilde{\hat{\mathcal{A}}(\omega)} \rangle = \frac{-\mathrm{i}}{\omega} \mathrm{F.T.} \left\langle e\hat{\bar{\psi}}_0 \gamma \hat{\psi}_0 \right\rangle$

∴ Solve the Dirac eq. & do the Fourier integral ⇒ Get harmonic spectrum

Numerical observation

✔ **Use numerics to get an exact harmonic spectrum of vacuum HHG**

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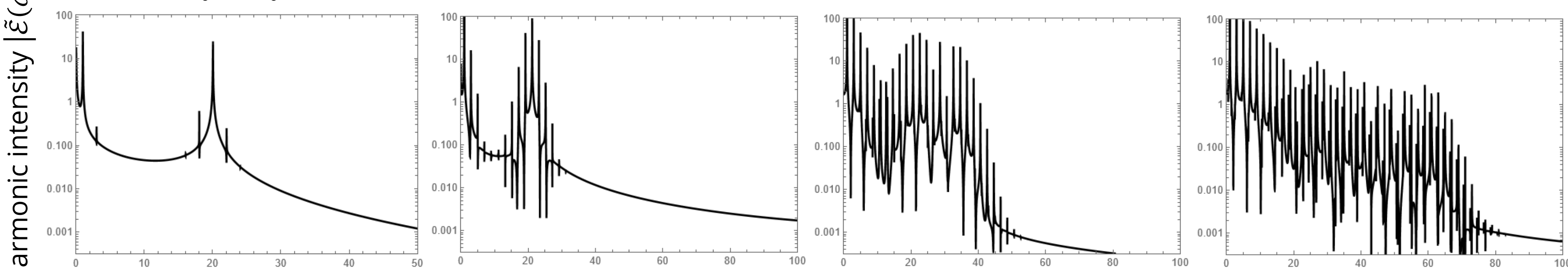
Harmonic spectrum for $\bar{E} = E_0 \cos \Omega t$

Weak field (fixed $\Omega/m = 0.2$) → **Strong field**

$eE_0/m^2 = 0.01$	$eE_0/m^2 = 0.05$	$eE_0/m^2 = 0.15$	$eE_0/m^2 = 0.30$
$(\gamma := \frac{1}{a_0} = \frac{m\Omega}{eE_0} = 10)$	$(\gamma = 2)$	$(\gamma = 0.666\ldots)$	$(\gamma = 0.333\ldots)$



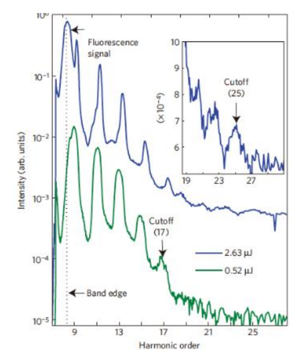
Harmonic order ω/Ω

Some features of vacuum HHG

(1) Plateau + cutoff + discretized peaks, similarly to solid-state HHG

(2) Cutoff increases with E_0 ↗

(3) Hamonic intensities do not necessary increase with E_0
(⇒ actually, it oscillates with E_0)



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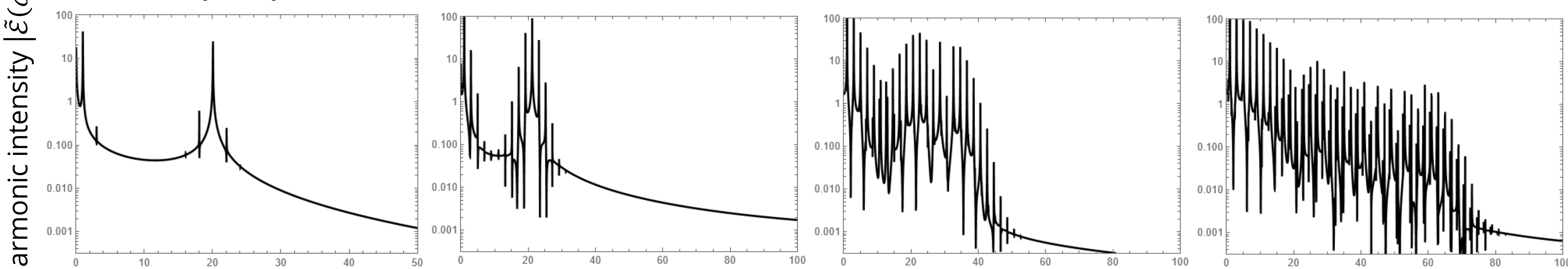

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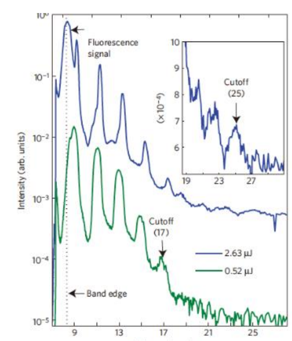

Origins of those features ? ⇒ Give you an answer with semi-classical analysis

Semi-classical analysis (1/3)

Two contributions to the current $J = \left\langle e\hat{\bar{\psi}}_0 \gamma \hat{\psi}_0 \right\rangle \propto \tilde{\mathcal{E}}$

- Strong E field ⇒ Pair production

  ⇒ Total mode function $\psi_0(t = \mathrm{in}) = \psi_- \;\; \rightarrow \;\; \# \times \psi_- + \# \times \psi_+$

- VEV has two contr.: $J = e\bar{\psi}_- \gamma \psi_- \;\rightarrow\; \# \times e\bar{\psi}_- \gamma \psi_-$ → Vacuum current (usually =0)

  $+\; \# \times e\bar{\psi}_+ \gamma \psi_+$ → Intraband current

  $+\; \# \times \mathrm{Re}[e\bar{\psi}_+ \gamma \psi_-]$ → Interband current

- Different time dynamical phases: $\psi_\pm \propto \mathrm{e}^{\mp imt}$

  ⇒ Interband current has the fastest time dependence $\propto e^{+2imt}$

  ⇒ the source of high harmonics !

  NB: same idea in solid-state HHG [Vampa et al., (2014)]

Semi-classical analysis (2/3)

More quantitative argument: Intermediate particle picture + Semi-classical analysis

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Step 1: Introduce an intermediate particle picture

・Identify $\psi_\pm$ as the instantaneous solutions to the Dirac eq.

$\psi_0(t) = \alpha(t) \times \psi_-(t) + \beta(t) \times \psi_+(t)$ where $H_{\rm Dirac}\psi_\pm = \pm\sqrt{m^2 + \left(\boldsymbol{p} + \frac{e\bar{E}_0}{\Omega}\sin\Omega t\right)^2}\,\psi_\pm =: E_\pm \psi_\pm$ ($\Rightarrow \psi_\pm \propto e^{-i\int {\rm d}t E_\pm}$)

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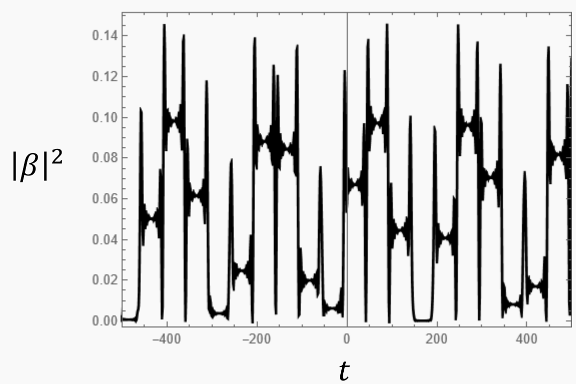
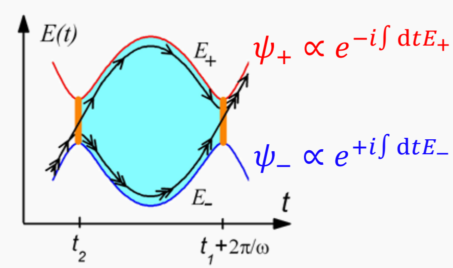
Step 2: Solve the evolution eqs. for the Bogoliubov coefficients α & β

- The evol. eqs. for ψ_0 and $\psi_\pm$ are known ⇒ can write down evol. eq. for α & β
- Complicated ⇒ Not exactly solvable but analytically with some approx.
  ⇒ semi-classical method [Dumlu, Dunne (2010)] [HT et al. (2020)]



⇐

Figure stolen from [Shevchenko, Ashhab, Nori (2010)]



(1) Periodic jumps w/ freq. Ω ⇐ Pair production occurs periodically at around the minimum gap

(2) β does not increase monotonically but oscillates
⇐ Quantum interference among pair productions (Stuckelberg-phase effect)

⇒ max β when most constructive ⇔ $2\pi\mathbb{Z} \ni E_+ - E_- \propto e\bar{E}_0/\Omega$ **⇒ determine harmonic intensity**

Semi-classical analysis (3/3)

More quantitative argument: Intermediate particle picture + Semi-classical analysis

Step 3: Calculate intra- and inter-band currents

$\mathcal{E}(t) \propto J(t) = J_{\rm intra}(t) + J_{\rm inter}(t)$

$J_{\rm inter}(t) \to\ \sim \alpha(t)\beta(t)e\bar{\psi}_+(t)\gamma\psi_-(t) \sim \beta(t)e^{+2i\int dt\sqrt{m^2+\left(\boldsymbol{p}+\frac{e\bar{E}_0}{\Omega}\sin\Omega t\right)^2}}$

$J_{\rm intra}(t) \to\ \sim |\beta(t)|^2 e\bar{\psi}_+(t)\gamma\psi_+(t) \sim |\beta(t)|^2$

$$\Rightarrow \tilde{\mathcal{E}}(\omega) \propto \tilde{J}(\omega) = \tilde{J}_{\rm intra}(\omega) + J_{\rm inter}(\omega)$$

$$\sim \int dt\, e^{-i\omega t}|\beta(t)|^2 + \int dt\, e^{-i\omega t}\, \beta(t) e^{+2i\int dt\sqrt{m^2+\left(\boldsymbol{p}+\frac{e\bar{E}_0}{\Omega}\sin\Omega t\right)^2}}$$

$$\sim |\beta|^2\delta(\omega-\Omega) + \beta\int dt\, \underbrace{e^{-i\omega t}\, e^{+2i\int dt\sqrt{m^2+\left(\boldsymbol{p}+\frac{e\bar{E}_0}{\Omega}\sin\Omega t\right)^2}}}$$

can be non-negligible only if $2\min\sqrt{m^2+\left(\boldsymbol{p}+\frac{e\bar{E}_0}{\Omega}\sin\Omega t\right)^2} < \omega < 2\max\sqrt{m^2+\left(\boldsymbol{p}+\frac{e\bar{E}_0}{\Omega}\sin\Omega t\right)^2}$

$\Rightarrow 2m < \omega < 2eE_0/\Omega$ **⇒ determine the plateau and the cutoff**

Semi-classical lessons v.s. numerics

Lessons from semi-classical analysis:

- Harmonic intensity oscillates with $\bar{E}_0$ due to quantum interference of the pair production
- High harmonics originate from the interband current, as it is very fast $J_{\rm inter} \propto e^{+2{\rm i}\int {\rm d}t\sqrt{m^2+\left(\boldsymbol{p}+\frac{e\bar{E}_0}{\Omega}\sin\Omega t\right)^2}}$
- Cutoff law $\min\,(E_+ - E_-) < \omega < \max\,(E_+ - E_-) \;\Rightarrow\; 2m < \omega < 2eE_0/\Omega$

<u>**NB:**</u> Time dependence of the driving field is crucial; otherwise, no interference and no plateau

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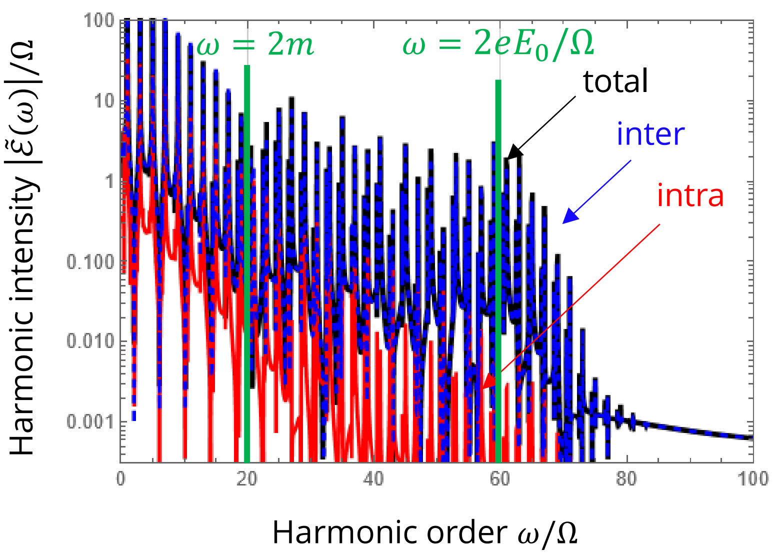
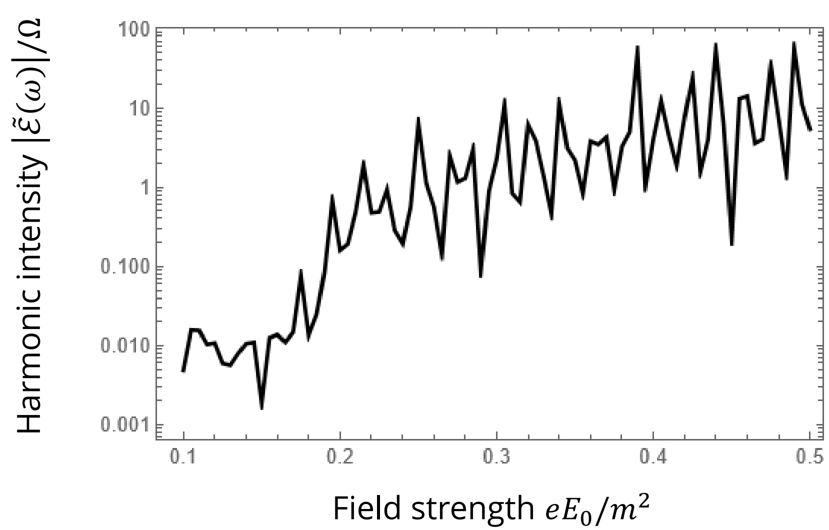
All of those lessons are consistent with numerics !

Harmonic spectrum for $\frac{eE_0}{m^2} = 0.3,\ \frac{\Omega}{m} = 0.1$



Harmonic intensity at $\omega/\Omega = 45$ (with $\frac{\Omega}{m} = 0.1$)



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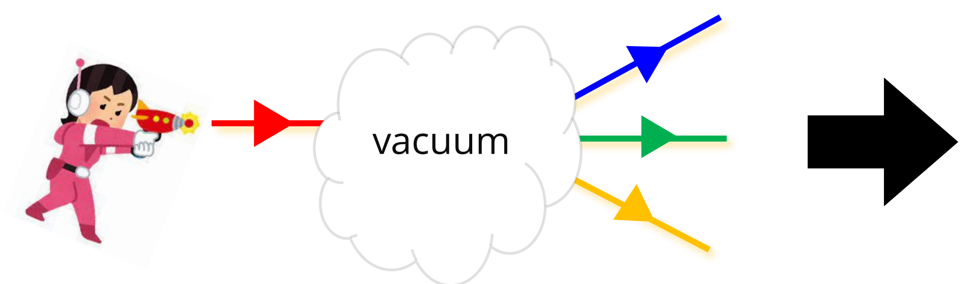
    - Semi-classical analysis ⇒ reveal the origin of HHG
3. **Summary**

Summary

Discuss vacuum high-harmonic generation (HHG) in strong-field QED

Vacuum HHG



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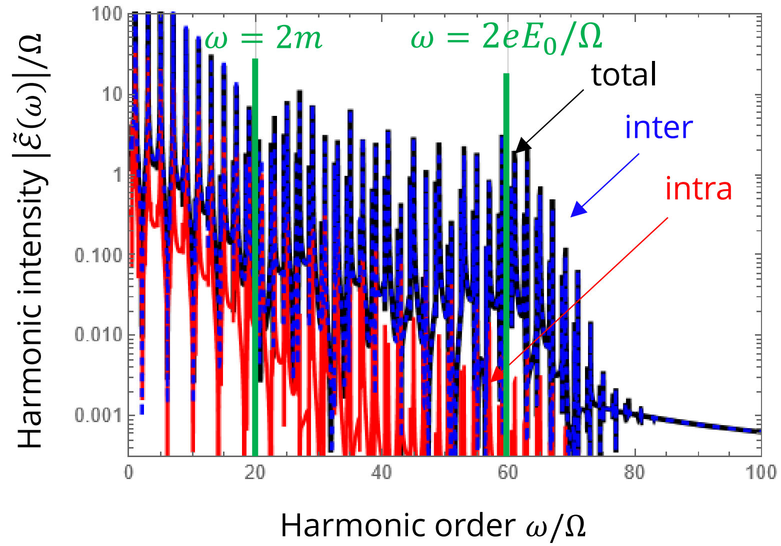

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